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MATRICULATION ARITHMETIC

WITH

NUMEROUS EXAMPLES AND SIMPLE GRAPHS

FOR THE USE OF SCHOOLS AND COLLEGES

BY

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PREFACE.

IN this book will be found all that is required in Arithmetic of the students of our Indian Universities. It will be useful to students who may afterwards have to serve in Mercantile offices. Any one, who intends to learn Arithmetic thoroughly, will find in it a safe and complete guide. It differs from the existing treatises in the greater prominence given to the *Unitary Method* and *Arithmetical Equations*. The Unitary Method (called *Subhankar's* method in this country) is practically more useful than the method of Rule of Three. The solution of a problem by the Unitary method gives a greater insight into it than the method of Rule of Three, the use of which in most cases is merely mechanical. The Arithmetical Equations require only certain axioms which are common to all branches of Mathematics.

The Examples in this book are more numerous and of greater variety in the belief that a sound knowledge of the analytical part of Mathematics requires a sound knowledge of Arithmetic, and this can be effected only by the student being drilled with home exercises of at least four sums of Arithmetic every day from the 8th to the 3rd class. The collection of examples in this book is sufficiently large to obviate the necessity of buying another book of Arithmetic. Typical examples of every variety have been worked out, and no pains have been spared to make them really instructive.

One-third of the more important examples in each set should be worked out in the class and the remaining two-thirds may be given as home exercises. The more difficult examples in each set and the Miscellaneous Examples may advantageously be left for a revisional course. The Oral Examples should not be neglected.

Typographical errors are likely to have crept in this the first edition. I shall, therefore, feel highly obliged if any one using this book would be good enough to point them out either to me or to the publishers.

In conclusion, I have to thank many friends who have assisted me in the verification of the Answers of the examples of this book, and especially Babu Chunilal Sil, late principal Mathematical Teacher of the General Assembly's Institution and author of several mathematical works, who has materially helped me in the preparation of this work and without whose help it would perhaps not have been possible for me to complete it.

38/2, NILMONY MITTER'S STREET,
Calcutta : the 15th December, 1897. } GAURI SANKAR DE.

PREFACE TO THE SECOND EDITION.

I AM very grateful to the Heads of Institutions, and the reading public for the very cordial reception given to this book, the first edition of which has been sold off in the very brief space of two months.

I also take this opportunity of acknowledging the help given me by several of my friends in pointing out errors, verifying answers of examples, and making valuable suggestions.

In this edition only slight alterations have been made here and there, and errors corrected. About 200 of the less important miscellaneous examples have been omitted from the latter part to reduce the size of the book.

38/2, NILMONY MITTER'S STREET, *20th April, 1898.* } GAURI SANKAR DE.

PREFACE TO THE THIRD EDITION.

IN this edition the book has been thoroughly revised and only slight alterations and additions have been made in certain places. Almost all the examples have been worked anew in the course of preparing the Key to this book which has been out about a month ago. I hope that few errors are left in this edition.

I have to tender my thanks to my friends and correspondents who have pointed out errors and communicated suggestions for the improvement of the book. Any communication for the improvement of the book will be thankfully received.

38/2, NILMONY MITTER'S STREET, *The 29th December, 1898.* } GAURI SANKAR DE.

PREFACE TO THE SEVENTH EDITION.

IN this edition the book has been thoroughly revised and several alterations and additions have been made. Many unimportant articles and examples have been omitted to reduce the bulk of the book and at the same time great pains have been taken to ensure accuracy in the examples and the answers.

8/2, NILMONY MITTER'S STREET, *The 2nd February, 1905.* } GAURI SANKAR DE.

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MATRICULATION ARITHMETIC.

CHAPTER I.

Definitions, Names of Numbers, Notation and Numeration.

I. DEFINITIONS AND PRELIMINARY NOTIONS.

1. Anything that is capable of increase or diminution is called **a magnitude.**

2. A magnitude may be *whole and undivided*, as the length of a stick, a period of time ; or it may consist of *separate and distinct* parts, as a heap of pebbles, a herd of oxen, a pack of dogs.

3. When a magnitude is whole and undivided, we select some well-marked magnitude of that kind which we call its **unit**, and by counting this unit a sufficient number of times, we make up the given magnitude ; but if the magnitude be made up of distinct objects, we select an object of that kind as our **unit**, and see how many of these units are to be taken to make up the given magnitude.

4. Hence, a **unit**, or as it is generally called **unity**, is the representation of a thing considered in its *individual* capacity, without regard to the *parts* of which it may be made up, and it is the *Base* or *Element* of all our computations.

Thus, each of the terms, *a* man, *a* house, *a* pound, &c. denotes one individual of its kind, being the same as *one* man, *one* house, *one* pound, &c. respectively; and these are the bases or elements by means of which *several* men, *several* houses, *several* pounds, &c., may be computed.

5. A magnitude represented as made up of one or more of its unit, is called a **quantity**, and the result of the comparison of the given magnitude with its unit respecting how many times it contains its unit is called a **number**.

Thus, the length of a stick, a heap of pebbles are *magnitudes* ; ten yards, a hundred pebbles are *quantities* ; ten and a hundred are *numbers*.

6. Hence, **number** signifies *one* or *more* units, or denotes one or more *distinct* objects of the same kind.

Thus, *one* man, *two* houses, *three* pounds, &c. which are represented by the numbers *one*, *two*, *three*, &c. denote one or more individuals of the same kind.

7. Numbers thus viewed or considered are termed **whole numbers** or **integers**; and the *unit* is considered as the *first* or *least* integer.

8. The **measure** or **numerical value** of any quantity is the *number* of times the quantity contains the unit.

Thus, when a foot is used as the unit of length, and we speak of a rod as four feet long, the number *four* represents the measure of the stick.

9. Hence the measure of a quantity represents its *relative* magnitude, but the measure and the unit together indicate its *absolute* magnitude.

10. Numbers are either **abstract** or **concrete**.

A *concrete* or *applicate* number is a number of objects or units of any kind; an *abstract* number is a number considered separately and without any relation to objects.

Thus, *five* apples, *ten* pounds, *four* men are *concrete* numbers; *five*, *ten*, *four* are *abstract* numbers.

11. Hence, an *abstract* number is a number in its literal sense, giving the idea of times or repetitions; but a *concrete* number is simply a quantity.

12. **Arithmetic** is the Science of numbers. It investigates their properties, and points out methods of calculations by means of them.

II. NAMES OF NUMBERS.

13. The *Symbol* or *Representation* of unit or unity is 1; but instead of other numbers being expressed by assemblages or multitudes of units *placed* together, which would soon become embarrassing, other characters or symbols have been invented, by means of which every number however great may be expressed; again, instead of a different symbol being adopted for every different number, which would soon become equally inconvenient, *all* numbers are expressed by means of the following *ten* symbols, or as they are usually termed **figures**, and sometimes **digits**, which have their names respectively annexed :—

1,	2,	3,	4,	5,	6,	7,	8,	9,	0
one,	**two,**	**three,**	**four,**	**five,**	**six,**	**seven,**	**eight,**	**nine,**	**zero.**

the first *nine* of which are all defined by their names: thus, one and one is **two**; two and one is **three**; three and one is **four**; four and one is **five**; and so on; and the last which is variously denominated **zero**, **cipher**, or **nought**, when standing by itself has no signification, or at most, denotes the absence of number, and is to be regarded merely as an *auxiliary* digit, for the purposes hereafter to be explained. These nine digits are called *simple numbers*, and *units of the first order*. Their names are perfectly arbitrary.

14. The next number is *nine* and *one*, and we give it the name **ten.** Ten forms a single *unit of the second order*, and by counting by *ten*, as we before counted by *one*, we have

one-ten, two-ten, three-ten, four-ten...nine-ten;

or more briefly, remembering that "ty" is equivalent to ten, and treating ten as a simple number, we say

ten, twenty, thirty, forty,......ninety.

The names of the nine numbers between ten and twenty, are eleven, twelve, thirteen, fourteen, fifteen,...nineteen.

The names of the nine numbers between twenty and thirty, thirty and forty, ..., as also the nine numbers that follow ninety are formed by placing in order the names of the first nine numbers after twenty, thirty,...., ninety. Thus we get at last ninety-nine.

15. The number which follows ninety-nine is ninety-nine and one, or *ten tens*, and is called a **hundred.** It is a single *unit of the third order*, and by counting by *hundreds* as we counted by simple units, we have

one-hundred, two-hundred, three-hundred,......nine-hundred.

The names of the ninety-nine numbers between one hundred and two hundred, two hundred and three hundred,......, as also the ninety-nine numbers that follow nine hundred, are formed by placing in order the names of the first ninety-nine numbers after one hundred, two hundred,......nine hundred. Thus we get at last nine hundred and ninety-nine.

16. The number which follows nine hundred and ninety-nine is nine hundred ninety-nine and one or *ten hundred*, and is called a **thousand.** It is a single *unit of the fourth order.* Proceeding as before, we get *ten-thousand* as forming a single *unit of the fifth order*, and *ten ten-thousands* or a *hundred thousand* as a single *unit of the sixth order*; but there being no independent names for these units we call a *thousand* as a *second* principal unit, and count by units, tens and hundreds of thousands.

The names of the nine hundred and ninety-nine numbers between one thousand and two thousand, two thousand and three thousand,......, as also the nine hundred and ninety-nine numbers that follow hundreds of thousand are formed by placing in order the names of the first nine hundred and ninety-nine numbers after one thousand, two thousand,......hundreds of thousands.

17. The next number is a *thousand thousands*, and forms a single *unit of the seventh order.* It has an independent name and is called a **million.** Considering a million as a *third* principal unit, we count by units, tens, hundreds, thousands, ten-thousands, and hundred-thousands of millions.

18. Lastly, we come to a *million millions*, which is called a **billion,** and forms a single *unit of the thirteenth order.* Proceeding

in this way, we get a *million billions*, which is called a **trillion**, a *million-trillions*, which is called a **quadrillion**, and so on.

The **periods** which follow the above in succession are **quintillion, sextillion, septillion, octillion, &c.**

In France and some of the United States of America a *thousand* millions is called a billion, a *thousand* billions a trillion, and so on ; hence a billion in England is a trillion in America, &c.

19. From what has been said above, it appears that we practically employ not more than *thirteen* independent words :—*one, two, three, four, five, six, seven, eight, nine, ten, hundred, thousand, million*, and that *ten* units of any order always make one unit of the next higher order.

III. NOTATION.

20. Notation is the method of expressing by certain symbols or characters, any proposed number expressed in words.

21. Beginners have already learnt from Art. 13 how to express the numbers from one to nine by **one** figure ; the following Article will teach them to express numbers from ten to ninety-nine by the use of **two** figures.

22. When a figure is placed on the *right* of the same or any other figure it has by *universal agreement*, the effect of increasing the value of the last mentioned figure *tenfold*, at the same time that it retains its own value.

Thus, beginning with the auxiliary digit 0, we have the following numbers and their representations :—

10 ten	29 twenty-nine	48 forty-eight
11 eleven	30 thirty	49 fotry-nine
12 twelve	31 thirty-one	50 fifty
13 thirteen	32 thirty-two	51 fifty-one
14 fourteen	33 thirty-three	52 fifty-two
15 fifteen	34 thirty-four	53 fifty-three
16 sixteen	35 thirty-five	54 fifty-four
17 seventeen	36 thirty-six	55 fifty-five
18 eighteen	37 thirty-seven	56 fifty-six
19 nineteen	38 thirty-eight	57 fifty-seven
20 twenty	39 thirty-nine	58 fifty-eight
21 twenty-one	40 forty	59 fifty-nine
22 twenty-two	41 forty-one	60 sixty
23 twenty-three	42 forty-two	61 sixty-one
24 twenty-four	43 forty-three	62 sixty-two
25 twenty-five	44 forty-four	63 sixty-three
26 twenty-six	45 forty-five	64 sixty-four
27 twenty-seven	46 forty-six	65 sixty-five
28 twenty-eight	47 forty-seven	66 sixty-six

67 sixty-seven	78 seventy-eight	89 eighty-nine
68 sixty-eight	79 seventy-nine	90 ninety
69 sixty-nine	80 eighty	91 ninety-one
70 seventy	81 eighty-one	92 ninety-two
71 seventy-one	82 eighty-two	93 ninety-three
72 seventy-two	83 eighty-three	94 ninety-four
73 seventy-three	84 eighty-four	95 ninety-five
74 seventy-four	85 eighty-five	96 ninety-six
75 seventy-five	86 eighty-six	97 ninety-seven
76 seventy-six	87 eighty-seven	98 ninety-eight
77 seventy-seven	88 eighty-eight	99 ninety-nine

which is the largest number that can be expressed by **two** digits.

23. The use of **two**, either the *same* or *different* figures, will not enable us to go beyond this number, but a repetition of the contrivance in the last Article, will by means of *more* figures supply the defect.

Thus, supposing the effect of any figure's being placed on the right of symbols formed as above, to be to increase all their values *tenfold*, we shall have

100 one hundred	200 two hundred
101 one hundred and one	201 two hundred and one
102 one hundred and two	202 two hundred and two
&c. &c.	&c. &c.

so likewise of succeeding numbers; thus, we have

345 three hundred and forty-five	750 seven hundred and fifty
586 five hundred and eighty-six	946 nine hundred and forty-six

and again 999 will be *nine hundred* and *ninety-nine*, which is the largest number capable of being expressed by **three** figures.

Here, the *first* figure on the right hand is said to occupy the *units' place*, the *second*, the place of *tens*, and the *third*, that of *hundreds*.

Of the auxiliary digit 0, the sole use is in the effect specified in the last two Articles; and all figures to the *right* of it will therefore be unaffected by it.

24. In estimating numerical magnitudes, we proceed in order from *hundreds*, to *thousands*, *tens of thousands*, and *hundreds of thousands*; *millions*, *tens of millions*, and *hundreds of millions*; in precisely the same manner as we have done above from *units* to *tens*, and from *tens* to *hundreds*.

25. Agreeably to the principle of Art. 22, it is *assumed* that "*any* figure placed on the right of one or more figures, has the effect of increasing every one of them tenfold without altering its own value"; and this enables us to express with facility any number whatever.

Thus, 1000 will represent one thousand.
5493 will represent five thousand, four hundred and ninety-three.
23456 will represent twenty-three thousand, four hundred and fifty-six.
729054 will represent seven hundred twenty-nine thousand and fifty-four.
1803205 will represent one million, eight hundred three thousand, two hundred and five.
32754081 will represent thirty-two million, seven hundred fifty-four thousand, and eighty-one.
473025004 will represent four hundred seventy-three million, twenty-five thousand and four.

26. *If* the first three figures beginning from the right-hand be denominated so many *units*, tens of *units* and hundreds of *units*, it follows that the next three figures taken in the same way will be *thousands*, tens of *thousands*, and hundreds of *thousands*; the next three in order will be *millions*, tens of *millions*, and hundreds of *millions*; and so on.

Whence, to express in figures any number proposed, we have only to consider in which of these divisions each part of it ought to be found, observing that *three* figures from the right must be taken to make each division *complete*, before we proceed to the next. Thus,

Ex. 1. Express by means of figures: *Thirty-five thousand eight hundred and nineteen.*

Here, eight hundred and nineteen belongs to the *first* division on the right; and is written 819: also, thirty-five thousand must be found in the *second* division from the right, and is 35: whence the proposed number will be expressed by 35,819.

Ex. 2. Write down in figures the number: *Five million, twenty-five thousand, six hundred and seven.*

In this case, the *first* division on the right will be 607; the *second* will be 025, the digit 0 being affixed to the left of the others without altering their values, to make up the required number of *three*; and the third is 5: so that the expression required will be 5,025,607.

Ex. 3. Express by figures the following number: *Five hundred and seventy million, two hundred six thousand and fifty four.*

Here, the *first* division is 054, the 0 altering only the values of the figures in the *subsequent* divisions; the *second* division is 206; and the *third* is 570; whence the number proposed is correctly expressed by 570,206,054.

27. This method of notation can never present any difficulty, provided it be carefully remembered that every division of figures, as we proceed from the right hand towards the left must be *completed* as far as it is possible; and by a little practice, we shall be enabled to write down any number by beginning at the *left hand.*

Ex. 1. To write down *Six hundred and thirteen million five hundred and twenty-seven*, we observe that the division of *millions* will be 613; that of *thousands* will be 000, and that of *units* 527, so that the number is expressed by 613,000,527.

Ex. 2. To represent *Ten thousand million* by figures; for the *fourth* division we have 10, and for each of the *third*, *second* and *first* 000, so that the representation required is 10,000,000,000.

Examples I.

Represent the following numbers in figures :—

1. Forty-three ; seventy-nine ; sixty-five ; eighty-four ; fifty-eight ; ninety-seven ; sixty ; eighty-seven.

2. Four hundred and forty-nine ; five hundred and ninety-eight ; seven hundred and four ; four hundred and five ; two hundred and thirty-five ; nine hundred and fifty-eight ; seven hundred and twenty-five ; eight hundred and thirty-five.

3. Four thousand ; seven thousand, eight hundred and four ; eighty-nine thousand and sixty-three ; fifty-three thousand, two hundred and twenty-three ; eight thousand and forty-six ; six hundred three thousand, two hundred and forty ; five hundred thousand, five hundred and five ; nine hundred nine thousand and nine.

4. Three hundred forty-one thousand, three hundred and twenty-three ; two hundred thousand and seventy-five ; seven hundred seven thousand and seventy ; five hundred thousand ; eighty thousand and eight ; four hundred two thousand and seven hundred.

5. Nine million, forty-three thousand, six hundred and two ; seven million, eight hundred fifty-nine thousand, six hundred and thirty-two ; three million, forty thousand and twenty ; one million, four hundred and three thousand ; five million, five hundred thousand, six hundred, and seventy-six ; eleven million and five ; one million, three hundred, seventy-eight thousand, two hundred and sixty-seven ; one million, ten thousand and one.

6. Forty-five million, three hundred eighty-seven thousand and twenty-five ; ninety-two million, five hundred sixty-eight thousand, nine hundred and eighty-five ; eleven million, five hundred sixty-five thousand, four hundred and thirty-seven ; forty million, forty thousand and five ; ninety-six million, ninety-six thousand and ninety-six.

7. Three hundred forty-nine million, four thousand and sixty-five ; one hundred million, thirteen thousand and one ; nine hundred nine million, nine thousand and ninety-nine ; eight hundred forty-two million, two hundred forty-six thousand, four hundred and eighty-four ; three million, four hundred fifty-two thousand, one hundred and sixty-one ; four hundred ninety-four million.

8. Ninety-nine million, ninety-nine thousand and ninety-nine ; one hundred eleven million, six hundred fifty thousand and fifty ;

six hundred forty million, sixty-four thousand and six hundred ; five hundred million, seven hundred three thousand and two ; six hundred nine million, one thousand, two hundred and eight.

9. Two thousand, eight hundred four million, two hundred fifty-two thousand and ninety-seven ; twelve thousand, thirty-six million, fifty-four thousand and seventy-nine ; four thousand million, nine hundred thousand and five ; six thousand, three hundred four million, five hundred six thousand, five hundred and six ; forty thousand, two hundred eighty million, five hundred thirty thousand, two hundred and fifty-nine.

10. Four hundred thousand million and ten thousand ; eight hundred thirty-six thousand, five hundred and seventy-three million, two hundred forty-four thousand and six ; nine hundred thousand, nine hundred million, nine hundred thousand and nine ; six hundred thousand, sixty million, six thousand and six.

11. Nine thousand, four hundred five million, four thousand, five hundred and fifty ; four hundred thirteen thousand, seven hundred twenty-three million, nine thousand and four ; five thousand, and eight hundred eight million, sixty-eight thousand and eighty.

12. Eight billion, two hundred seven thousand and five ; three billion, four thousand, seven hundred two million, one hundred sixty-four thousand, seven hundred and twenty-two ; one billion, three hundred thousand and five.

13. Ninety-nine billion, ninety million, ninety-nine thousand, nine hundred and nine ; one hundred billion, one hundred ninety-six thousand, four hundred million, ten thousand and nine.

14. Six hundred fifty-four thousand, three hundred twenty-three billion, four thousand, twenty-one million, fifty thousand, three hundred and one ; forty-seven thousand, five hundred twenty-six billion, eight hundred seventy thousand, seven hundred forty-four million, one hundred three thousand, two hundred and eighty-four.

15. Nine trillion, four billion, six hundred forty million, three hundred and sixty-five.

16. Write in figures the least number of six digits and the greatest number of eight digits. *How many numbers are represented by three digits ?*

17. Write down in figures all the numbers between eighty-seven and ninety-three, between six hundred and eleven to six hundred and twenty, and between nine hundred and forty-seven to nine hundred and seventy.

18. When told to write five million, five hundred five thousand, five hundred and five in figures, one boy wrote 550555, and another wrote 50550505 ; what mistakes did they commit ?

---

28. It will be observed, from what has been said, that each of the nine figures or digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, has a **simple,**

absolute or **intrinsic** value of its own, whereas the auxiliary digit o has no such value; and on this account the former are termed **significant** figures, in contradistinction to the last. It will moreover have occurred to the reader, that every one of these significant digits, in addition to its **simple** value, which is fixed and certain, possesses also a **local** or **accidental** value dependent upon the situation in which it is placed.

Thus, in the expression of the number, *Four thousand three hundred and twenty-one*, which will be 4321, the 1 in the first place on the right hand, retains its *simple* value; the second figure 2, in its situation denotes two *tens* or *twenty*; the third is three *hundreds*, and the fourth is four *thousands*; so that the *local* values of 2, 3 and 4 here, are respectively, *ten* times, a *hundred* times and a *thousand* times, as great as their *simple* values; and it is the circumstance of assigning to each of the significant figures a **local** as well as a **simple** value, which confers upon the system, the immense powers it possesses.

29. The characters 1, 2, 3, 4, 5, 6, 7, 8, 9, o and the mode of representing numbers by their combinations were first invented by the Hindus. The word *Digit* (denoting a *Finger*) usually applied to these characters, seems to point out the means originally employed in estimating numerical magnitudes; the number 10, which is called the *Base* or *Radix* of the system, and by which the *local values* of the *digits* are regulated, being that of the *Fingers* of *both* hands. Thus came the name **Decimal System** of Notation. The system was borrowed from the Hindus by the Arabs, who introduced it into Europe about the 11th century. Hence the Europeans call it the *Arabic Notation*. The Notation appears to be as *complete* and *convenient* as can well be imagined, and in its present state may certainly be regarded as one of the greatest and most successful efforts of human ingenuity ever exhibited to the world.

IV. NUMERATION.

30. Numeration is the art of reading or estimating the value of a number expressed by figures, and is therefore the *reverse* of Notation.

31. From the circumstance of every figure possessing a *local* as well as a *simple* value, it follows that the value of each figure must be estimated by the place which it occupies: hence, a figure standing by itself expresses so many *units*; a figure in the second place from the right, denotes so many *tens*; a figure in the third place, so many *hundreds*, and so on: consequently, if we suppose any numerical expression to be divided into **periods**, or portions each consisting of three figures as far as they go, the figures of the **period** on the right will be **units**, and tens and hundreds of **units**; those of the next will be units, tens and hundreds of **thousands**; those of the third will be units, tens and hundreds of **millions**; and so on.

Thus, 25 is read twenty-five.
304 is read three hundred and four.
5,287 is read five thousand, two hundred and eighty-seven.
60,539 is read sixty thousand, five hundred and thirty-nine.
207,385 is read two hundred seven thousand, three hundred and eighty-five.
1,739,204 is read one million, seven hundred thirty-nine thousand, two hundred and four.
35,024,376 is read thirty-five million, twenty-four thousand, three hundred and seventy-six.
275,008,005 is read two hundred seventy-five million, eight thousand and five.

32. In each of the above instances, we conceive the expression to be separated into *periods* of three figures each as far as they go, beginning at the right hand. But if the number contains more than nine figures, then instead of supposing that each division consists of *three* figures, if we include *six* figures as far as we can in each division from the right hand, the first may be regarded as so many hundreds of thousands of **units**; the next as so many hundreds of thousands of **millions**; the next as so many hundreds of thousands of what are called **billions**, and the succeeding divisions, of so many hundreds of thousands of what are termed **trillions, quadrillions**, &c.

Thus, 34,567008,093402 is read thirty-four billion, five hundred sixty-seven thousand and eight million, ninety-three thousand four hundred and two.

33. The last two Articles will be rendered more clear by the following scheme, called the **Numeration Table.**

&c. &c. &c.	hundreds of thousands of billions	tens of thousands of billions	*thousands* of billions	hundreds of billions	tens of billions	**billions.**	hundreds of thousands of millions	tens of *thousands of millions*	*thousands* of millions	hundreds of millions	tens of millions	**millions.**	hundreds of thousands	tens of *thousands*	**thousands**	hundreds	tens	**units**
	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1

34. In reference to what was said in Art. 32, it may be proper to observe that the method of proceeding differs from that adopted by the *French* and some other *European* Arithmeticians, who adhere throughout to divisions of *three* figures, according to the principle of Art. 31, and after the division of *millions*, proceed directly to that of *billions*, tens of *billions*, and hundreds of *billions*: then to *trillions*, tens of *trillions*, and hundreds of *trillions*, and so on: and this method certainly possesses some advantages in point of simplicity; but as numbers of these magnitudes are not of very frequent occurrence, it has not been thought necessary to depart from the *Notation* and *Nomenclature* established in England.

Examples II.

Write down in words the following numbers :—

1. 17 ; 24 ; 35 ; 46 ; 27 ; 48 ; 59 ; 76 ; 84 ; 95 ; 66 ; 75 ; 89.

2. 217 ; 319 ; 583 ; 695 ; 725 ; 308 ; 406 ; 846 ; 932 ; 725.

3. 3406 ; 5260 ; 4236 ; 3298 ; 5678 ; 2405 ; 9286.

4. 43201 ; 87054 ; 34002 ; 49803 ; 58030 ; 76503.

5. 903756 ; 903284 ; 827109 ; 319420 ; 243065 ; 123456.

6. 2714325 ; 8047328 ; 4010010 ; 8004640 ; 1234007.

7. 12870045 ; 20084216 ; 79030284 ; 43002005.

8. 321408653 ; 408076032 ; 314159265 ; 123456789.

9. 571268405 ; 3179040601 ; 319680209078.

10. 1234567654321 ; 5020040003060 ; 4302500764009.

11. 200900600002 ; 43287000006321 ; 64000002646002.

12. 319080259417 ; 236045978213478.

13. 1327875430029 ; 5432176989007.

14. Write the largest and the smallest numbers possible with the symbols 5, 4, 9, 2, 7.

15. Give the local value of each of the significant digits in the following numbers :—

95 ; 64 ; 575 ; 8297 ; 40276 ; 3205 ; 478296 ; 40302605 ; 50003029 ; 70300006 ; 9786002030.

16. Express in words the greatest number of five figures and the least number of seven figures.

17. Write down *all* the numbers that can be formed by the digits 2, 3, 4, taken all together.

V. THE HINDU METHOD OF NUMERATION.

35. The following is the **Indian Numeration Table** in common use :—

&c.	&c.	hundreds of crores বৃন্দ	tens of crores অর্বুদ	crores কোটি	tens of lacs নিযুত	lacs লক্ষ	tens of thousands অযুত	thousands সহস্র	hundreds শত	tens দশ	units এক
		4	5	6	7	5	2	8	3	6	4

The above number is read thus :—Four hundred and fifty-six crores, seventy-five lacs, twenty-eight thousand, three hundred and sixty-four.

The Hindu names of places of figures are as follow :—**eka**, *dasha*, *shata*, **sahasra**, *ajuta*, **laksha (lac)**, *nijuta*, **koti (crore)**, *arbuda*, *brinda*, *kharva*, *nikharva*, *sankha*, *sagara*, *padma*, *padmanava*, *mohapadma*, *kshuni*, *akshuhini*, *dhuli*, *mohadhuli*, *antya*, *parardha*.

Examples III.

Write down in words the following numbers according to the Indian Numeration :—

1. 19237 ; 60081 ; 49027 ; 167208 ; 200753 ; 830005.

2. 7090709 ; 8001025 ; 3905086 ; 24050008 ; 4001745.

3. 402178l5 ; 4030024340 ; 4780230016 ; 23456000.

4. 123456789 ; 6450300000 ; 760242900.

5. 4500002430 ; 8000785000 ; 4020504008.

Express in figures :—

6. Four lacs, fifteen thousand, two hundred and eight ; fifty-six lacs, four thousand and twenty-nine ; eight hundred forty-three lacs, seventy-four thousand, two hundred and nine ; eight lacs and five ; seventy-five lacs ; thirty lacs, seven hundred and eight.

7. Two crores, fifteen lacs and four ; thirty-seven crores, seven lacs, four thousand and twelve ; one hundred and forty-five crores, nineteen lacs and seven ; five thousand and ninety-nine crores, four lacs, five thousand, six hundred and seven.

8. Eighty crores, thirty lacs, one thousand and eleven, four thousand two hundred and ninety-five crores, fourteen lacs, and eighty-five; seventy-five thousand four hundred and ten crores, fourteen lacs, nine thousand and nine.

9. How many lacs are there in twenty millions? How many thousands are in ten lacs? How many millions in forty crores?

10. Read according to the Indian numeration the number—four hundred five million, seventy-five thousand, nine hundred and four.

11. Express a *billion* in Indian, and a *akshuhini* in English Notation.

12. A boy was told to write nine crores, five lacs, four thousand, seven hundred and fifty-six, and he wrote 905407056. Find out his mistakes.

VI. THE ROMAN SYSTEM OF NOTATION.

36. A different system of Notation was in use among the Romans, long before the introduction of the Arabic Notation into Europe by the Moors in Spain.

In this system the characters chiefly used are I, V, X, L, C, D and M which denote respectively the numbers 1, 5, 10, 50, 100, 500 and 1000 in the Arabic system. Again when a *bar* or *line* is placed over a character, it increases its value a *thousandfold*.

Thus $\overline{\text{V}}$ stands for 5000, $\overline{\text{C}}$ represents 100000.

The following table gives a full view of the method of expressing numbers in the **Roman System**:—

I	1	XV	15	CC	200
II	2	XVI	16	CCC	300
III	3	XVII	17	CD	400
IV	4	XVIII	18	D	500
V	5	XIX	19	DC	600
VI	6	XX	20	DCC	700
VII	7	XXX	30	DCCC	800
VIII	8	XL	40	CM	900
IX	9	L	50	M	1000
X	10	LX	60	MCD	1400
XI	11	LXX	70	MCM	1900
XII	12	LXXX	80	MM	2000
XIII	13	XC	90	MDCCCLXXXVI	1886
XIV	14	C	100	$\overline{\text{DLX}}$DCCCXLIV	560844

Examples IV.

Express in Arabic Notation each of the following numbers :—

1. VII, XVII, XXI, LIV, XXVIX, XXXIX.

2. LXLV, XLVIII, XCV, CCXIV, DXIV, CDXIX.

3. MIX, MDCCCIV, MDCL, MDCCLXVI, $\overline{\text{MC}}$, DCV.

4. $\overline{\text{V}}$DLV, $\overline{\text{V}}$IDL, $\overline{\text{CCX}}$CDXL, $\overline{\text{CCXC}}$DXL, $\overline{\text{M}}$X, $\overline{\text{MM}}$DMC.

Express in Roman Notation each of the following numbers :—

5. 9, 16, 35, 46, 68, 75, 89, 99, 105, 148.

6. 32, 28, 49, 69, 78, 95, 215, 327, 433, 549.

7. 745, 923, 567, 1234, 1567, 1853, 1918.

8. 1231, 1262, 1862, 1877, 1999, 2001, 1769

9. 15497, 20015, 200150, 651002, 1000001, 2003450.

CHAPTER II.

The Four Fundamental Operations.

37. We now proceed to the consideration of the **Four Fundamental Operations** that can be performed upon numbers, which are those of **Addition, Subtraction, Multiplication** and **Division**, each of which will be defined, explained and exemplified in its order.

I. ADDITION.

38. Addition consists in finding a number equal to *two* or *more* numbers taken together.

The several numbers given to be added are called **summands**, and the single number obtained by adding them is called their **sum** or **amount.**

In addition the several numbers to be added must be either all *abstract* numbers or all *concrete* numbers of the *same kind.*

39. *Addition* is of two kinds, **simple** and **compound.**

(i) *Simple Addition* is one in which the numbers to be added together are either all *abstract* numbers, or all *concrete* numbers *of the same denomination* (*e. g.*, all *rupees*, or all *pounds*, or all *miles*, &c.)

(ii) *Compound Addition* is the method of collecting into one sum several *concrete* numbers of the same kind, but not expressed in *one* denomination of that kind.

40. It is usual, in the applications of Arithmetic, to express the operation of *Addition* by the *sign* + invented for the purpose. It is read **plus.**

Thus, the sum of 4 and 5 is expressed in the form $4+5$, wherein the sign + between 4 and 5 denotes the addition of the latter number to the former, and is read four *plus* five.

The sign = is called **the sign of equality.** It is read **equals** or **is equal to.**

Thus, $4+5=9$ expresses the result of the addition of 4 and 5 to be 9, or the *equality* between the *sum* of the numbers 4 and 5 and the *number* 9. It is read four *plus* five *equals* nine.

41. To effect the operation of *Addition*, it is merely necessary to know from *memory* or by *practice*, the sums of every two single figures. The following Table, called the **Addition Table,** should be carefully committed to memory by beginners :—

1 and	2 and	3 and	4 and	5 and	6 and	7 and	8 and	9 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10
2......3	2......4	2......5	2......6	2......7	2......8	2......9	2......10	2......11
3......4	3......5	3......6	3......7	3......8	3......9	3......10	3......11	3......12
4......5	4......6	4......7	4......8	4......9	4......10	4......11	4......12	4......13
5......6	5......7	5......8	5......9	5......10	5......11	5......12	5......13	5......14
6......7	6......8	6......9	6......10	6......11	6......12	6......13	6......14	6......15
7......8	7......9	7......10	7......11	7......12	7......13	7......14	7......15	7......16
8......9	8......10	8......11	8......12	8......13	8......14	8......15	8......16	8......17
9......10	9......11	9......12	9......13	9......14	9......15	9......16	9......17	9......18

This Table can easily be carried on for numbers larger than 10; for instance, since 4 and 1 make 5, 4 and 11 make 10 more than 4 and 1, *i.e.*, make 15. Again, since 8 and 7 make 15, 8 and 17 will make 25, and so on, the result in each case, being 10 more than in the corresponding case in the Table. Also 3 and 46 make 49, 9 and 56 make 65, 8 and 87 make 95, and so on, the results in the several cases being respectively 40, 50, 80, more than the corresponding results in the Table.

Ex. 1. Add together 4, 8, 5, 0, 9.
We add thus, 4 and 8 make 12, 12 and 5 make 17,
17 and 0 make 17, 17 and 9 make 26,
$\therefore 4+8+5+0+9=26.$

or thus,

$$\begin{array}{r} 4 \\ 8 \\ 5 \\ 0 \\ \underline{9} \\ 26 \end{array}$$

Ex. 2. Find the sum of 24, 13, 15, 42.
24 and 13 make 37, 37 and 15 make 52,
52 and 42 make 94,
$\therefore 24+13+15+42=94.$

or thus,

$$\begin{array}{r} 24 \\ 13 \\ 15 \\ \underline{42} \\ 94 \end{array}$$

Examples V. (MENTAL ADDITION.)

1. Write down the sums of :—
(1) 2 and 4 ; 2 and 10 ; 3 and 5 ; 4 and 7 ; 5 and 9 ; 8 and 7.
(2) 9 and 10 ; 8 and 8 ; 7 and 3 ; 7 and 6 ; 9 and 1 ; 5 and 9.
(3) 2 and 9 ; 0 and 7 ; 4 and 9 ; 9 and 7 ; 4 and 11 ; 9 and 14.
(4) 7 and 7 ; 7 and 9 ; 8 and 10 ; 9 and 6 ; 4 and 12 ; 7 and 13.
(5) 8 and 2 ; 8 and 5 ; 9 and 14 ; 8 and 13 ; 7 and 15 ; 6 and 14.
(6) 10 and 6 ; 10 and 9 ; 11 and 5 ; 13 and 6 ; 14 and 3.
(7) 4 and 17 ; 3 and 19 ; 12 and 12 ; 13 and 13 ; 16 and 12.
(8) 8 and 0 ; 12 and 13 ; 12 and 15 ; 11 and 16 ; 10 and 19.
(9) 15 and 8 ; 11 and 15 ; 18 and 12 ; 16 and 15 ; 13 and 16.
(10) 18 and 16 ; 15 and 15 ; 14 and 14 ; 16 and 16 ; 11 and 17.
(11) 10 and 11 ; 10 and 12 ; 11 and 13 ; 11 and 18 ; 12 and 19.
(12) 17 and 17 ; 18 and 19 ; 16 and 18 ; 19 and 19 ; 16 and 19.

2. (1) Add 6 to 28, to 38, to 48, to 58, to 68, to 78, to 88, &c.
(2) Add 8 to 25, to 35, to 45, to 55, to 65, to 75, to 85, &c.
(3) Add 15 to 39, to 49, to 59, to 69, to 79, to 89, to 99.

3. Add together :—
(1) 12 and 37 ; 13 and 25 ; 14 and 84 ; 14 and 26 ; 14 and 76.
(2) 19 and 75 ; 17 and 87 ; 16 and 56 ; 18 and 75 ; 18 and 52.
(3) 26 and 64 ; 36 and 85 ; 49 and 24 ; 39 and 75 ; 27 and 31.
(4) 39 and 42 ; 49 and 99 ; 26 and 37 ; 75 and 94 ; 53 and 84.
(5) 16 and 85 ; 17 and 54 ; 45 and 33 ; 64 and 89.

4. Count aloud by increments of 7 up to 100, starting at 6, at 9, at 13, at 15, at 17, at 19, at 21, at 23, at 25, and at 29.

5. Find the sums of :—
(1) 1, 3 and 5 ; 2, 5 and 3 ; 3, 9 and 7 ; 8, 4 and 6 ; 7, 7 and 7.
(2) 9, 9 and 2 ; 7, 3 and 6 ; 8, 5 and 9 ; 5, 5 and 9 ; 7, 5 and 9.
(3) 3, 3, 3 and 3 ; 4, 6, 1 and 9 ; 8, 0, 9 and 6 ; 8, 8, 8 and 8.
(4) 5, 5, 8 and 4 . 9, 8, 7 and 6 ; 4, 7, 2 and 6 ; 6, 7, 8 and 9.
(5) 4, 0, 3, 5 and 9 ; 6, 0, 5, 0 and 9 ; 7, 2, 8, 8 and 5.

6. Find the values of :—
(1) $3+4+9+3+3+5$; $3+6+8+5+6+4$; $6+0+4+7+0+5$.
(2) $9+5+7+8+3+4$; $6+9+9+7+7+5$; $5+8+9+7+5+6+3$.

7. Ram has 6 books, and his brother 5 ; how many books have they together ?

8. A boy has 8 marbles in one pocket, and 5 in another ; how many marbles has he ?

9. Bepin has 4 marbles, Gopal 7 and Bejoy 5 ; how many have they together ?

10. In a garden there are 4 mango trees, 6 cocoanut trees, 5 jack trees and 8 plum trees ; how many trees are there in all ?

11. Shyam paid 3 pice for a loaf, 4 pice for sugar, and 2 pice for butter. How much did he pay altogether?

12. One boy gained 3 prizes, another 2, and another 5. How many prizes did the three boys gain?

13. Hari has 8 marbles, and Bhuban 7 more than Hari. How many have they both together?

14. One dovecot has 8 pigeons, another has 10, and a third has 12. How many pigeons have the three dovecots?

15. A boy paid 4 pice for a pencil, 2 pice for a pen-holder, 14 pice for a slate and 7 pice for quills; how many did he pay for the whole?

16. Ram's age is 4 years, Gopal is 2 years older than Ram; Shyam's age is the sum of the ages of the other two. Find the sum of all their ages.

17. In a school there are four classes. In the first class there are 6 boys; in the second class 7 boys; in the third 2 more than in the first class; in the fourth 5 more than in the second class. How many boys are there in the school?

18. Ram, Hari, and Gopal went to fish. Ram caught 9 lobsters, Hari caught none, and Gopal caught 12. How many lobsters did the three boys catch?

19. Ram has a line 6 feet long, Shyam one 10 feet long, and Bhuban one 9 feet long. If the three lines were joined, how long a line would they make?

20. Jogin got a prize of 5 rupees, Upendra got 6 rupees more than Jogin; how many rupees did they get altogether?

21. A farmer has 8 cows, 6 calves, and 5 sheep. How many animals has he altogether?

22. Hari got from his father 9 pice, his two brothers 7 and 8 pice respectively, and his sister 5 pice; how much did the father give in all?

23. A man's age is 38 years; how old will he be after 12 years?

24. From a rope are cut off 15 yards and there are 6 yards left; what was the length of the rope?

25. After giving away 15 rupees, I have 8 rupees still left; how many rupees had I in all?

26. What number is that from which if I take first 8, and then 5, there will remain 24?

27. A man has a son whose age is 10 years; he is older than his son by 26 years; what is his age?

28. I have 25 nuts in my pocket, and my father gives me 15 more; how many have I in all?

29. A rupee contains 64 pice; how many pice are there in two rupees?

30. A woman sold 4 mangoes to A, to B 5 more than to A, to C as many as to A and B, to D 9 more than to B; had C bought as many more mangoes as he did buy, the woman would have sold all her mangoes; how many mangoes had she to sell?

SIMPLE ADDITION.

42. The principle usually termed **carrying** in the Rule given below is "*that the tens of any order in a partial sum may be* **carried** *as units to the next higher order*," for ten units of any order are equivalent to one unit of the next higher order.

43. The following is the Rule for the addition of large numbers :—

RULE. Place the numbers under one another in such a manner that units may stand under units, tens under tens, hundreds under hundreds, and so on, and draw a line below all the horizontal rows of figures. Then add up the figures in the first vertical row on the right-hand, find the numbers of *tens* and *units* in their sum, and put down the number of *units*, whether it be zero or any of the nine other digits. **Carry** as many *units* as there are *tens* thus found to the next vertical row and add them up as before, observing the numbers of *tens* and *units* contained in the sum. Place the number of *units* under the row added, and carry the number of *tens*, to the next; proceed in the same manner till the last row is added, when put down the numbers both of *tens* and *units*, as there are no more figures of higher denominations. The entire sum thus put down will be the **sum** of the separate numbers.

Ex. Add together 6254, 893, 48 and 5487.

Arrange the numbers according to the Rule given above, and proceed to add the columns beginning from the column of units.

$$\begin{array}{r} 6254 \\ 893 \\ 48 \\ 5487 \\ \hline 12682 \end{array}$$

The sum of 4, 3, 8 and 7 is 22. Place the **2** units under the row of units, and carry on the 2 tens units to the row of tens.

The sum of 2, 5, 9, 4 and 8 is 28. Place the **8** tens under the row of tens, and carry on the 2 hundreds units to the row of hundreds.

The sum of 2, 2, 8 and 4 is 16. Put down the **6** hundreds under the row of hundreds, and carry on the 1 thousand units to the row of thousands.

The sum of 1, 6 and 5 is 12. Put down the **12** under the row of thousands. Thus the entire sum is 12682.

44. A **Proof** is a second operation which serves as a test of the correctness of the first.

The *Proofs of Addition* depend on this principle—The **sum** of several numbers is not affected by the **order** in which they are added together; thus $4+8=8+4$.

45. To ascertain whether the operation is correctly performed, various expedients might be resorted to :—first, that of adding the numbers *downwards* instead of *upwards*, which, because the *same* set of numbers cannot have two *different* sums, must give the same result : second, that of omitting one of the horizontal rows of figures in a *second* operation, and afterwards adding it to the result of the rest obtained by the Rule : third, that of *casting out the nines* from the sum of the digits in the *summands* and the sum of the digits in the amount; if the two results coincide the operation may be *presumed* to be correct. (*Casting out the nines* is explained in Art. 79.)

Examples VI.

1. Add together :—

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
37	90	57	24	98	68	79	12	87	97
42	45	68	56	55	48	27	56	68	59
23	73	75	35	60	99	94	48	59	68

(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
85	78	310	704	345	2969	787	347	3214
92	69	46	450	902	4868	678	238	6786
99	75	147	979	450	6787	425	410	2345

(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
889	654	8888	6748	415	293	814	325	4028
803	546	5173	5555	278	75	326	748	354
519	465	7421	7864	614	409	628	493	95
745	824	7643	5408	932	3	459	869	2076

(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)
736	9806	785	6045	736	8076	459	9542	17384
402	1932	8756	4500	405	432	3687	876	12345
4159	6580	9540	8068	8159	5431	7468	4093	5432
47	9889	8559	9647	49	458	243	7777	946
2468	7885	386	9407	7204	9327	5907	4685	24607

(38)	(39)	(40)	(41)	(42)	(43)	(44)
71407	15161	96748	33456	67895	796210	894142
90781	8098	25003	84771	56789	34728	378523
68943	958	84067	66854	98765	514344	66666
32605	41978	95674	72984	87956	858521	8524
72777	78368	98765	99999	78965	936266	981234

(45)	(46)	(47)	(48)	(49)	(50)
659873	5678912	2345678	1234567	4893054	9876543
487	4567891	3192463	2345671	9876543	9999999
6935	3456789	7283642	8742015	2483109	4602
70415	2144124	9234925	8888888	7834510	341025
8796	7878787	8364774	4310943	3469146	46

(51)	(52)	(53)	(54)	(55)
466779	897654	9466495	768402	27591046
878987	987763	7545478	95320089	5768004
365363	123456	29099	6949	39039587
432698	789099	2988607	84982759	596459
756545	789789	9292929	700897	78534842
487988	437977	7833210	78563412	19827634

2. Find the values of :—

(1) 567+90+48+39+4728+1000+6489+327+4578.

(2) 37045+6879+3724+4562+82971+37256+409.

(3) 5971096+7266440+5846666+5325863+5755621 + 5656219+2754013+4036957.

(4) 48678+53232+48214+87292+93246+37527+40752 + 53033+35002+15382+1128+5404.

(5) 1541061+1891484+1817881+2265380+2323979+379153+2010958+1476985+1774013+1764304+1076539+847590.

(6) 795824+1049700+1279605+593411+949908+8204 + 208513+1250687+974983+1267694+2038505 + 801986 +608592+1007740+7292.

3 Find the sum of :—

(1) 774145, 999455, 1016062, 1797223, 1854905, 1681274, 74952, 3467035 and 1226612.

(2) 5971096, 1756856, 2124682, 1964909, 2582060, 2633447, 51027, 2280382 and 1721608.

(3) 36530, 4179, 1899, 52773, 130079, 17801, 15235, 118940, 101665, 35584, 5057 and 12162.

(4) 925682143, 832563297, 4327568, 98526342, 753291424, 643263, 71952875 and 2147397.

(5) 441698853, 37519162, 599678437, 4840, 5128697, 20304009, 679821345, 172564 and 4263721.

4. Add together seven hundred and six ; twenty-five thousand and eighty-four ; nineteen thousand and ninety-nine ; seven thousand, four hundred and three ; ten thousand ; ninety-nine thousand and ninety-nine ; and eight hundred and eighteen.

5. Add together five hundred sixty thousand, two hundred and eighteen ; ninety thousand and eighty-five ; three hundred six thousand, five hundred and sixty-seven ; seventeen thousand, eight hundred and nine : seventy-eight thousand and eight ; twelve thousand and fifty ; six hundred twenty thousand, six hundred and twenty-six ; and nine thousand and twelve.

6. Add together seven hundred seven thousand, four hundred and fifty-nine ; ninety eight thousand and seventy-four ; six thousand, eight hundred and seven ; five hundred thousand, three hundred and nine ; seven thousand, nine hundred and seventy-eight ; and nine hundred nine thousand, nine hundred and ninety-nine.

7. Add together fifty-five millions, seven hundred thousand and five; seven hundred millions, nine hundred eight thousand, two hundred and five; seventy-six millions, fourteen thousand and fifty-nine; eight hundred seventy-seven millions, nine hundred two thousand and forty-seven; seven millions, eight hundred four thousand, five hundred and twelve; and five hundred seventy-five millions, eight hundred one thousand and ninety-nine.

8. Add together three hundred nine millions, four hundred seventeen thousand and eighty-seven; six hundred seventy-five thousand and forty-nine; seven thousand ninety-seven millions, eight hundred fourteen thousand, three hundred and five; seventy-nine millions, five hundred four thousand and forty-nine; six thousand seventy-eight millions, four hundred thirty-nine thousand, six hundred and forty-seven; and seven thousand millions, eight hundred seventy-six thousand, four hundred and twenty-nine.

9. Find the amount of five thousand, six hundred and ninety-two; four lacs, thirty-five thousand and eleven; eighty-five lacs, four hundred and ninety-nine; forty-three lacs and forty-three; and five hundred and four.

10 Find the total of six lacs, six thousand and six; four crores, twenty-five lacs, six hundred and thirty-five; nine hundred and ninety-three crores, seventy-five lacs, and seventy-five; eighty-five crores, eighty-five lacs, eighty-five thousand and eighty-five; twenty-three crores, five lacs, fifty-eight thousand and eighty-nine; and four hundred sixty-three crores, nineteen lacs, four thousand and ninety-five.

11. One apple-tree had 816 apples on it, and another had 638; how many apples were on both trees?

12. There are 129 boys, 308 girls, and 60 infants in a school; how many children are there altogether in the school?

13. A train contains 63 first-class passengers, 120 second-class and 154 third-class; how many passengers are in the train?

14. A man has been working five days. On Monday he earns 25 annas, on Tuesday 34, on Wednesday 16, on Thursday 38, and on Friday 27; how much does he earn in the five days?

15. In one book there are 525 pages, in another 144, and in another as many as in the other two; how many pages are there in the three books?

16. Figures were used by the Arabs in the year 890 and decimal fractions were invented 574 years later; in what year were they invented?

17. Five mango-trees produced as follows: the first 657; the second 231 more than the first; the third 892; the fourth 11 more than all the first three; the fifth as many as all the others. How many mangoes were there on all the trees?

18. A *gentleman left* his property by will, thus : to his wife nine thousand and eighty rupees ; to each of his two younger sons, five thousand, eight hundred and ninety-four rupees ; the rest of his property in two equal shares between his three daughters, and eldest son : the eldest son's share was fifteen hundred and twenty rupees more than the mother's share ; what did the gentleman die worth ?

19. Europe contains 3807195 square miles, Asia 17805146, Africa 11647428, America 13542400, and Oceania 3347840, what does this make the extent of the land on the surface of the globe ?

20. The number of Mahomedans in the Burdwan division is 957630, in the Presidency division 4063137, in the Rajshahye division 4885165, in the Dacca division 5531869, and in the Chittagong division 2425610 ; find the total Mahomedan population for Bengal Proper.

21. Bought a lot of ground for 675 rupees ; erected a house upon the same, at a cost for carpenter's works 2540 rupees, mason's works 637 rupees ; painter's works 242 rupees and for grading the lot 293 rupees ; what was the cost of the whole ?

22. A man bought four chests of oranges. In the first chest there were 589 oranges ; in the second 215 more than in the first ; in the third 197 more than in the first ; in the fourth as many as there were in the first and third. How many oranges did he buy ?

23. A man has two thousand and eighty-one sovereigns, three thousand and sixty-eight rupees, one thousand, one hundred and eleven dollars, and two hundred and sixty-nine half-rupees. How many *coins* has he altogether ?

24. *Find the sum of six numbers each equal to 7903856.*

25. A man was born in 1764; in what year was he 83 years old ?

26. In a dictionary there are 869 words beginning with the letter A, 742 with the letter B, 1061 with the letter C, and 1154 with the letter D. How many words begin with the letters A, B, C and D ?

27. Add together the sum of five numbers each equal to 4597, and the sum of four numbers each equal to 89796.

28. January has 31 days, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30 and December 31. How many days are there in the whole year ?

29. From a sum of money I first took away 71407 rupees, and then 90781 rupees and had still 69843 rupees left ; what was the sum ?

30. The number of soldiers in an army of six regiments are 895, 976, 884, 937, 949 and 982 respectively ; the first, third and fifth regiments are respectively joined by 246, 145, and 102 soldiers. Find the whole number of soldiers in the six regiments.

II. SUBTRACTION.

46. **Subtraction** is the method by which we find what number is left when a smaller number is taken from a greater.

The greater number is called the **minuend**, the smaller one the **subtrahend**, and the number left the **remainder**.

47. The number left is the **difference** between the two given numbers; it is also the **excess** of the greater number over the less; it is also the number which must be **added** to the less number to make it equal to the greater. Hence **Subtraction** is sometimes called **complementary addition**.

48. Like *Addition*, *Subtraction* is of two kinds, **simple** and **compound**.

(i) *Simple Subtraction* is one in which the numbers are both *abstract* numbers or both *concrete* numbers of the *same* kind.

(ii) *Compound Subtraction* is the method of finding the difference between two *concrete* numbers of the same kind, but of *different* denominations of that kind.

49. The operation of *Subtraction*, is indicated or expressed by the sign $-$, which is read **minus**, with the use of the sign $=$.

Thus, the *excess* of 7 above 3 will be expressed in the form $7-3=4$, which is read seven *minus* three *equals* four: where the sign $-$ between 7 and 3 denotes the subtraction of the latter from the former and the sign $=$ between 3 and 4 shows the *equality* of the excess to 4.

50. To effect the operation of *Subtraction*, it is necessary to *recollect* the difference of every two numbers less than 20. The following Table, called the **Subtraction Table**, should be committed to memory by beginners.

1 from	2 from	3 from	4 from	5 from	6 from	7 from	8 from	9 from
1 leave 0	2 leave 0	3 leave 0	4 leave 0	5 leave 0	6 leave 0	7leave 0	8leave 0	9leave 0
2...... 1	3...... 1	4...... 1	5...... 1	6...... 1	7...... 1	8..... 1	9..... 1	10.... 1
3...... 2	4...... 2	5...... 2	6...... 2	7...... 2	8...... 2	9..... 2	10.... 2	11.... 2
4...... 3	5...... 3	6...... 3	7...... 3	8...... 3	9...... 3	10.... 3	11.... 3	12.... 3
5...... 4	6...... 4	7...... 4	8...... 4	9...... 4	10..... 4	11.... 4	12.... 4	13.... 4
6...... 5	7...... 5	8...... 5	9...... 5	10..... 5	11..... 5	12.... 5	13.... 5	14 ... 5
7...... 6	8...... 6	9...... 6	10 6	11..... 6	12..... 6	13.... 6	14.... 6	15.... 6
8 7	9...... 7	10..... 7	11..... 7	12..... 7	13 7	14.... 7	15.... 7	16.... 7
9... . 8	10..... 8	11 .. 8	12..... 8	13..... 8	14..... 8	15.... 8	16 ... 8	17.... 8
10..... 9	11..... 9	12..... 9	13..... 9	14..... 9	15..... 9	16.... 9	17.... 9	18.... 9
11 . ..10	12.....10	13.....10	14 10	15.....10	16.....10	17....10	18....10	19... 10

This Table can easily be extended further; for instance, since 2 from 3 leave 1, 2 from 13, *i.e.* from $3+10$, leave $1+10$, or 11, the result being 10 more than in the corresponding case in the Table. Also since 7 from 15 leave 8, 7 from 45, *i.e.* from $15+30$, leave $8+30$, or 38, the result leaving 30 more than in the corresponding case in the Table. Also since 9 from 14 leave 5, 9 from 54, *i.e.* from $14+40$

leave $5+40$, or 45, and 9 from 99, *i.e.* from $19+80$, leave $10+80$ or 90; and so on.

Examples VII. (Mental Subtraction.)

1. (1) Take 2 from 4, from 7, from 11, from 6, from 12, &c.
(2) Take 3 from 4, from 3, from 6, from 8, from 13, &c.
(3) Take 4 from 6, from 9, from 13, from 15, from 19, &c.
(4) Take 8 from 12, from 15, from 19, from 21, from 25, &c.
(5) Take 9 from 15, from 18, from 20, from 24, from 36, &c.

2. (1) Subtract 6 from 20, 47, 32, 70, 63, 55, 81, 71 and 99.
(2) Subtract 7 from 18, 22, 49, 33, 84, 51, 94, 88 and 38.
(3) Subtract 5 from 18, 25, 53, 61, 70, 82, 67, 93 and 90.

3. How many does

(1) 9 leave from 15; 5 from 14; 7 from 12; 9 from 71; 8 from 21?
(2) 7 leave from 44; 8 from 38; 9 from 88; 6 from 94; 5 from 47?

4. Find the difference between :—

(1) 13 and 18; 3 and 14; 20 and 25; 30 and 45; 15 and 11.
(2) 89 and 47; 46 and 12; 34 and 68; 14 and 31; 14 and 95.

5. What must be added to 11 to make 15, 7 to make 18, 6 to make 15, 4 to make 11, 9 to make 17, 21 to make 49, 31 to make 44 and 30 to make 82?

6. By how much does 13 exceed 7, 17 exceed 8, 19 exceed 8, 26 exceed 14, 29 exceed 13, 69 exceed 26, 95 exceed 32, 98 exceed 36, 82 exceed 64, and 89 exceed 72?

7. Count by decrements of 3, 5 and 7, commencing at 100.

8. How much is 33 less 7; 84 less 5; 49 less 6; 67 less $5+2$; 96 less $4+0+4$; and $67+16$ less $15-4$?

9. Take $5+3$ from 11; $7+2$ from 17; 12 from $14+11$; 25 from $48+11$; $9+6$ from $12+5$; $3+8$ from $2+9$; and $1+4$ from $2+7$.

10. How many times can 5 be taken from 15; 6 from 18; 9 from 27; and 12 from 48?

11. A girl has 8 oranges. She gives 3 to her sister. How many has she left?

12. Shyam has 6 pice. He pays 1 pice for a top, 2 pice for a whistle, and 2 pice for a kite. How many has he over?

13. A boy has 18 pice in his pocket. He loses 7 and spends 4. How many pice has he left?

14. If you buy 18 yards of ribbon, and find that you have 3 yards too much, how many yards should you have bought?

15. A man planted 25 trees; 8 of them died. How many lived?

16. Jadu has 19 apples, and Bhuban has 8. How many has Jadu more than Bhuban?

17. I bought 6 pice worth of apples, and 4 pice worth of pears. What money had I over out of 15 pice?

18. A baker's boy sets out with 21 rolls. He leaves 5 in one house, 4 in another, 6 in a third and 5 in a fourth. How many rolls has he left?

19. Ram is 19 years old; Gopal is 8 years old. How many years is Gopal younger than Ram?

20. A man had 26 sheep; he sold 10, and 6 were stolen. How many were left?

SIMPLE SUBTRACTION.

51. The following are the Rules for the subtraction of large numbers.

(i) When none of the figures of the *Subtrahend* exceeds the corresponding figures of the *Minuend*.

RULE. Place the less number under the greater, so that units may stand under units, tens under tens, hundreds under hundreds, and so on; then draw a line below the lower number. Begin at the units' place and subtract each figure in the lower line from the corresponding figure in the upper, taken by itself, and put down the remainder below the line just drawn, units under units, tens under tens, hundreds under hundreds, and so on. The entire difference or remainder, so put down, will be the **difference** or **remainder** of the proposed numbers.

Ex. 1. Subtract 425 from 1679.

$$\begin{array}{r} 1679 \\ 425 \\ \hline 1254 \end{array}$$

Place the smaller number 425 under the greater 1679, and draw a line below it. First take 5 from 9, and place the difference **4** under the units' figure below the line drawn; next take 2 from 7 and set down the remainder **5** in the tens' place below the line; next take 4 from 6 and put down the difference **2** in the hundreds' place under the line. Lastly bring down **1** since there is nothing below it. Thus the remainder is 1254.

Ex. 2. Subtract 5634 from 9657.

$$\begin{array}{r} 9657 \\ 5634 \\ \hline 4023 \end{array}$$

As before, put 5634 below 9657, and draw a line. Take 4 from 7, the remainder is **3**; 3 from 5 leaves **2** as remainder; 6 from 6 leaves *nothing* or **0** as remainder; lastly 5 from 9 leaves **4** as remainder. Thus the entire remainder is 4023.

(ii) If the units of any order in the *Subtrahend* exceed those of the *Minuend*.

In this case we avail ourselves of the following principle, usually termed **borrowing**:—"*The Minuend and Subtrahend may be increased by the same number without altering their difference.*

Hence we may increase the number of units in any order of the *Minuend* by 10, if we increase that of the next higher order in the *Subtrahend* by 1.

RULE. Place the numbers as in (i) and draw a line below. Begin at the units' figure, but if the said figure in the lower line exceed that in the upper, increase the upper figure by **ten** and then subtract the lower figure from the upper *figure thus increased.* Put down the remainder as in (i), and **carry 1** to the *next higher* figure in the lower line. Proceed with the remaining figures as in (i), observing that whenever **ten** units have been **borrowed**, or added to the upper line, **one** unit must be **carried**, or added to the next higher denomination in the lower line.

Ex. Subtract 5634 from 7483.

7483
5634
1849

Since 4 is greater than 3, 3 is made 13 by adding 10 to it; from 13 take 4 and put down the remainder **9**. Now add 1 to the next lower figure 3; the sum is 4, which subtracted from 8 leaves 4. Put down **4**. Next 6 is greater than 4; so 10 is added to 4, and from the sum 14, subtract 6. The remainder is **8**. Lastly, add 1 to the next lower figure 5; the sum is 6, which subtracted from 7 leaves the remainder **1**. Thus the difference is 1849.

52. In the preceding Example, the same result would be obtained, if we have **borrowed** ten units of the *next* denomination from the *Minuend*, as is usual in France. For whether we suppose 1 to be *added* to the *lower* line, or *subtracted* from the *upper*, the remainder will evidently be the same on both suppositions. In *practice*, however, the former method is convenient.

53. Subtraction being the *reverse* of Addition, it follows, that if we add together the remainder and the less of the numbers proposed, the sum ought to be equal to the greater; and the operation of subtraction may be *presumed* to be correct when this is the case. Another method of testing the correctness of the result is this: *Cast out the nines* from the sum of the digits in the *minuend*, and also from the sum of the digits in the *subtrahend* and the *remainder*; if the two results coincide, the operation may be *presumed* to be correct.

Examples VIII.

1. Perform the following subtractions :—

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
59	79	85	70	98	428	526	702	650	912
42	45	69	54	89	274	317	504	56	707

(11)	(12)	(13)	(14)	(15)	(16)	(17)
7046	7825	4286	9821	8943	6789	5959
807	4976	3097	6935	4573	697	999

(18)	(19)	(20)	(21)	(22)	(23)	(24)
23456	56785	76325	62831	708001	542657	204087
4987	39876	59876	48072	39508	214958	76498

(25)	(26)	(27)	(28)	(29)	(30)
6829019	1531335	1287657	78602045	493827156	8539410
6599341	1456516	1000958	59763567	246913578	3438148

(31)	(32)	(33)	(34)	(35)	(36)
74147863	370489000	68539582	650030042	13456789	352100435
9701297	269579235	45947895	94090096	8765432	79213679

(37)	(38)	(39)	(40)	(41)
777722233	909009099	453870250	1000100010	765007005
38945635	842248484	39004065	999999999	400827054

2. Find the difference between :—

(1) 75011 and 6012 ; 3095 and 80131 ; 8019 and 18018.

(2) 110111 and 11012 ; 916553 and 1683452 ; 251483 and 77777.

(3) 20470932 and 80476325 ; 613020303 and 420536075.

(4) 12785462 and 1842567 ; 92603745298 and 25402987609.

3. Find the values of :—

(1) 5124060 − 5083959 ; 1056789 − 967899 ; 4060124 − 3951035.

(2) 6284503 − 4995629 ; 7014062 − 6985172 ; 6001004 − 5480018.

(3) 1010102 − 956784 ; 3601020 − 3598642 ; 5490206 − 4301218.

(4) 500120456 − 499296845 ; 4060213697 − 2846545789.

4. What is the excess of 12795 above 8096? How much greater is 2600509050 than 433418175?

5. By how much is 87719808 greater than 68440260?

6. What is the excess of 9497605 above 8688516?

7. By how much is a lac greater than ninety-five thousand, nine hundred and nine, and less than a million?

8. What number must be added to each of the following numbers to make the sum equal to ten millions?—8423458, 457685, 9032401, 7612345, 5040289, 904507 and 9003465.

9. What number must be taken from each of the numbers 999999, 425078, 8725900, 6420587 and 428905 to leave 245678?

10. Required the excess of three hundred five million, two hundred and four, above seventy-five thousand, three hundred and eighty-six.

11. From seven hundred eighteen million, fourteen thousand and fifty-six take ninety-eight million, seven hundred three thousand, six hundred and seventeen.

12. Subtract thirteen lacs, four thousand and fifty-six from seventy-five crores, two hundred and three.

13. Take eleven thousand eleven hundred and eleven from twelve thousand one hundred and twelve.

14. A box contains 4074 oranges; 2386 of them were sold. How many remained?

15. In 1882 a man was 86 years old. In what year was he born?

16. William the Conqueror began to reign in the year 1066; how many years elapsed between that period and the battle of Waterloo, which was fought in 1815?

17. A tea merchant has 4680 maunds of tea. He sells 1000 maunds to one customer, 999 to a second, and 354 to a third. How many maunds of tea has he left?

18. Jadu has 829 marbles; he gives away 618 and then buys 206. How many has he now?

19. A man was born in 1845; what was his age in 1896?

20. A man was 25 years old at the birth of his son; what is the son's age when the father is 74 years old?

21. A merchant bought a certain quantity of goods for 6246 rupees and sold them for 7137 rupees. How much did he gain?

22. One mountain is 15732 feet high, another is 3571 feet high. How much is the one higher than the other?

23. A railway receives in a year 2684040 rupees. Of this sum 1786064 rupees are for goods and the rest for passengers. How much was received for passengers?

24. Of 17254120 Hindu population for Bengal Proper, 8624022 are males and the rest females; find their number.

25. Queen Victoria was born in 1819. How old was she in 1895?

26. Three boys A, B and C at marbles won together 105; if the numbers that B and C won be added together they will make 82, and of this number B won 47. What did each boy win at play?

27. A gentleman gave 12462 rupees for a house and some land; the house alone was worth 9375 rupees; what was the value of the land?

28. The answer to a subtraction sum is 1026 and the top line 4387. What is the second line?

29. A man has 826 sovereigns in one box and 682 in another; he takes 176 from the former and puts them in the latter. How many are in each box now?

30. When will the Prince of Wales, who was born in the year 1841, be as old as the Queen was in the year 1878, who was born in the year 1819? How old will the Queen then be?

54. A number preceded by the sign + (*plus*), is called a **positive** number, and a number preceded by the sign − (*minus*) is called a **negative** number. When *no* sign is affixed to a number, it is considered as *positive*.

55. An **expression** is one in which two or more numbers are connected by the sign + or − ; and the numbers thus connected are called its **terms**.

Thus, $4-3+2+1$ is an *expression* ; 4, 3, 2 and 1 are *terms* ; 4, 2 and 1 are *positive* ; and 3 is *negative*.

56. If an addition and a subtraction, or *vice versâ*, have to be performed in succession, we may invert their **order**, provided the resulting expression be possible.

Thus, since $9+5-3=11$ and $9-3+5=11$; $\therefore 9+5-3=9-3+5$.

57. Hence it is easily shewn that additions and subtractions may be performed in any *order* ; and that the value of an expression made up of additions and subtractions may be obtained by taking the *difference* of the sums of *all* the positive and the negative numbers separately.

Ex. Find the value of $365-101+2+18-267$.

Here, $365+2+18=385$; $101+267=368$; also $385-368=17$.

Therefore the value reqiured $=\underline{17}$.

58. The **complement** of a number is its defect from 10 units of the number's highest order.

Thus, the *complement* of 6 is 4 and of 659 is 341, for $10-6=4$, and $1000-659=341$.

Examples IX.

Find the value of each of the following expressions :—

1. $16-4+12-25+7-2$. **2.** $751-9+1786-235-12-672$.

3. $18+6-31+537-628-19+209$. **4.** $467-84+49-36$.

5. $1246-362-371+495+156-386+256$.

6. $3210-67+59+401-342+491-382+459-87$.

7. What number must be added to the sum of 750 and 3287 to make the result equal to the sum of 505, 650, 19 and 9003?

8. What is the difference between $23047+175-368+495-132$ and $10000-8406-704+7305$?

9. From the difference between 3285 and 456 subtract the difference between 19011 and 17455.

10. A basket contained oranges, nuts and mangoes, in all 1769; there were 1696 oranges and nuts, and 1262 nuts and mangoes. How many more nuts were there than oranges?

11. Gopal goes up 16 steps of a ladder, which has 45 steps, then down 7 steps, then up 10, then down 2, then down 4, then up 11, then down 9, then up 7, then up 5, then down 8; what step from the top and bottom will he then be standing upon?

12. Write down the complements of 4; 7; 43; 86; 574; 998.

III. MULTIPLICATION.

59. **Multiplication** is the method by which we find the sum of a given number repeated as many times as there are units in another given number.

60. The number to be *repeated* is called the **multiplicand**, the other the **multiplier**, and the sum found the **product**. The *multiplicand* and the *multiplier* are both called **factors** or *makers* of the *product*.

61. From the mode in which results are obtained in multiplication, it is manifest that Multiplication is merely a *compendious* method of performing the addition of two or more *equal* numbers.

Thus, to multiply 7 by 4 being the sum arising from the number 7 repeated *four* times, we may determine the product as $7+7+7+7$ or 28 Here 7 is the *multiplicand*, 4 the *multiplier*, and 28 the *product*; also 7 and 4 are *factors* of 28.

62. *Multiplication* is either **simple** or **compound**.

(i) When the multiplicand is either an *abstract* number, or a *concrete* number of *one* denomination, it is called *Simple Multiplication.*

(ii) When the multiplicand is a *concrete* number of *more than one* denomination, but all of the *same* kind, it is called *Compound Multiplication.*

63. The operation of *Multiplication* is expressed by the sign $\times$, which is read **into** or **times** or **multiplied by**. Sometimes a dot is used instead of a $\times$.

Thus, 5×7 denotes the product of 5 and 7, and is read 5 *into* 7, or 5 *times* 7, or 5 *multiplied by* 7. Also $5.7=5\times7$. This must not be confounded with a dot placed near the top, as $5{\cdot}7$. (Art. 332.)

64. The operation intended by the word *Multiplication*, is defined in Art. 59; and in the first place we will shew that the conclusions which it leads to, may be safely depended upon, as far as the *order* of the *factors* may influence the *product*.

Thus, to multiply 7 by 5, write down 1 in a horizontal line 7 times, and repeat this line 5 times. The sum of each horizontal line is 7, and there are 5 such lines, therefore the sum of all the ones is 7×5. Again, the sum of each vertical line is 5 and there are 7 such lines, therefore the sum of all the ones is 5×7; that is, 7×5 is the same as 5×7.

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

By reasoning of this kind, it is made to appear that the product has a *similar* or *symmetrical* relation to both its factors, because it remains the same if we interchange the *Multiplicand* and the *Multiplier*.

65. A number multiplied by 0 is 0, as also 0 multiplied by a number is 0; for a number taken *no* number of times is *nothing*, also *nothing* taken any number of times is *nothing*.

Thus, $5 \times 0 = 0$, as also $0 \times 5 = 0$.

66. The following Tables, which are termed the **Multiplition Tables**, present at one view the product arising from the multiplication of any two numbers not exceeding 20; and though the products of the *nine digits* form the *basis* of those of all numbers whatever, it is here extended for the sake of *practical* convenience, and should be carefully committed to memory.

Table 1.

	1	2	3	4	5	6	7	8	9	10
Once ...	1	2	3	4	5	6	7	8	9	10
Twice ...	2	4	6	8	10	12	14	16	18	20
Thrice ...	3	6	9	12	15	18	21	24	27	30
4 times ...	4	8	12	16	20	24	28	32	36	40
5 times ...	5	10	15	20	25	30	35	40	45	50
6 times ...	6	12	18	24	30	36	42	48	54	60
7 times ...	7	14	21	28	35	42	49	56	63	70
8 times ...	8	16	24	32	40	48	56	64	72	80
9 times ...	9	18	27	36	45	54	63	72	81	90
10 times ...	10	20	30	40	50	60	70	80	90	100

Table 2.

	1	2	3	4	5	6	7	8	9	10
11 times ...	11	22	33	44	55	66	77	88	99	110
12 times ...	12	24	36	48	60	72	84	96	108	120
13 times ...	13	26	39	52	65	78	91	104	117	130
14 times ...	14	28	42	56	70	84	98	112	126	140
15 times ...	15	30	45	60	75	90	105	120	135	150
16 times ...	16	32	48	64	80	96	112	128	144	160
17 times ...	17	34	51	68	85	102	119	136	153	170
18 times ...	18	36	54	72	90	108	126	144	162	180
19 times ...	19	38	57	76	95	114	133	152	171	190
20 times ...	20	40	60	80	100	120	140	160	180	200

Table 3.

	11	12	13	14	15	16	17	18	19	20
11 times ...	121	132	143	154	165	176	187	198	209	220
12 times ...		144	156	168	180	192	204	216	228	240
13 times ...			169	182	195	208	221	234	247	260
14 times ...				196	210	224	238	252	266	280
15 times ...					225	240	255	270	285	300
16 times ...						256	272	288	304	320
17 times ...							289	306	323	340
18 times ...								324	342	360
19 times ...									361	380
20 times ...										400

67. In Multiplication, one of the factors, namely, the *multiplier* must necessarily be an *abstract* number.

Thus, if the factors are 7 rupees and 8 rupees, we could easily multiply together the abstract numbers 7 and 8, whose product is 56; but the *denomination* of this result as the product of 7 rupees and 8 rupees cannot be ascertained, and the *operation* is altogether *absurd*. Hence, the multiplication of *concrete* numbers *as such*, is altogether impossible. We can, however, multiply 7 rupees by the *abstract* number 8, and interpret the product 56 rupees as how many rupees there are in 8 times 7 rupees.

It is also absurd to speak of 7 multiplied by 8 rupees, but not of 7 times 8 rupees. Of the two factors that make 56 rupees, one must be *abstract*, the other *concrete*, but it does not matter which, for 7 times 8 rupees = 8 times 7 rupees. In no case do we multiply by rupees.

In certain cases, however, as will be seen hereafter, the meaning of multiplication may be so extended as to include some concrete multipliers. (Art. 378.)

Examples X. (Mental Multiplication.)

1. How much is

(1) 7 times 6; 11 times 8; 9 times 7; 11 times 11; 8 times 9; 7 times 15?
(2) 10 times 3; 9 times 12; 7 times 7; 12 times 14; 4 times 18; 6 times 8?
(3) 8 times 11; 5 times 12; 11 times 12; 5 times 17; 6 times 19?

2. What is the product of—

(1) 13 by 12; 8 by 9; 15 by 14; 18 by 17; 0 by 4; 12 by 4; 11 by 15?
(2) 15 by 19; 17 by 12; 6 by 0; 0 by 11; 20 by 15; 16 by 18; 14 by 18?

3. How many are 16×19; 13×15; 19×19; 12×12; 17×19; 20×13; 13×14; 14×18; 17×15; 15×20?

4. One book has 12 pages. How many pages will 8 such books have?

5. There are 11 boys in a class; each works 8 sums in an hour. How many sums do they all work together?

6. If one knife costs 14 pice, how many pice will 9 knives cost?

7. If there are 9 desks in a room, and 6 boys at each desk, how many boys will there be in the room?

8. What will 9 stools cost at 9 rupees each?

9. How many trees are in 18 rows, each row having 9 trees?

10. If I give 5 boys 8 marbles each, how many will be left out of 81, and out of 100?

11. A boy wrote 12 lines of dictation and there were 9 words in a line; how many words did he write altogether?

12. How many more are 9 tens than 4 twenties? 10 tens than 6 tens? 9 nines than 5 nines?

13. In one foot there are 12 inches; how many inches are there in 6, 8, 9, 11 feet?

14. There are 7 days in a week; how many days are there in 8, 11, 12 weeks?

15. A boy walks 3 miles in an hour. How many miles will he walk in 6 hours?

16. How many legs have 14 horses? How many feet have 9 ducks?

17. Ram is 8 years of age; his father is 4 times as old. How old is his father?

18. A man walked 4 miles in one hour. How many miles would he walk at the same rate in 16 hours?

19. Multiply 8 by 4 and take away 10; how much remains?

20. A window has 9 rows of panes, and 12 panes in each row. How many panes are there in the window?

SIMPLE MULTIPLICATION.

68. When the Multiplier does not exceed 20, the multiplication is called **Short Multiplication.**

69. When the Multiplicand is a large number and the Multiplier a number of one figure, we have the following Rule :—

RULE. Write down the multiplier under the units' figure of the multiplicand, and draw a line underneath. Begin at the units' figure of the multiplicand, and multiply each figure in succession by the multiplier, setting down and *carrying* precisely as in Addition.

Ex. Multiply 3468 by 7.

$$\begin{array}{r} 3468 \\ 7 \\ \hline 24276 \end{array}$$

Here 7 times 8 is 56. Set down **6** in the units' place and carry 5; 7 times 6 is 42, and $42+5=47$; set down **7** in the tens' place and carry 4; 7 times 4 is 28, and 4 carried is 32; put down **2** in the hundreds' place and carry 3; lastly 7 times 3 is 21, and $21+3=24$; set down **24**. The product is therefore 24276.

70. Writing down the *local* values of the figures, the process will stand thus :—

$$\begin{aligned} 3468 &= 3 \text{ thousands} + 4 \text{ hundreds} + 6 \text{ tens} + 8 \text{ units}; \\ \therefore 3468 \times 7 &= 7\times 3 \text{ thousands} + 7\times 4 \text{ hundreds} + 7\times 6 \text{ tens} + 7\times 8 \text{ units}, \\ &= 21 \text{ thousands} + 28 \text{ hundreds} + 42 \text{ tens} + 56 \text{ units}, \\ &= 21 \text{ thousands} + 28 \text{ hundreds} + 47 \text{ tens} + \mathbf{6} \text{ units}, \\ &= 21 \text{ thousands} + 32 \text{ hundreds} + \mathbf{7} \text{ tens} + \mathbf{6} \text{ units}, \\ &= \mathbf{24} \text{ thousands} + \mathbf{2} \text{ hundreds} + \mathbf{7} \text{ tens} + \mathbf{6} \text{ units}, \\ &= \mathbf{24{,}276}. \end{aligned}$$

71. When the multiplier is greater than 9 but does not exceed 20, the multiplication can be effected easily in one line, by the help of the Rule in Art. 69.

Ex. 1. Multiply 59867 by 15.

59867
15
898005

Here, 15 times 7 is 105; put down **5** *and carry 10;* then 15 times 6 is 90, and $90+10=100$; put down **0** and carry 10; then 15 times 8 is 120, and $120+10=130$; put down **0** and carry 13; then $15\times 9=135$ and $135+13=148$; set down **8** and carry 14; lastly, $15\times 5=75$ and $75+14=89$; set down **89**. Thus the product is 898005.

Ex. 2. Multiply 350974 by 18.

350974
18
6317532

Here, $18\times 4=72$; put down **2** and carry 7; then $18\times 7=126$ and $126+7=133$; put down **3** and carry 13; then $18\times 9=162$, and $162+13=175$; set down **5** and carry 17; then $18\times 0=0$ and $0+17=17$; place **7** and carry 1; then $18\times 5=90$, and $90+1=91$: put down **1** and carry 9; lastly $18\times 3=54$, and $54+9=63$; put down **63**. Thus the product is 6317532.

72. When the multiplier is a simple number followed by **one** or **more** ciphers, we have the following Rule :—

RULE. Multiply the multiplicand by the simple number and to the right of the product place as many ciphers as there are ciphers to the right of the multiplier.

Ex. Multiply 5867 by 70; and by 18000.

5867
70
410690
5867
18000
105606000

(1) *Here* $5867\times 70=5867\times 7$ *tens,*
$=41069$ tens,
$=410690$.

(2) Here $5867\times 18000=5867\times 18$ thousands,
$=105606$ thousands,
$=105606000$.

Examples XI.

1. Multiply 284 by 2; 1475 by 3; 2867 by 4; 9048 by 2; 6057 by 4; 80965 by 5; 49508 by 8; 33069 by 7; 91537 by 12.

2. Multiply

(1) 5849 separately by 2, 3, 4, 5, 6, 7, 8, 9 and 11.
(2) 38476 separately by 3, 5, 7, 9, 11, 13, 14, 15 and 19.
(3) 3870492 separately by 2, 5, 3, 7, 4, 9, 6, 8, 11, 12 and 15.
(4) 6508794 separately by 8, 7, 9, 11, 13, 15, 17 and 19.
(5) 987654321 separately by 2, 3, 4, 5, 6, 7, 8, 9, 11 and 12.

3. Find the values of

(1) 48508×8; 69360×6; 49216×11; 69432×12.
(2) 38476×9; 876549×12; 378125×16; 456932×18.
(3) 43275×17; 46059×16; 30748×19; 600954×20.
(4) 4609758×14; 4609758×19; 56380477×18.

4. Multiply

(1) 980989 separately by 10, 100, 1000 and 10000.
(2) 72051 separately by 30, 40, 70, 90 and 100.

(3) 91357 separately by 20, 200, 300, 5000 and 9000.
(4) 790785 separately by 120, 1500, 17000, 1300 and 190000.
(5) 900968 separately by 800, 1600, 14000 and 180000.

5. By how much does 18 times 1118 exceed 17 times 1010?

6. Find the sum of 19 times 2304 and 15 times 2045.

7. Multiply 123456789 separately by 1, 2, 3, 4, 5, 6, 7, 8 and 9, and add the several products together.

8. And estate contains 45068 bighas. Each bigha is worth 18 rupees. What is the value of the whole estate?

9. A railway train consists of 17 trucks. Each truck carries 12644 maunds weight. How many maunds does the whole train carry?

10. A man bought 305 cows at 12 rupees a head, and having spent on them for food 95 rupees, sold them at 16 rupees a head; what did he gain by his bargain?

11. Ram bought of Judu 15 books at 13 annas each, and Judu bought of Ram 19 books at 19 annas each; how many annas had Judu to give to Ram?

12. Two persons start from the same place, and travel (i) in the same direction, (ii) in opposite directions. One travels at the rate of 93 miles a day and the other at 79 miles a day. How far will they be apart at the end of 7 days?

---

73. When the multiplier is greater than 20, the multiplication is called **Long Multiplication.**

74. When the multiplicand and multiplier are both large numbers, we have then the following general Rule :—

RULE. Place the multiplier under the multiplicand, so that units of the same order may be under one another and draw a line under the whole. Begin at the units' figure of the multiplier, and multiply by each of its figures in order, writing down each *partial* product so that its first figure shall be under the figure of the multiplier that produces it. *Add* together these *partial* products and the *sum* is the product required.

Ex. Multiply 7823 by 645.

```
   7823
    645
  39115
 31292
46938
5045835
```

Here, first multiply 7823 by 5 and set down the product 39115. Then multiply 7823 by 4, and put down the product 31292, so that 2 may come under the tens' place of the first partial product, 9 in the hundreds' place and so on. Lastly, multiply 7823 by 6 and set down the product 46938 so that 8 may be in the hundreds' place of the first partial product, and so on. Add up the three lines of figures already obtained and their sum 5045835 is the required product.

75. The reasoning above employed can be rendered more clear by the following consideration.

Since the above product is the sum of 7823 repeated 645 times

and 645 = 600+40+5 ; therefore by the use of Arts. 69 and 72 we have the following process :—

7823	645 = 600+40+5.
645	
39115	7823 × 5 = 39115
312920	7823 × 40 = 312920
4693800	7823 × 600 = 4693800
5045835	5045835

76. If one or more of the figures of the multiplier be 0, it is evident that the corresponding *partial product* will be 0 (Art. 65) and the lines may be entirely omitted after placing down each 0 *once*, to give the proper value to the product arising from the next figure.

Ex. Multiply 4968 by 709.

4968	Here, in multiplying by 709, we first multiply by 9 and put down the result ; then when we multiply by 7, we really multiply by 700, but not by 70, and so place the first figure of the second partial product under the hundreds' figure of the first, affixing one cipher in the ten's place.
709	
44712	
347760	
3522312	

77. If the multiplicand, or multiplier, or both, end in ciphers, the ordinary process of Multiplication may be *shortened* or *facilitated* by the following Rule :—

RULE. Suppose the ciphers at the right of multiplicand, or multiplier or both omitted, find the product of the resulting numbers, and to the right of this product place as many ciphers as were supposed to be omitted in multiplicand, or multiplier or both together.

Ex. Multiply 47600 by 47 ; 257 by 64000, and 7900 by 83000.

Here, omitting the ciphers on the right, or *supposing* them to be omitted, we have

47600	257	7900
47	64000	83000
3332	1028	237
1904	1542	632
2237200	16448000	655700000

where the ciphers are *annexed* at last to the right of the products obtained in the ordinary way, to give the other figures their proper local values.

Thus, in the first case, when we multiply 6 by 7, we really multiply 600 by 7, and 600 multiplied by 7 gives 4200 ; therefore two ciphers are annexed after 2 in the product.

In the second case, when we multiply 7 by 4, we really multiply 7 by 4000, and 7 multiplied by 4000 gives 28000 ; therefore three ciphers are annexed after 8 in the product.

In the third case, when we multiply 9 by 3, we really multiply 900 by 3000, and 900 multiplied by 3000 gives 2700000, therefore five ciphers are annexed after 7 in the product.

78. If the multiplicand and multiplier change places, the product must be the *same* as before, otherwise the *same* numbers would have *more* products than *one* (Art. 64). Hence, it is convenient to make the larger number the multiplicand and the smaller number the multiplier.

79. The following **Proofs** are generally adopted in Multiplication.

(1) Interchange multiplicand and multiplier; the product ought to be the *same*.

(2) **By Casting out the nines.** We cast the nines out of a number thus: add together all its figures, omitting every 9, and if the sum be greater than 9, replace it by the sum of its figures, and if the new sum be greater, replace it by the sum of its figures, and so proceed till we have a sum less than 9.

Cast the nines out of multiplicand and *multiplier*. Multiply the results, and *cast the nines out* of their product, noting the new result: now *cast the nines out* of the *product*, and if the result *coincide* with the one previously noted, we presume that the work is *correct*.

80. We may mention here that the above test fails in the *three* following cases:—

(1) If the order of figures in the product be misplaced, as 86 for 68.

(2) If the errors made compensate each other, so far as the sum of the digits is concerned, as 65 for 83.

(3) If 9 be written for 0, or 0 for 9, or either be inserted or omitted too often; as 59 for 50, or 597 for 57, or 708 for 78, and so on.

Ex. Multiply 5867 by 3478, annexing the proofs.

5867...8		Multd. and Multr.	3478
3478...4		5	5867
——.5			
46936			24346
41069	Multd. 8	Multr. 4	20868
23468			27824
17601		5	17390
20405426...5		Prodt.	20405426

Beginning at the left hand, we cast the *nines* out of the

(1) Multiplicand thus:—13, 19, 26; replace 26 by the sum of 2 and 6 or **8**.

(2) Multiplier thus:—7, 14, 22; replace 22 by the sum of 2 and 2 or **4**.

Now multiply 8 by 4, giving 32, which replace by the sum of 3 and 2 or **5**; and note this result.

(3) Product thus:—6, 11, 15, 17, 23; replace 23 by the sum of 2 and 3 or **5**.

As this result coincides with the previous one, we presume the work is correct.

Examples XII.

1. Multiply 946 by 61 ; 869 by 89 ; 917 by 46 ; 909 by 88 ; 463 by 608 ; 417 by 739 ; 3259 by 497, and 692 by 73.

2. Multiply

(1) 47691 by 27 ; 28573 by 35 ; 716281 by 48 ; 39265 by 39.
(2) 129385 by 66 ; 138476 by 81 ; 480765 by 97 ; 829741 by 59.
(3) 8241763 by 123 ; 921846 by 158 ; 827941 by 376.
(4) 5086927 by 495 ; 254037 by 2980 ; 4785328 by 7802.
(5) 56380477 separately by 35, 48, 72 and 132.
(6) 67836479 separately by 356, 4378 and 78539.
(7) 70870096 separately by 404, 3009 and 900807.
(8) 279420 by 7350 ; 678000 by 87600 ; 80108 by 7770.
(9) 56348 by 50601 ; 876000 by 678000 ; 896385 by 6687400.
(10) 57483000 by 40, 900, 430, 24500 and 4670000.

3. Find the product of :—

(1)	(2)	(3)	(4)	(5)
45678	3124791	436712	1100785	4532815
9128	89023	45678	71053	751283
(6)	(7)	(8)	(9)	(10)
447002	8913243	110375009	110200570	275642
578648	234567	198075	200570	125255
(11)	(12)	(13)	(14)	(15)
447529123	4465348	79094451	84964270	123456789
8901234	7000608	7640950	8743590	123456789

4. Find the values of :—

(1) 704745×615 ; 469830×369 ; 391525×861.
(2) 1174575×2214 ; 3523725×2583 ; 926196×7896.
(3) 920685×7098 ; 4465348×7000608 ; 7650329×600509.
(4) $400905703206 \times 7008130502$; $8070906050493 \times 6403200905$.
(5) $6709802607508 \times 2005032057$; $1310275031496 \times 20456300170$.

5. What is the difference between 23456 multiplied by 996, and the remainder in subtracting 4 times 23456 from 23456000 ?

6. A bigha of land costs 784 rupees, what will 203 bighas cost ?

7. If there are 432 pages in a book, how many will there be in 80704 such books ?

8. If I give 125 boys 79 marbles each, how many shall I have left out of 10000 ?

9. 79432 copies of a newspaper are printed daily. How many are printed in a year of 314 days ?

10. The cost of constructing a Railway is 61303 rupees per mile ; what will 701 miles cost ?

11. An army consists of 295 battalions of 34618 men each ; what is the whole number of men in the army ?

12. In a town there are 734 houses; 345 of them contain, on an average, 11 persons each and the rest 15 each. How many persons reside in the town?

13. If a master employs 73 workmen, each of whom receives 34 rupees per month, how many rupees does he pay away per month?

14. If of 20000 shells used in war, 3648 are 36 pounders, 11275 are 24 pounders, and the rest 18 pounders; what is the total weight (in pounds) of the whole?

15. A clock strikes 114 times in a day. How often will it strike in 365 days?

16. A town has 436 streets. Each street contains on an average 6422 inhabitants. What is the population of the town?

17. A directory contains 798 pages. There are 72 names in each page. How many names are in the directory?

18. 343 paving-stones are required for every yard in a street. There are 18742 yards in the street. How many paving-stones will the whole street require?

19. The distance of the Earth from the Sun is found to be 11608 times the Earth's equatorial diameter, and that diameter is 7926 miles. Required the distance between the Earth and the Sun.

20. India contains about 1466576 square miles and the population is reckoned to be about 189 persons to every square mile; what is the whole population of the country?

---

31. To find the product of *more* than two numbers, multiply the product of two of the numbers by the third, the result by the fourth, and so on. The final result is called the **continued product** of so many **factors**.

Thus, the *continued product* of 3, 5, 8 and 47 $= 3 \times 5 \times 8 \times 47 = 15 \times 8 \times 47 = 120 \times 47 = 5640$, and 3, 5, 8 and 47 are *factors* of 5640.

32. The continued product of any numbers will remain the *same*, however we may change the *order* of its factors.

Thus, since $4 \times 2 \times 5 \times 7 \times 3 = 8 \times 5 \times 7 \times 3 = 40 \times 7 \times 3 = 280 \times 3 = 840$, and $5 \times 4 \times 2 \times 3 \times 7 = 20 \times 2 \times 3 \times 7 = 40 \times 3 \times 7 = 120 \times 7 = 840$;

$$\therefore\ 4 \times 2 \times 5 \times 7 \times 3 = 5 \times 4 \times 2 \times 3 \times 7.$$

Ex. Find the continued product of 3471, 7 and 52.

$$\begin{array}{r} 3471 \\ \underline{7} \\ 24297 \\ \underline{52} \\ 48594 \\ \underline{121485} \\ 1263444 \end{array}$$

Here, we first multiply 3471 by 7, and the product is 24297; again multiply 24297 by 52, and the product is 1263444; thus the continued product of the several factors is 1263444.

83. If *one* or *more* of the factors in any continued product be 0, the whole product is 0. (See Art. 65.)

Examples XIII.

1. Find the continued products of :—

(1) 4, 7, 25. (2) 13, 15, 17. (3) 18, 19, 20. (4) 407, 18, 5.
(5) 729, 8, 61. (6) 7184, 6, 12. (7) 35, 32, 14, 29. (8) 35, 29, 43, 87.
(9) 33, 13, 15, 4, 56. (10) 27, 57, 35, 1277. (11) 156, 13, 365, 78.
(12) 18, 19, 35, 24, 12, 17. (13) 340,255, 783. (14) 675, 225, 180, 125.

2. A library contains 3275 volumes, and each volume on the average 493 pages, and each page 39 lines. How many lines are there?

3. If the earth moves round the Sun at the rate of 68000 miles an hour, how far will it move in 365 days of 24 hours each?

4. If every page of a book contains 36 lines, and each line on an average 11 words, how many words would there be in 157 pages?

5. If each of 36 trucks in a luggage train contains 18 barrels of cement, and each barrel 36 maunds, how many maunds is the train carrying?

6. How many yards of silk are there in 9 packages, each containing 8 parcels, each parcel 26 pieces, and each piece 53 yards?

7. In a school there are 10 classes; each class has 4 desks, *each desk holds 18 boys; how many boys are there in the school?*

8. If 37 labourers earn 39 rupees each per day; how many rupees do they all earn in 36 working days?

9. If every man lived to marry and have 8 male children, how many great-great-grand children of the male sex could every one expect to have?

10. A Railway passenger train consists of 32 carriages; each carriage is divided into 12 compartments; in each compartment there are 5 benches and on each bench there is space for 8 persons; how many persons can the train carry?

---

84. When a number is multiplied by itself *once, twice, thrice four, &c.*, times, the product is called the **second, third, fourth, fifth,** &c., **power** of that number respectively. The *second* and *third powers* of a number are commonly termed its **square** and **cube** respectively. The number itself is called its **first power**.

85. These *powers* are often indicated by small numerals 2,3,4,5, &c., placed above the number to its right, which express how often the number is repeated in the product. The small numerals so used are therefore called the **indices** or **exponents** of the several *powers*.

Thus, $5^2 = 5 \times 5 = 25$; ∴ 25 is the *second power* or *square* of 5.

$5^3 = 5 \times 5 \times 5 = 125$; ∴ 125 is the *third power* or *cube* of 5.

$5^4 = 5 \times 5 \times 5 \times 5 = 625$; ∴ 625 is the *fourth power* of 5, and so on.

86. If the three signs +, −, ×, occur in an expression, the

operation of Multiplication is to be performed first and then that of Addition or Subtraction.

Thus, $4\times4\times3+3\times3\times2-4\times2\times1+2\times1\times0=48+18-8+0$
$=66-8=58.$

Examples XIV.

1. Find the squares of :—

(1) 1, 2, 3, 4, 5,...25 ; 39, 46, 54, 86, 99. (2) 172, 237, 906, 987.
(3) 729, 873, 1043, 5496. (4) 7342, 9384, 8796, 1234.

2. Find the cubes of :—

(1) 1, 2, 3, 4, 5,...25 ; 37, 48, 68, 77. (2) 88, 97, 123, 456.
(3) 308, 876, 765, 999. (4) 987, 5386, 9876, 1234.

3. Find the fourth powers of :—

(1) 678, 305, 987, 988. (2) 908, 3271, 8004, 9999.

4. Find the values of :—

(1) $1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2$. (2) $23^2+15^2-3^3$.
(3) $5^3-4^2-8^2$. (4) $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3$.
(5) $2^4+3^4-1^4$. (6) $25^2+28^2-20^2-18^2+15^2$.

5. Simplify the following expressions :—

(1) $8\times4-3\times6+4\times3-2\times1+5\times2+3\times7$
(2) $5\times6\times3+4\times3\times0-2\times1\times4+3\times6\times4-2\times2$.
(3) $8\times6\times3\times1-3\times6\times2\times4+4\times6\times7\times4-7\times8\times2\times0$.
(4) $-9\times6\times2\times3+7\times4+4\times6\times3\times5-3\times6\times7\times0\times5+2\times3\times4$.
(5) $7^2+2\times3^2+3\times5^2+4\times9^2$. (6) $3^2\times2+2\times3-2^2\times3+6\times1^3$.
(7) $23^2-11^2+115\times11^2-110^2+112^2$.
(8) $3^3+3\times4\times5+5^3-4^3-2\times4-2^3+6^2-3^3$.

IV. DIVISION.

87. **Division** is the method of finding *how many times* one given number is contained in another given number. The former of these numbers is called the **divisor**, the latter the **dividend**, and the number telling *how many times* the **quotient**. The number left after the operation is finished, is termed the **remainder**.

88. In dividing one number by another, we obviously take the latter number from the former, as often as we are able, according to the principle of Subtraction before explained. Hence *Division* bears the same relation to *Subtraction*, as *Multiplication* bears to *Addition*.

Thus, to divide 26 by 8, means that we are to find how many times 26 contains 8, and the operation at the side shews that 26 contains 8, 3 times with a remainder 2. Here 26 is called the *dividend*, 8 the *divisor*, 3 the *quotient* and 2 the *remainder*.

$$\begin{array}{r} 26(1,\ 1,\ 1 \\ \underline{8} \\ 18 \\ \underline{8} \\ 10 \\ \underline{8} \\ 2 \end{array}$$

89. Hence, by division we break up a given number into as many equal parts as there are units in another given number, and thus find one of these parts.

90. *Division* is of two kinds, **simple** and **compound**.

(i) When the dividend and divisor are both *abstract* numbers, or both *concrete* numbers of one and the same denomination ; or when the divisor is an *abstract* number, and the dividend a *concrete* number of one denomination, it is called *Simple Division*.

(ii) When the dividend is a *concrete* number of the same kind, but of different denominations of that kind, and the divisor an *abstract* number ; or when both the dividend and divisor are *concrete* numbers of the same kind but of different denominations of that kind, it is called *Compound Division*.

91. When there is no remainder, the division is said to be **exact** and since the Quotient tells how many times the Dividend contains the Divisor, it follows that *Dividend* = *Divisor* × *Quotient*. But when there is a Remainder, the division is called **inexact**, and the *Dividend* = *Divisor* × *Quotient* + *Remainder*.

92. The operation of *Division* is expressed by means of the sign ÷ and sometimes /, which is read **divided by** or simply **by**. It is also denoted by writing the dividend above the divisor with a line between them.

Thus $42 \div 7$ denotes that 42 is to be divided by 7, and is read 42 *divided by* 7 or simply 42 *by* 7. Also $42/7$ and $\frac{42}{7}$ mean $42 \div 7$.

93. In division, the quotient is an *abstract* number, if the dividend and divisor are both abstract or both concrete numbers, but the quotient is a *concrete* number, if the dividend is a concrete number and the divisor an abstract number. The divisor, if concrete, must be of the same kind as the dividend.

Thus, 45 divided by 5, or 45 rupees divided by 5 rupees, gives the *abstract* number 9 as quotient, for 5 or 5 rupees taken 9 times gives 45 or 45 rupees, and 45 rupees divided by 5 gives the *concrete* number 9 rupees as quotient, for if 45 rupees be divided into 5 equal parts, each of these parts will contain 9 rupees. Also 45 rupees divided by 5 yards has no meaning, according to the definition of Division in Art. 93.

94. As Division is the *reverse* of Multiplication, it follows that, by a reversed process, the Multiplication Tables must furnish the means of obtaining the quotient, when the divisor does not exceed 20 and the dividend 400.

Ex. 1. Divide 96 by 8.

Since $8 \times 12 = 96$; therefore $96 \div 8$ gives 12 as quotient.

Ex. 2. Divide 259 by 17.

Since $17 \times 15 = 255$, and $259 - 255 = 4$; therefore $259 \div 17$ gives 15 as quotient and 4 as remainder.

Examples XV. (Mental Division.)

1. How many times does 8 contain 2 ? 36 contain 3 ? 20 contain 4 ? 35 contain 5 ? 24 contain 6 ? 56 contain 7 ? 81 contain 9 ?

2. Divide 14 by 2; 12 by 3; 48 by 4; 20 by 5; 42 by 6; 49 by 7; 32 by 8; 108 by 9; 90 by 10; 77 by 11; 96 by 12.

3. Divide

(1) 56 separately by 2, 3, 4, 5, 6, 7, 8, 9, 12 and 14.
(2) 98 separately by 2, 5, 7, 9, 13, 15, 17, 18 and 19.
(3) 168 separately by 2, 7, 8, 9, 6, 12, 11, 15 and 18.
(4) 288 separately by 4, 7, 9, 10, 6, 8, 12, 15 and 17.
(5) 342 separately by 3, 6, 8, 9, 4, 11, 13, 15, 16 and 18.
(6) 172 by 9; 141 by 11; 128 by 14; 257 by 16; 195 by 19.

4. In 54, how many times is 8, and how many over? How many times is 15 contained in 195? In 240, how many times is 18, and how many over?

5. If 16 be taken 14 times from 228, what is left?

6. What is the 9th part of 36, 54, 108 and 144?

7. To how many boys can I give 9 marbles if I have 153?

8. At a cricket match 11 players make 132 runs. If each made the same number of runs, how many did each make?

9. A Patsala consists of 128 boys and they are made to stand in 8 rows; how many are there in each row?

10. If 320 rupees are shared equally among 16 men; how many does each man receive?

11. Divide 132 oranges equally among 7 girls and 5 boys.

12. Divide 96 pencils equally among 8 boys.

13. Bhuban spent 180 pice in oranges, buying them at the rate of 6 for 3 pice; how many oranges did he buy?

14. A boy, having a basket containing 214 oranges, distributed them equally between his 8 school-fellows and himself; the number which remained he gave to his school-master; how many did the school-master receive?

15. A man bought 11 cows at 18 rupees each, and sold them so as to gain 99 rupees; what did he sell each cow for?

16. How many seers of sugar at 5 annas each can be bought for 330 annas?

17. A woman bought 180 eggs at 3 for 2 pice and 275 more at 5 for 3 pice, and sold the whole lot at 13 for 19 pice; what does she gain or lose?

18. If 5 men can do a piece of work in 18 days, how long will it take 9 men to do the same work?

19. How many penknives, worth 8 annas each, ought to be exchanged for 144 pen-holders at one anna each?

20. A man walked 306 miles in 18 days; how many miles did he walk per day?

SIMPLE DIVISION.

95. When the dividend is a large number, but the divisor less than 20, the division is called **Short Division** and can be done by the following Rule.

RULE. Place the divisor and dividend thus :
divisor) dividend.

From the left of the dividend cut off a number not less than the divisor but less than 10 times the divisor, giving the first partial dividend. Find by the aid of the Multiplication Tables how often the divisor is contained in this dividend ; put down the quotient under the units' figure of this dividend, and take notice of the remainder (whether it be any number or 0). On the right of this remainder, conceive in your mind to be placed the least number of the figures next following in the dividend which, affixed to the remainder, will make a number not less than the divisor. Proceed, as above, with this new partial dividend to find the next figure of the quotient; taking care to place after the first figure in the quotient a cipher for every figure brought down from the dividend which, affixed to the remainder, makes a number less than the divisor.

Continue this process till all the figures of the dividend have been thus brought down ; and if there be any remainder at the end of the operation, write it as a remainder distinct from the quotient.

Ex. 1. Divide 612459 by 7.

7)612459
  87494 rem. 1.

From the left of the dividend cut off a number not less than 7 but less than 70 : that is, cut off 61, our first partial dividend. Now 7 is contained in 61, 8 times and 5 over; put the **8** under the 1 in 61, and to the right of the remainder 5 affix the next figure of the dividend 2, making 52, the second partial dividend. But 7 is contained in 52, 7 times and 3 over; put **7** in the quotient, and to the right of the remainder 3 affix the next figure 4 making 34, the third new dividend ; and so proceed.

The above operation is usually performed in saying :—
7 in 61, **8** and 5 over ; in 52, **7** and 3 over; in 34, **4** and 6 over ; in 65, **9** and 2 over ; in 29, **4** and **1** over (as remainder).

Thus the quotient is 87494, and the remainder 1.

Ex. 2. Divide 61245 by 15.

15)61245
   4083

Here 15 in 6 goes no times, but 15 in 61 goes 4 times and 1 over ; write **4** under the 1. Then 15 in 12 goes no times, but 15 in 124 goes 8 times and 4 over; write **0** under the 2 and **8** under the 4; lastly 15 in 45 goes 3 times write **3** under the 5.

Thus the quotient is 4083.

96. The truth of the above method may be shewn thus :—

Since 61245 = 61 thousands + 2 hundreds + 4 tens + 5 units,
= 60 thousands + 12 hundreds + 4 tens + 5 units,
= 60 thousands + 124 tens + 5 units,
= 60 thousands + 120 tens + 45 units.

∴ 61245 divided by 15 gives as quotient **4** thousands + **8** tens + **3** units or 4083.

Examples XVI.

1. Divide

(1) 462 separately by 3, 6, 8, 9, 10, 11 and 12.
(2) 682 separately by 3, 4, 6, 8, 9, 11, 14 and 15.
(3) 8425 separately by 5, 7, 8, 10, 13, 16, 17 and 19.
(4) 6876 separately by 2, 3, 7, 9, 11, 12 and 14.
(5) 35298 separately by 3, 5, 9, 7, 10, 12 and 18.
(6) 348 by 2 ; 4596 by 3 ; 276284 by 4 ; 84375 by 5.
(7) 53844 by 5 ; 536074 by 7 ; 95832417 by 8 ; 3158367 by 10.
(8) 7163253651 by 9 ; 1234567890 by 11 ; 9876543 by 12.
(9) 27643532 by 14 ; 35762445 by 15 ; 47623554 by 18.
(10) 34672352 by 16 ; 987654321 by 17, by 18, by 19, by 20.

2. If 1674 men are drawn up in 18 columns, how many men are there in each column ?

3. I distributed 2160 marbles among a number of boys, and gave each boy 12 marbles ; how many boys were there ?

4. What is the 15th part of 135090 ? the 11th part of 101112 ?

5. A farmer has 1786 sheep divided into 19 equal flocks. How many sheep are there in each flock ?

6. A farmer spent 1872 rupees in the purchase of oxen. Each ox cost 12 rupees. How many oxen did he buy ?

7. If the sum of 18 and 30 be divided by their difference, and the quotient be multiplied by the product of 16 and 27, what is the result ?

8. A man gives 14 cows and 35 sheep for 55 bags of potatoes worth 7 rupees per bag ; if each sheep was worth 3 rupees, what did he get for each cow ?

---

97. When the dividend and divisor are both large numbers, the division is called **Long Division** and can be performed by the following general Rule.

RULE. On either side of the dividend draw curved lines ; place the divisor on the left and the figures of the quotient as they arise on the right ; thus

divisor)dividend(quotient

Then try to find how often the first one or two figures on the left hand of the divisor are contained in the first one or more of those of the dividend, and place the result on the right as the first figure of the quotient ; and the product arising from the multiplication of the divisor by this figure being subtracted from the dividend, *bring down* or *annex* to the right of the remainder the next figure of the dividend. Proceed as before, and continue the process till all the figures of the dividend have been brought down ; then the quotient, and the remainder if any, will be obtained.

If at any stage of the process, the divisor is greater than the *partial* dividend, affix a *cipher* to the quotient and bring down the next figure of the dividend. Continue this process till the partial dividend is greater than the divisor and then proceed as before.

Ex. 1. Divide 75035 by 349.

```
349)75035(215
    698
    ---
     523
     349
     ---
     1745
     1745
```

Here, the first figure **2** in the quotient is obtained by inquiring how often 3 is contained in 7, or 34 in 75; then, after multiplying 349 by 2, which, from the places of the figures, represents 2 *hundreds*, and subtracting the product which is 698, from 750, we have a remainder 52; to this the next figure 3 of the dividend is *annexed* to form the partial dividend 523. Now seek how often 3 is contained in 5, or 34 in 52, and the quotient being **1**, 1 *ten* is annexed to the 2 *hundreds* already obtained: multiplying 349 by 1, which means 1 *ten*, and subtracting the product 349 from 523, we get the remainder 174. Bring down the last figure 5 of the dividend to form the partial dividend 1745, and we find the corresponding quotient to be **5** units exactly, for 349 multiplied by 5 produces 1745, and the operation is then completed, leaving no remainder. Therefore the whole quotient is 215.

98. Supplying the auxiliary digits omitted in the above operation the process would stand thus :—

```
349)75035(200+10+5
    69800
    -----
     5235
     3490
     ----
     1745
     1745
```

Ex. 2. Divide 39875365 by 8654.

```
8654)39875365(4607
     34616
     -----
      52593
      51924
      -----
        66965
        60578
        -----
         6387
```

Here 3987 is less than 8654, but 39875 is greater; therefore take 39875 for the first partial dividend. It contains the divisor 4 times; put **4** in the quotient, multiply 8654 by 4, placing the product 34616 under 39875, and subtract, leaving 5259. To the remainder 5259 *annex* the next figure of the dividend 3, giving 52593, the second partial dividend. It contains the divisor 6 times; put **6** in the quotient, multiply 8654 by 6, placing the product 51924 under 52593, and subtract, leaving 669. Again, to 669 bring down the next figure 6, giving 6696 the third partial dividend. It contains the divisor 0 times; put **0** in the quotient, and the remainder is now 6696. Lastly to 6696 bring down the last figure 5, giving 66965, the fourth partial dividend. It contains the divisor 7 times; put **7** in the quotient, multiply 8654 by 7, placing the product 60578 under 66965, and subtract, leaving a remainder 6387. Thus the quotient is 4607 and the remainder is 6387.

99. When the divisor is terminated by one or more ciphers, we use the following Rule.

RULE. Cut off all the ciphers on the right of the divisor and as many figures from the right of the dividend :—for the quotient, divide the remaining figures of the dividend by the remaining figures of the divisor (Arts. 95, 97), and for the final remainder bring down to the particular remainder the figures cut off from the dividend.

Ex. Divide 20573296 by 80 and by 345000.

(1)
```
8,0)2057329,6
    257166 - 16
```

(2)
```
345,000)20573,296(59
        1725
         3323
         3105
          218296
```

In the first example, in dividing by 8 the remainder is **1**, to which we bring down the figure cut off **6**, giving 16 for the final remainder, and 257166 for quotient.

In the second example, the remainder in dividing by 345 is **218**, to which we bring down the figures cut off **296**, giving 218296 for the final remainder and 59 for quotient.

100. The **Proofs** usually adopted in division are the following :—

(1) To the product of the divisor and quotient add the remainder (if any) ; if the result coincides with the dividend, we presume that the work is correctly performed.

(2) By **casting out the nines.**

(*a*) From the sums of the digits in the *divisor* and the *quotient* subtract 9 as many times as possible, and set down the remainders to the left and right of a cross sign.

(*b*) Multiply the two remainders and from the product subtract 9 as often as possible. Put down the remainder below the cross sign.

(*c*) Lastly subtract the *remainder* from the *dividend* and from the sum of the digits of this difference subtract 9 as many times as possible and set down the remainder above the cross sign. If the upper and lower figures agree, it is presumed that the operation is correct.

Ex. Find the quotient and remainder when 275487 is divided by 736.

Division.

```
736)275487(374
    2208
     5468
     5152
      3167
      2944
       223
```

Thus the quotient is 374 and the remainder 223.

Proofs.

(1)
```
   374
   736
  2244
 1122
2618
275264
   223
275487
```

(2) $7+3+6=16$, *rem.* **7**
and $3+7+4=14$, *rem.* **5**.

```
   8
7  ×  5
   8
```

$7\times5=35$, *rem.* **8**.
Also $275487-223=275264$,
and $2+7+5+2+6+4=26$,
rem. **8**.

101. If all the four signs $+$, $-$, $\times$, $\div$ are used together in an expression, the operations of *Division* and *Multiplication* are to be performed first and next those of *Addition* and *Subtraction*.

Ex. Find the value of $14+12\div 6\times 4-3\times 2+6\times 72\div 12$.

The expression $=14+2\times 4-3\times 2+6\times 6=14+8-6+36$

$=58-6=\underline{52}$.

Examples XVII.

1. Divide :—

(1) 92483 by 23. (2) 79958 by 39. (3) 79796 by 79. (4) 588168 by 84.
(5) 79512587 by 43. (6) 69637856 by 32. (7) 67001228 by 49.
(8) 144157246 by 83. (9) 47073256 by 37. (10) 7417784 by 88.
(11) 579826952 by 76. (12) 9009196416 by 96. (13) 58762347 by 99.
(14) 17587694293 by 54. (15) 14528340631 by 84.
(16) 3708501975 by 81. (17) 96790123458 by 98.

2. Find the values of :—

(1) $419352633\div 123$. (2) $1721034655\div 144$. (3) $47123419361\div 132$.
(4) $3577926\div 506$ (5) $27291888\div 478$. (6) $87624792\div 843$.
(7) $48310567\div 549$. (8) $6430776444\div 876$. (9) $137090807\div 996$.
(10) $630762540981\div 652$. (11) $632798014\div 7243$.
(12) $519387042\div 2731$. (13) $140167329\div 7038$.
(14) $395494875\div 6007$. (15) $2106144185\div 2375$.
(16) $25413286\div 7960$. (17) $8327976\div 5730$.
(18) $64157660\div 1480$. (19) $935384767\div 4836$.
(20) $900370575\div 54321$. (21) $183920748\div 37246$.
(22) $2828882701578\div 38706$. (23) $2919333978682\div 76913$.
(24) $6119085281767 4\div 873156$. (25) $163034794788\div 321567$.
(26) $487264325876\div 5678909$. (27) $876824985621\div 90956845$.
(28) $5640000 3227\div 76589451$. (29) $32899438654\div 100104325$.
(30) $191776658604\div 68589649$. (31) $4676705026675\div 154321235$.
(32) $12193263111263 5269\div 123456789$.
(33) $163018805310364920 3285\div 2837154309$.
(34) $560211975014967053000\div 700002030506$.
(35) $163018805310364920 3285\div 574585614865$

3. Divide :—

(1) 237876093 by 5605, by 9089, by 40857, and by 57085.
(2) 81229 separately by 10, 20, 30, 40, 50, 80, 90.
(3) 342604 separately by 100, 400, 600, 800, 900.
(4) 78534826 separately by 800, 12000, 3200, 475000.
(5) 3854269734 separately by 310, 5900, 587000, 90900.
(6) 25413286 by 7900; 19054832 by 83000; 26799534687 by 7890000

4. Find the values of :—

(1) $192\div 16+720\div 18+795\div 15-1786\div 19$.
(2) $3871\div 49+6935\div 95-5432\div 56-1375\div 25+4590\div 45$.
(3) $56+81\div 3+8\times 7\times 9\div 12\times 136\div 17-72\div 18+6\times 3$.
(4) $12\times 16\div 8+17\times 6-18\times 32\div 8-27\div 9\times 7+8\times 30\div 15+56\div 14$.
(5) $15\times 37153+73474-67152\div 4+40734\times 2-5485\times 75$.

5. If a bag contains 103 potatoes, how many will be required to hold 7432274 potatoes ?

6. If each carriage contains 57 passengers, how many carriages are there in a train carrying 969 passengers ?

7. Each of 156 boys uses 12 pen-nibs, and a box contains 144 nibs. How many boxes are required ?

8. A confectioner sells 23475 maunds of sweetmeats in a year of 313 days ; how many maunds does he sell in a day ?

9. Supposing a Railway train to travel from Calcutta to Delhi, a distance of 924 miles, in 44 hours, what is the average speed per hour ?

10. The population of a country is 3083220 and its area is 7341 square miles. How many people are there on an average to each square mile ?

11. Find the number of pages in a book which has on an average 207 words on a page, and contains 201411 words altogether ?

12. How many minutes will a wheel be in turning round 895702 times, if it turn 158 times in a minute ?

13. What number multiplied by 79 will give the same product as 257 multiplied by 553 ?

14. A shopkeeper sold 267 shawls for 4005 rupees, gaining thereby 4 rupees on each shawl ; what had each shawl cost him ?

15. The population of a certain village is 21510, and one out of 45 dies annually. How many die in a year ?

16. Find how many times the numbers 11, 15, 19, and 23 must be equally repeated to make 13668.

17. Find the 532nd part of 1004416. What is the 365th part of 36865365 ?

18. How many pages contain 30888 words, every page having 52 lines of 9 words each ?

19. If 168465 maunds of rice be distributed equally among 11231 famine-stricken men, how many maunds will each receive ? and if the family of each consist of 5 persons, what will be the share of each person?

20. The rays of light comes from the Sun to the Earth in 498 seconds ; at what rate does light move per second, the distance of the Sun from the Earth being 93000000 miles ?

V. THE USE OF BRACKETS.

102. **Brackets,** which are of several kinds, as ( ), { }, [ ], are used to denote that all numbers included within any pair of them are to be considered as forming but one number, and are therefore to be equally affected by any number not included within the same pair of brackets.

Thus, $(2+3+7)$ denotes that 2, 3 and 7 are to be taken as making one number, *i. e.* whatsoever, outside the brackets, affects 2 in any way, must also affect 3 and 7 in the same way.

A **vinculum** is a sign sometimes used instead of brackets. It consists of a *line* drawn over the numbers to be considered as forming one number.

Thus, $\overline{2+3}$ express the same thing as $(2+3)$.

103. When two or more numbers, connected by the signs of operation are enclosed in a pair of brackets, the operations of arithmetic indicated inside the brackets are to be performed before the brackets are removed. Thus,

Ex. 1. $7-\overline{5-3}=7-2=\underline{5}.$

Ex. 2. $22-(4\times 3+5-6\div 2)=22-(12+5-3)$
$=22-(17-3)=22-14=\underline{8}.$

104. When a number immediately precedes an expression included in a pair of brackets, this number is to be multiplied by the number obtained after removing the brackets.

Thus, $7+4(5-2)-6\times 3=7+4\times 3-18=7+12-18=19-18=1.$

105. When an expression is included in more than one pair of brackets, it is convenient to remove the innermost bracket first, then the innermost of those that remain, and so on, till all the brackets are removed.

$$
\begin{aligned}
\text{Thus, } 25&-[\{15\times 10-2\times 12-8(2\times 12-10)\}\div 2]\times(15-\overline{10+2})\\
&=25-[\{150-24-8(24-10)\}\div 2]\times(15-12)\\
&=25-[\{150-24-8\times 14\}\div 2]\times 3\\
&=25-[\{126-112\}\div 2]\times 3\\
&=25-[14\div 2]\times 3=25-7\times 3=25-21=4.
\end{aligned}
$$

106. If the sign $+$ (*plus*) precedes a bracket, the bracket may be removed without affecting the result.

Thus, since $7+(5-3)=7+2=9$, and $7+5-3=12-3=9$,
therefore, we have $7+(5-3)=7+5-3$.

107. If the sign $-$ (*minus*) precedes a bracket, the bracket may be removed, provided the signs of all the numbers, inside the bracket be changed from $+$ to $-$, and from $-$ to $+$.

Thus, since $29-(7-5+3)=29-(2+3)=29-5=24$,
and $29-7+5-3=34-10=24$,
therefore, we have $29-(7-5+3)=29-7+5-3$.

108. The sign $\therefore$ signifies *therefore*, and is often used in stating a method by which an answer has been obtained. The sign $\because$ stands for *because* or *since*, and is used in stating a reason.

Examples XVIII.

1. Find the values of :—

(1) $10+(5-3)-(17-8)+(16-11)+25-(6-3+4)$.
(2) $20-\overline{10-3-6}+(15-3)-(16-9)-(5+6)+(4+9)$.
(3) $8+4(12-7)-3(9-5)+7(16-19+5)-(18-\overline{6+7})$.
(4) $3\{8+25-3(20-12)\}$. (5) $3\{8+(25-3)\overline{20-12}\}$.

(6) $287-[\{15\times 10-2(12-8)(2\times 12-10)\}\div 2]\times 15-\overline{10-2}$.

(7) $1520-\{610+\overline{703-608}\}$. (8) $605-\{(95-11-19)+237\}$.

(9) $86-\{(59-48)+16-(59-49)\}$. (10) $168-\{(70-39)+(90-83)\}$.

(11) $1246-(362-156)-\{371-(495-386)\}$.

2. Find the values of :—

(1) $(1536-487)-1392\div 29+7\times 5$. (2) $5880\div(167-132)\times 6$

(3) $(194+65)\times 7+(352-220)\div 11-952\div(91-35)$.

(4) $(67893-8637)\div 823+7546\times(2356-945)-(9870\times 170)$.

(5) $\{(312570\times 598+76125\times 47+318\div 3)-155146\}\div(6139\times 15)$.

3. If the sum of 274 and 108 be multiplied by their difference and the product be divided by 166, what will be the quotient ?

4. If the sum of 103, 29, and 267 be divided by 19, and the quotient be multiplied by 57, and the product be diminished by 197, what will be the remainder ?

5. Multiply $(325-293)$ by $(306\div 17)$ and to the product add $(1000+99)$.

6. From 34856 subtract (763×41) and to the remainder add $\{1998\div(663-441)\}$.

7. Find the difference between 876 and $459-368+149$.

8. What number subtracted from $(2471+56)$ will leave $(3863-1498)$ as remainder ?

9. Find the difference between

$$3210+401-(67-59) \text{ and } 342-(491-382).$$

10. From the sum of the greatest number of 9 and 10 digits subtract the difference of the least numbers of 10 and 11 digits.

11. From the sum of the greatest numbers of 4, 5 and 6 digits subtract the sum of the least numbers of 3, 4 and 5 digits.

12. Find the values of :—

(1) $6+8[3\times 6+\{3+7-(8+3-6)-(2\times 6\div 3+3-2)\}]$.

(2) $66\times 37-8[(9-7)\times 6-(27+12)\div 13+(17+15+39-50)\times 5-9\times 7]$.

(3) $\{(7+75)\times 43+(4698+171)\div 9\}\div\{(73+14-2)-(16\div 2+4\times 7)\}$.

(4) $8[4\times\{(360\times 120)-(47+13)\div 3\}]\div\{(360\times 120)+(65-25)\div 5\}\div 5401$.

(5) $108\div 9\times[76-9\{63-7(9\times 3-4\times 8+5^2-10\times 2)-(2^3\times 9-2^4)\}]$
-11×12.

(6) $23\times 11\times 3+7[206\times(8+6-13)-\{(14-8)\times 7-(15+5-11)\times 2^3$
$+(6^3-13\times 2^3)+(6\times 8\times\overline{15\div 5})\}]+(2\times 5+3^2-3\times 4\times 7)$.

(7) $84-7[-11-4\{-17+3(8-\overline{9-5})\}]$.

(8) $5\times\{4-2[4-2(4+3)]\}-4\times\{4-2[4-2(4+3)]\}$.

(9) $19+12\times 15-120-4+\{29-13\times 2+(14-9)\times 3\}$.

(10) $9\times[125\div 5(7-2)\times 8(9-7)+4\{7+2(3+8)\}]$.

VI. MISCELLANEOUS PROPOSITIONS.

(IN THE FUNDAMENTAL OPERATIONS.)

109. Sum, difference, &c.

(1) Given the difference between two numbers and the greater, to find the smaller number.

RULE. *Subtract the given difference from the greater number, and the result is the required smaller number.*

Ex. If 34060 be the difference between two numbers, and the greater number is 48752; what is the less number?

The less number $= 48752 - 34060 = \underline{14692}$.

(2) Given the difference between two numbers and the smaller, to find the larger number.

RULE. *Add together the given difference and the smaller number, and the sum is the required larger number.*

Ex. The difference between two numbers is 14610 and the less is 4007; what is the larger number?

The larger number $= 14610 + 4007 = \underline{18617}$.

(3) Being given the sum and difference of two numbers, to find the numbers.

RULE. *To find the larger number, add together the given sum and difference, and divide the result by 2. To find the smaller number, subtract the given difference from the given sum and divide the result by 2.*

Ex. 1. The sum of two numbers is 25264, and their difference is 736; what are the numbers?

The larger number $= (25264 + 736) \div 2 = 26000 \div 2 = \underline{13000}$.

The smaller number $= (25264 - 13000) = \underline{12264}$,

or, the smaller number $= (25264 - 736) \div 2 = 24528 \div 2 = \underline{12264}$.

Ex. 2. The price of a carriage with horse is 1590 rupees, and the price of the carriage is 324 rupees more than that of the horse. Find the price of each.

Here, the sum of the two prices is 1590 rupees and the difference 324 rupees.

$\therefore$ the price of the carriage $= (1590 + 324) \div 2 = \underline{957}$ rupees.

And the price of the horse $= (1590 - 957)$ or $\underline{633}$ rupees.

(4) Being given the sums of every two of three given numbers, to find the numbers.

RULE. *Add together the three given sums, divide the result by 2, and from the quotient subtract separately the three given sums. The several differences are the required numbers.*

Ex. 1. The sum of the first and second of three numbers is 59; that of the first and third is 53; and that of the second and third is 42. Find the numbers.

$$(59+53+42)\div 2=77,$$

$$\left.\begin{aligned} \therefore \text{ the first number} &= 77-42=35, \\ \text{the second number} &= 77-53=24, \\ \text{and the third number} &= 77-59=18. \end{aligned}\right\} \quad \textit{Ans.}$$

Ex. 2. At a game of cricket A and B together score 75 runs; B and C together score 90 runs; and A and C together score 51 runs; find the number of runs scored by each of them.

Here, A, B and C together score $(75+90+51)\div 2$ or 108 runs.

$$\left.\begin{aligned} \therefore\ A \text{ scored } (108-90) \text{ runs} &= 18 \text{ runs}, \\ B \text{ scored } (108-51) \text{ runs} &= 57 \text{ runs}, \\ \text{and } C \text{ scored } (108-75) \text{ runs} &= 33 \text{ runs}. \end{aligned}\right\} \quad \textit{Ans.}$$

(5) Having given the sum of three numbers, the excess of the first over the second, and the excess of the second over the third, it is required to find the numbers.

RULE. *Subtract the sum of the excess of the second over the third and of the first over the third (which may be obtained by adding the two given excesses) from the given sum, and divide the result by 3. The quotient is the least of the three required numbers.*

Ex. Divide 53 rupees among A, B and C, so that A may receive 7 rupees more than B, and B 8 rupees more than C.

Here, the sum of the three shares is 53 rupees, and the excess of A's share over C's is $8+7$ or 15 rupees,

$$\text{and } 53-(8+15)=53-23=30.$$

$$\left.\begin{aligned} \therefore\ C\text{'s share} &= (30\div 3) \text{ rupees} = 10 \text{ rupees}, \\ B\text{'s share} &= (10+8) \text{ rupees} = 18 \text{ rupees}, \\ \text{and } A\text{'s share} &= (18+7) \text{ rupees} = 25 \text{ rupees}. \end{aligned}\right\} \quad \textit{Ans.}$$

110. Product, Quotient, Remainder, &c.

(1) Given the product of two numbers and one of them, to find the other.

RULE. *Divide the product by the given number, and the quotient thus obtained is the other required number.*

Ex. The product of two numbers is 90368, and the smaller number is 256; what is the larger number?

The larger number $= 890368\div 256 = \underline{3478}$.

(2) Given the divisor, the quotient and the remainder, to find the dividend.

RULE. *Multiply together the divisor and the quotient, and to the product add the remainder. The result is the dividend.*

Ex. If the divisor be 3857, the quotient 489, and the remainder 1305, what is the dividend?

$$\text{The dividend} = 3857 \times 489 + 1305 = \underline{1887378}.$$

(3) Given the dividend and the quotient, to find the divisor.

RULE. *Divide the dividend by the quotient, and the result is the divisor.*

Ex. The dividend is 342604 and the quotient 883, find the divisor.

$$\text{The divisor} = 342604 \div 883 = \underline{388}.$$

(4) Given the dividend, the quotient, and the remainder, to find the divisor.

RULE. *From the dividend subtract the remainder, and divide the difference by the quotient. The result is the divisor.*

Ex. 1. The dividend is 119376, the quotient 25 and the remainder 2076; what is the divisor?

$$\text{The divisor} = (119376 - 2076) \div 25 = 117300 \div 25 = \underline{4692}.$$

Ex. 2. A farmer having 2316 sheep, on putting an equal number of them into each of 25 fields, had 16 remaining. How many did he put into each of the fields?

$$\text{The required number} = (2316 - 16) \div 25 = 2300 \div 25 = \underline{92}.$$

(5) To find the *least number* which must be added to a given number to make it exactly divisible by a second given number.

RULE. *Divide the first given number by the second, and subtract the remainder from the second given number. The difference is the required number.*

Ex. What least number must be added to 4856752 to make it exactly divisible by 2163?

$4856752 \div 2163$ gives 2245 as quotient and 817 as remainder.

$\therefore$ the number to be added $= 2163 - 817 = \underline{1346}$.

(6) To find the *least number* which must be subtracted from a given number to make it exactly divisible by a second given number.

RULE. *Divide the first given number by the second, and the remainder is the required number.*

Ex. What least number must be subtracted from 90625 that it may be divisible by 727?

$90625 \div 727$ gives 124 as quotient and 477 as remainder.

$\therefore$ the number to be subtracted $= \underline{477}$.

(7) To find the *greatest* number of a given number of digits which is divisible by a given number.

Proceed as in the following example:—

Ex. Find the greatest number of five digits which is divisible by 529.

The greatest number of 5 digits is evidently 99999.

99999 divided by 529 gives 189 as the quotient and 18 as the remainder.

$\therefore$ the reqd. greatest number $= 99999 - 18 = 99981$.

(8) To find the *least* number of a given number of digits which is divisible by a given number.

Proceed as in the following example :—

Ex. Find the least number of six digits which is divisible by 4325.

The least number of 6 digits is evidently 100000.

100000 divided by 4325 gives 23 as the quotient and 525 as the remainder, and $4325 - 525 = 3800$,

$\therefore$ the reqd. least number $= 100000 + 3800 = 103800$.

111. Equidifferent series.

The numbers 1, 2, 3, 4, 5, etc., are called *natural* numbers, of which 1, 3, 5, etc., are *odd*, and 2, 4, 6, etc., are *even* numbers.

(1) To find the sum of any number of the *natural* numbers beginning with 1.

RULE. *Multiply the last number by the next higher number, and divide the result by 2. The quotient is the required sum.*

Ex. Add together $1 + 2 + 3 + 4 + 5 + \ldots\ldots \ldots + 40$.

Here, the last number is 40, and the next higher number is 41.

$\therefore$ the required sum $= 40 \times 41 \div 2 = \underline{820}$.

(2) To find the sum of any number of *odd* numbers beginning with 1.

RULE. *The square of the number of times the numbers are repeated, is the required sum.*

Ex. Add together $1 + 3 + 5 + 7 + 9 + \ldots\ldots \ldots + 25$.

Here, the number of times the numbers are repeated is 13.

$\therefore$ the sum required $= 13^2 = \underline{169}$.

(3) To find the sum of any number of *even* numbers beginning with 2.

RULE. *Multiply the number of times the numbers are repeated by the same increased by 1. The product is the required sum.*

Ex. Add together $2 + 4 + 6 + 8 + \ldots\ldots + 30$.

Here, the number of times the numbers are repeated is 15.

$\therefore$ the sum required $= 15 \times 16 = \underline{240}$.

(4) To find the sum of any given numbers increasing or decreasing by a fixed number.

RULE. *Multiply the sum of the two extreme numbers by the number of terms (or times repeated), and divide the result by 2. The quotient is the required sum.*

Ex. Add together $2+5+8+11+\dots\dots+47$.

Here, the number of terms will be found to be 16.

$\therefore$ the sum $=16\times(2+47)\div2=16\times49\div2=\underline{392}$.

Examples XIX.

1. What number subtracted from 850967 will leave 3876?

2. The difference between two numbers is 84489 and the larger is 123456, what is the smaller?

3. The smaller of two numbers is $3087+56299$ and their difference is 22371; what is the larger number?

4. The greater of two numbers is the sum of 505, 650, 19 and 9003 and the difference between them is $3287-750$. What is the less number?

5. *The sum of two numbers is 12640 and their difference 1608*; what are the numbers?

6. The sum of the ages of two men is 173 years and the difference between them is 15 years; what are their ages?

7. The sum and difference of two numbers are 1426 and 384 respectively; find the numbers?

8. A man bought a pair of horses and a carriage for 857 rupees; the carriage was worth 165 rupees more than the horses; what was the price of each?

9. Two men having met on a journey, found that they had travelled 1200 miles, and that one had travelled 360 miles more than the other; what distance had each travelled?

10. Divide 168 marbles between two boys, giving to one 42 more than the other.

11. Ram, Gopal and Hari begin to play at marbles. Ram and Gopal have 77 marbles between them, Gopal and Hari 63, and Ram and Hari 70. How many marbles has each?

12. A basket containing oranges, apples and plums, has 15 more oranges than apples, and 8 more apples than plums. The whole number of fruits in the basket is 112. Find the number of each kind in the basket.

13. Three persons *A*, *B* and *C*, are possessed of certain sums of money, such that *A* and *B* together have 120 rupees; *A* and *C* together have 140 rupees; and *B* and *C* together have 150 rupees. What is the sum possessed by each?

14. Divide 4680 rupees, after giving away 180 rupees to the poor, between *A*, *B* and *C*, giving *B* 216 rupees more than *A*, and *C* 336 rupees more than *B*.

15. The product of two numbers is 17037006 and one of them is 4858, what is the other?

16. If the divisor be 3857, the quotient 489 and the remainder 1305, what is the dividend ?

17. A dividend is 16322853, the quotient is 1754 and the remainder is 129 ; what is the divisor ?

18. The quotient arising from the division of 183926157 by a certain number is 4938 and the remainder is 5409. Find the divisor.

19. What least number must be added to 34568135 that the sum may be exactly divisible by 357 ?

20. What least number must be subtracted from 56854327 that the difference may be exactly divisible by 7323 ?

21. By what number must 109109109 be divided so that the quotient may be 51784, and 221 over ?

22. What number multiplied by 1617 will give 50696184 ?

23. What least number must we subtract from 57385 so that it can be exactly divided by 387 ? and what least number must we add ?

24. The sum of the product of two numbers and 355 is 87403 ; one of the numbers is 216, find the other number.

25. What number must be added to 30984051, that the sum may be exactly divisible by 288 ?

26. Add together :—

(1) $1+2+3+4+\ldots\ldots+60$.   (2) $1+2+3+4+\ldots\ldots+100$.

(3) $2+5+8+11+\ldots\ldots+29$.   (4) $1+3+5+7+\ldots\ldots+31$.

(5) $2+4+6+8+\ldots\ldots+30$.   (6) $2+6+10+14+\ldots+78$.

(7) $5+8+11+14+\ldots\ldots+53$.   (8) $100+97+94+\ldots\ldots+43$.

27. A debt can be discharged in 52 weeks by paying one rupee the first week, 3 rupees the second week, 5 rupees the third week and so on. Required the amount of the debt.

28. A person goes 3 miles on the first day, 5 miles on the second, 7 miles on the third, and so on. How far has he travelled in a month of 30 days ?

29. How many times will a clock strike in a day of 24 hours ?

30. Write down 576987, and under it write the eighth succeeding number, and under this latter the next eighth succeeding number and so proceed till nine numbers have been written down ; find their sum.

31. Find the greatest and least numbers of 5 digits which are divisible by 327.

32. Find the least number of 6 digits which is divisible by 273.

33. Find the product of the two greatest numbers of 5 digits.

34. Divide the greatest number of 7 digits by the least number of 4 digits.

35. Find the sum of the greatest and the least number that can be formed by the digits 3, 2, 0, 1, 5, 8 and 9 taken all together.

112. Addition, Subtraction, &c.

(1) To subtract a number from another consisting of 1 followed by ciphers only.

RULE. *Put down as many nines as there are ciphers in excess of the number of figures in the subtrahend; then (beginning from the left) write down in order the differences of each of the figures from 9 except the units' figure, which subtract from 10.*

Ex. Subtract 5736428 from 10000000000.

Here are 10 ciphers in the minuend, and 7 figures in the subtrahend; hence put down **999**. Again 5 from 9 is **4**, 7 from 9 is **2**, 3 from 9 is **6**, 6 from 9 is **3**, 4 from 9 is **5**, 2 from 9 is **7**, and 8 from 10 is **2**. Therefore the required difference is 9994263572.

(2) To subtract *mentally* the sum of several numbers from a given number.

Proceed as in the following example :—

Ex. Subtract the sum of 1286, 495, 4758, 984 from 15812.

15812	
1286	*Mentally* thus : 4, 12, 17, 23 and **9** = 32 ;
495	*carry* 3, 11, 16, 25, 33 and **8** = 41 ;
4758	*carry* 4, 13, 20, 24, 26 and **2** = 28 ;
984	*carry* 2, 6, 7 and **8** = 15.
8289 *Ans.*	

(3) To subtract *mentally* from a number the product of two other numbers one of which is less than 20.

Proceed as in the following example :—

Ex. Subtract 8×549 from 6567.

6567	*Mentally* thus : $8 \times 9 = 72$, and **5** = 77 ;
549	*carry* 7, add 8×4, 39, and **7** = 46 ;
8	*carry* 4, add 8×5, 44, and **1** = 45 ;
2175 *Ans.*	*carry* 4, 4, and **2** = 6.

113. Multiplication by factors.

To multiply one number by another which can be resolved into factors each less than 20.

RULE. *Multiply the given number by each of the factors in succession, and the final product is the required one.*

Ex. 1. Multiply 31729 by 648.

$648 = 9 \times 9 \times 8$,

31729	285561	2570049
9	9	8
285561	2570049	20560392 *Ans.*

Ex. 2. Multiply 43896 by 357, and by 735; making in each case only two partial multiplications.

	(1)		(2)
	43896		43896
	357		735
	307272		307272
$35=7\times5$	1536360	$35=7\times5$	1536360
	15670872 *Ans.*		32263560 *Ans.*

Ex. 3. Multiply 567224 by 48872: and 48872 by 567224; making in each case only three partial multiplications.

	(1)		(2)
	567224		48872
	48872		567224
	4537792		342104
$48=8\times6$	27226752	$56=7\times8$	2736832
$72=8\times9$	40840128	$224=56\times4$	10947328
	27721371328 *Ans.*		27721371328 *Ans.*

Examples XX.

1. Subtract 57364 from 1000000; 542056 from 1000000000; 7859064 from 1000000000; and 79854 from 10000000.

2. Subtract

(1) $3671+45+467+2073$ from 10608.
(2) $469+10876+2468+13972$ from 38709.
(3) $1234567+1234+123+12345$ from 4567208.
(4) $3843+396+428+1543+2897$ from 12964.

3. Subtract *mentally* :—

(1) 4×2016 from 8124; 6×1632 from 9798; 8×4506 from 46325.
(2) 9×18764 from 198765; 7×53197 from 3690756.
(3) 15×14567 from 3567824; 18×51987 from 37373784.

4. Add 4×123 to 878; 9×2345 to 4675; 8×1071 to 8795.

5. Multiply by factors :—

(1) 98989 by 44; 98909 by 72; 89088 by 96; 79797 by 63.
(2) 9785643 by 128; 6301246 by 256; 8725364 by 432.
(3) 9457283 by 792; 8465729 by 512; 5374896 by 588.
(4) 13245 by 1188; 246785 by 1872; 989045 by 15015.

6. Multiply in *two* lines :—

(1) 4016 by 637; 3543 by 648; 47862 by 1629; 31127 by 14412.
(2) 324567 by 486, by 936, and by 13212; 617635 by 1089.

7. Multiply in *three* lines :—

(1) 765389 by 64164, by 189279, and by 83256.
(2) 92135 by 10813212; 459896 by 864729; 1234567 by 4321089.
(3) 7893261 by 5678109; 5710987 by 105613212.

8. Multiply 876043 by 1449117 and by 28917136 in *three* lines.

114. Abbreviated methods of Multiplication.

(1) To multiply a number by 5.

RULE. *Annex one cipher to the right of the multiplicand, and divide the result by 2. The quotient is the required product.*

Ex. 1. Multiply 879324 by 5.

2)8793240
4396620 = the required product.

Ex. 2. Multiply 6508 by 15.

2)65080 = product by 10............(1)
32540 = product by 5............(2)
97620 = the required product, adding (1) and (2).

(2) To multiply a number by 25.

RULE. *Annex two ciphers to the right of the multiplicand, and divide the result by 4. The quotient is the required product.*

Ex. 1. Multiply 57943 by 25.

4)5794300
1448575 = the required product.

Ex. 2. Multiply 7575 by 35.

4)757500
189375 = product by 25.........(1)
75750 = product by 10.........(2)
265125 = the required product, adding (1) and (2).

Ex. 3. Multiply 6213 by 75.

4)621300 = product by 100......(1)
155325 = product by 25......(2)
465975 = the reqd. prod., subtracting (2) from (1).

(3) To multiply a number by 125.

RULE. *Annex three ciphers to the right of the multiplicand, and divide the result by 8. The quotient is the required product.*

Ex. Multiply 860978 by 125.

8)860978000
107622250 = the required product.

(4) To multiply a number by a number all the figures of which are *nines.*

RULE. *Annex as many ciphers to the right of the multiplicand as there are nines in the multiplier, and from the result subtract the number itself. The difference is the required product.*

Ex. Multiply 6875 by 999.

6875000 = product by 1000......(1)
6875 = product by 1........(2)
6868125 = the reqd. prod. subtracting (2) from (1).

(5) To multiply a number by a number which differs by a small number from 100, 1000, 10000, &c., or from 50, 500, 5000, &c.

Proceed as in the following examples :—

Ex. 1. Multiply 423571 by 98 and by 9997.

(1) $98 = 100 - 2$.	(2) $9997 = 10000 - 3$.
$423571 \times 100 = 42357100$	$423571 \times 10000 = 4235710000$
$423571 \times \ \ 2 = \quad 847142$	$423571 \times \quad 3 = \quad 1270713$
$\therefore$ the product $= \underline{41509958}$.	$\therefore$ the product $= \underline{4234439287}$.

Ex. 2. Multiply 6854 by 496.

Here, $496 = 500 - 4$.

$6854 \times 5{,}00 = 68540{,}00 \div 2 = 3427000$

$6854 \times 4 \qquad\qquad\qquad = \quad 27416$

$\therefore$ the required product $= \underline{3399584}$.

(6) To multiply a number by 11.

RULE. *Add each figure to the figure on its left, beginning with 0 on the right, carrying 1 when necessary. The number thus formed is the required product.*

Ex. Multiply 75384 by 11.

$$\begin{array}{r} 75384 \\ 11 \\ \hline 829224 \end{array}$$

Here, $0+4=4$; $4+8=12$, carry 1; $1+8+3=12$, carry 1; $1+3+5=9$; $5+7=12$, carry 1; $1+7=8$; but all the necessary wordings are 4, 12, 12, 9, 12, 8.

(7) To multiply a number by 625.

RULE. *Annex four ciphers to the right of the multiplicand and divide the result by 16. The quotient is the required product.*

Ex. Multiply 4837 by 625.

16)48370000

$\underline{3023125}$ = the required product.

115. Squares, Cubes, &c.

(1) To find the square of a number of two figures.

RULE. *Increase and diminish the number by the complement of its units' figure, and to the product of the two results thus obtained add the square of the complement. The number thus formed is the required square.*

Ex. 1. Find the square of 84 and 95.

Here, the complement of 4 is 6, and of 5 is 5.

(1) $84+6=90$ and $84-6=78$.	(2) $95+5=100$ and $95-5=90$.
$\therefore$ the reqd. square $= 90 \times 78 + 6^2$	$\therefore$ the reqd. square $= 100 \times 90 + 5^2$
$= 7020 + 36$	$= 9000 + 25$
$= 7056$.	$= 9025$.

Ex. 2. Find the square of 467.

$467+67=534$; $467-67=400$
$\therefore 467^2=534\times400+67^2$
$=213600+67^2$.

Again, $67+7=74$; $67-7=60$
$\therefore 67^2=74\times60+7^2$
$=4440+49=4489$.

Hence $467^2=213600+4489=218089$.

(2) To find the difference of the squares of two numbers.

RULE. *Multiply the sum of the numbers by their difference, and the product is the required difference.*

Ex. Find the value of $(339)^2-(319)^2$.

Here, $339+319=658$ and $339-319=20$.

$\therefore$ the required difference $=658\times20=\underline{13160}$.

(3) To express the product of two numbers as the difference of two squares.

RULE. *Find the sum and difference of the numbers and divide each result by 2. The difference of the squares of the two quotients is the required difference of two squares.*

Ex. Express 81×53 as the difference of two squares.

Here $(81+53)\div2=134\div2=67$ and $(81-53)\div2=28\div2=14$,

$\therefore$ the required difference $=(67)^2-(14)^2$.

Examples XXI.

1. Multiply :—

(1) 879326 separately by 5, 25, 75, 125 and 625.
(2) 63945 separately by 15, 35, 75 and 125.
(3) 87911365 separately by 5, 25, 75, 125 and 625.
(4) 4439854 separately by 99, 999, 9999 and 99999.
(5) 5792 separately by 96, 996, 9994 and 9998.
(6) 8734652 separately by 11, 121, 1331 and 99994.

2. Find the squares of :—

(1) 37, 45, 48, 55, 65, 75, 64, 71, 83, 96 and 125.
(2) 108, 149, 156, 183, 215, 391, 478, 456 and 524.

3. Express the following products as the difference of two squares :—65×53; 96×74; 126×84; 245×197; 478×316.

4. Find the values of :—

(1) $(575)^2-(425)^2$; $(101)^2-(99)^2$; $(1639)^2-(739)^2$; $(1811)^2-(689)^2$.
(2) $(753)^2-(625)^2$; $(1723)^2-(277)^2$; $(2731)^2-(269)^2$; $(678)^2-(638)^2$.

5. Divide :—

(1) $(8133)^2-(8131)^2$ by 16264; $(5874)^2-(3795)^2$ by 2079.
(2) $(2259)^2-(1759)^2$ by 4018; $(3156)^2-(968)^2$ by 2188.

6. Find the greatest number of 8 digits which is divisible by 5293.

7. Find the least number of 7 digits which is divisible by 7293.

8. Find the least number of 9 digits and the greatest number of 8 digits which are divisible by 37213.

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116. Division by factors.

When the divisor is the product of two or more factors, we use the following Rule:—

RULE. *The quotient is obtained by dividing in succession by each of the factors of the divisor, and the final remainder at each step is obtained by multiplying its particular remainder by all the divisors preceding its own, and adding the preceding final remainder.*

Ex. 1. Divide 25872 by 56.

$56 = 7 \times 8.$

```
7)25872
8) 3696
    462
```

Dividing in succession by 7 and 8, the quotient is 462.

Ex. 2. Divide 96500093 by 105.

$105 = 3 \times 5 \times 7.$

```
3)96500093
5)32166697...2
7) 6433339...2...8
    919048...3...53.
```

Dividing in succession by 3, 5 and 7, the particular remainders are 2, 2 and 3. The final remainder at the first step is 2; the final remainder at the second step is found thus: $2 \times 3 + 2 = 6 + 2 = 8$. The final remainder at the third step is found thus: $3 \times 5 \times 3 + 8 = 45 + 8 = 53$.

Thus the quotient is 919048 and the remainder 53.

117. Abbreviated methods of Division.

(1) To divide a number by 5, 15, 35, 45, 55 or 65.

RULE. *Multiply the number by 2 and divide the product respectively by 10, 30, 70, 90, 110 or 130, as in Art. 99. The result in each case gives the quotient and for the true remainder divide the remainder so obtained by 2.*

Ex. Divide 86246 by 5 and 15623 by 45.

(1)

```
      86246×2
1,0)17249,2
    17249...2
```

Thus the quotient is 17249, and the remainder $2 \div 2 = 1$.

(2)

```
    15623×2
9,0)3124,6
     347.....16
```

Thus the quotient is 347, and the remainder $16 \div 2 = 8$.

(2) To divide a number by 25, 75, 175, 225, 275 or 325.

RULE. *Multiply the number by 4, and divide the result by 100, 300, 700, 900, 1100, or 1300 as in Art. 99. The result in each*

case gives the quotient and for the true remainder divide the remainder so obtained by 4.

Ex. Divide 37057 by 25, and 905785 by 175.

(1) $\underline{37057 \times 4}$
1,00)1482,28
1482...28
Thus the quotient is $\underline{1482}$, and the true remainder $28 \div 4 = \underline{7}$.

(2) $\underline{905785 \times 4}$
7,00)36231,40
5175...640
Thus the quotient is $\underline{5175}$, and the remainder $640 \div 4 = \underline{160}$.

(3) To divide a number by 125, 375 or 875.

RULE. *Multiply the number by 8, and divide the product respectively by 1000, 3000, or 7000, as in Art. 99. The result in each case gives the quotient and for the true remainder divide the remainder so obtained by 8.*

Ex. Divide 905785 by 125, and 1607708 by 375.

(1) $\underline{905785 \times 8}$
1,000)7246,280
7246 ..280
Thus the quotient is $\underline{7246}$, and the remainder $280 \div 8 = \underline{35}$.

(2) $\underline{1607708 \times 8}$
3,000)12861,664
4287...664
Thus the quotient is $\underline{4287}$, and the remainder $664 \div 8 = \underline{83}$.

(4) To divide a number by 625.

RULE. *Multiply the number by 16, and divide the result by 10000, as in Art. 99. The quotient is the required quotient and for the true remainder divide the remainder so obtained by 16.*

Ex. Divide 3023173 by 625.

$\underline{3023173 \times 16}$
1,0000)4837,0768
4837...768

Thus the quotient is $\underline{4837}$, and the remainder $768 \div 16 = \underline{48}$.

(5) The method of Long Division may be much shortened by the use of Art. 112 (3).

Ex. Divide 15218125 by 3854.

3854)15218125(3948 *Ans.*
36561
18752
33365
2533 rem.

Mentally thus : $3 \times 4 = 12$ and $\mathbf{6} = 18$,
carry 1, add 3×5, 16 and $\mathbf{5} = 21$;
carry 2, add 3×8, 26 and $\mathbf{6} = 32$;
carry 3, add 3×3, 12 and $\mathbf{3} = 15$.

Then bringing down the next digit in the dividend, repeat the process for the next digit in the quotient.

Examples XXII.

1. Divide by factors :—

(1) 1461408 by 32 ; 347808 by 56 ; 1556334 by 162.

(2) 7825687 by 64 ; 6598769 by 84 ; 8791605 by 88.

(3) 7654325 by 96 ; 12345678 by 68 ; 36925814 by 82.

(4) 76538959 separately by 28, 64, 72, 96 ; 39541234 by 256.

(5) 87625432 by 726 ; 17927618 by 476 ; 5213742 by 1142.

(6) 3790603808 separately by 132, 196, 378 ; 3246541 by 792.

2. Divide :—

(1) 37964 separately by 5, 50, 500, 5000 and 25.

(2) 8754316 separately by 5, 15, 25, 35, 45, 55, 65 and 75.

(3) 90273189 separately by 125, 175, 225 and 275.

(4) 154725876 separately by 125, 375, 625 and 875.

(5) 68015637 by 8654 ; 57300652 by 5129, and 36942536 by 4204.

---

118. Average, Shares, Barter, &c.

(1) To find the average of two or more numbers.

RULE. *Divide the sum of the numbers by their number and the quotient is the required average.*

Ex. 1. The attendance at a school was 254 on Monday, 326 on Tuesday, 204 on Wednesday and 192 on Thursday. Find the average daily attendance of the 4 days.

On Monday the attendance was	254.	$976 \div 4 = 244$.
Tuesday	326.	
Wednesday	204.	∴ the average was 244.
Thursday	192.	
∴ in 4 days	976.	

Ex. 2. The yearly expenses of a person during the first 4 years was *Rs.*675, during the next 5 years *Rs.*825, and during the following 7 years *Rs.*977. What was his average expenses?

In 4 years the exp. amt. to	(*Rs.*675 × 4) or *Rs.*2700,	16)*Rs.*13664.
5	(*Rs.*825 × 5) or *Rs.*4125,	*Rs.*854.
7	(*Rs.*977 × 7) or *Rs.*6839,	∴ the average
∴ in 16 years ...	*Rs.*13664.	was *Rs.*854.

(2) To divide a given number into parts, having certain given relations among them.

Proceed as in the three following Examples.

Ex. 1. Divide 184 oranges between Ram and Gopal, giving Ram 7 times as many as Gopal.

If Gopal gets 1 orange, Ram gets 7 oranges ; and $1+7=8$.

$$\left.\begin{aligned}\therefore\ \text{Gopal's share}&=(184\div 8)\text{ or }\underline{23}\text{ oranges,}\\ \text{and Ram's share}&=(23\times 7)\text{ or }\underline{161}\text{ oranges.}\end{aligned}\right\}$$

Ex. 2. Divide 384 rupees among A, B, C and D, so that for every 5 rupees given to A, B gets 7 rupees, C 8 rupees, and D 12 rupees.

$5+7+8+12=32$. $\quad$ $Rs.384\div 32=Rs.12$.

$\therefore$ A gets $Rs.12\times 5=$ R$\underline{60}$, $\quad$ C gets $Rs.12\times 8=$ R$\underline{96}$,

B .. $Rs.12\times 7=$ R$\underline{84}$, $\quad$ and D ... $Rs.12\times 12=$ R$\underline{144}$.

Ex. 3. Divide 1351 nuts among 13 men, 17 women, and 30 children, giving each woman 5 times the share of each child, and each man the share of a woman and a child.

If each child gets 1, a woman gets 5 and a man $5+1$ or 6. Therefore 30 children get 30, 17 women get 5×17 or 85, and 13 men 6×13 or 78.

Now, $30+85+78=193$; and $1351\div 193=7$.

$\therefore$ *the children will have* 7×30 *or* $\underline{210}$ *nuts,*
the women ...7×85 or $\underline{595}$ nuts,
and the men ...7×78 or $\underline{546}$ nuts.

Ex. 4. How many horses worth 132 rupees each, must be given for 1476 sheep worth 11 rupees each ?

The cost of 1476 sheep at $Rs.11$ each $=Rs.1476\times 11=Rs.16236$. And $16236\div 132=123$. $\quad\therefore$ the required no. of horses $=\underline{123}$.

Ex. 5. If a man can travel 2440 miles in 4 weeks, how many miles can he travel in 9 weeks ?

In 4 weeks the man travels 2440 miles.

$\therefore$ in 1 week ... ... $2440\div 4$ or 610 miles.
$\therefore$ in 9 weeks ... ... 610×9 or $\underline{5490}$ miles.

119. Backward process.

In a backward process, beginning from the last number, we change Addition into Subtraction, Subtraction into Addition, Multiplication into Division and Division into Multiplication.

Ex. What number is that which if I divide by 6, to the quotient add 25, from the sum take 36 and multiply the remainder by 4, the product is 40 ?

$$\text{The required number}=(40\div 4+36-25)\times 6=(46-25)\times 6$$
$$=21\times 6=\underline{126}.$$

Examples XXIII.

1. What is the average age of 4 men whose ages are 47, 55, 29 and 77 respectively?

2. In a school register of daily attendance the numbers for a certain week were—Monday 83, Tuesday 80, Wednesday 75, Thursday 80, Friday 78, Saturday 72. What was the average daily attendance?

3. At a competitive examination there were 4 candidates at the age of 19, 3 at 20, 2 at 22 and 3 at 24. Find the average age.

4. A man's income for 3 years is *Rs.*250 a year, for the next 5 years it is *Rs.*294 and for the next 4 years *Rs.*309. What is his average income for the 12 years?

5. In the month of April, a man slept 7 hours on each of 16 nights, 6 hours on each of 8 nights, 8 hours on each of 5 nights and 10 hours on the last night. How long did he sleep each night on an average during the month?

6. Divide 1008 rupees among A, B and C, so that for every 2 rupees A gets, B shall get 3 rupees and C 4.

7. Divide 2624 apples among A, B and C, so that for every 5 apples given to A, B may get 11, and C 16.

8. The price of a carriage with horse is 1590 rupees, and the price of the carriage is 5 times that of the horse. Find the price of the horse.

9. If 23 men earn 1380 rupees in a month, how many men will earn 1980 rupees in the same time?

10. A gentleman left 225,000 rupees to be divided amongst his 4 sons and 3 daughters in such a way that each son would receive three times as much as each daughter. How much did each son and each daughter receive?

11. Divide 33775 rupees among 13 men, 17 women and 30 children, giving each woman 5 times the share of each child, and each man the share of a woman and a child.

12. 24 cows are worth 864 rupees, and 45 horses are worth 2835 rupees; how many of such horses ought to be exchanged for 2520 of such cows?

13. Divide 2954 rupees among A, B, C and D, so that for every 2 rupees given to A, B shall get 3 rupees, C 4 and D 5.

14. A farmer had a horse worth 375 rupees and exchanged it for a yoke of oxen and three cows; the oxen he sold for 125 rupees, two of the cows at 85 rupees each and the other for 76 rupees. How much did he lose by the bargain?

15. Find a number such that if I divide it by 3, and then add 4, then divide the result by 2 and add 3, then multiply the result by 4 and subtract 5, the result of the whole will be 19.

Examples worked out.

Ex. 1. I have to divide 750 rupees among a number of boys and girls, giving 3 rupees to each boy and 2 rupees to each girl; there are as many boys as girls; how many boys are there?

Here, 1 boy + 1 girl receive $(3+2)$ or 5 rupees.

$\therefore$ $150\times(1$ boy $+1$ girl) receive 5×150 or 750 rupees,

for $750\div5=150$. $\therefore$ the number of boys $=$ 150. *Ans.*

Ex. 2. A man living at the rate of 750 rupees a year for 6 years finds that he is exceeding his income, and reduces his expenditure to 540 rupees a year; at the end of 4 years he finds that he is just out of debt; what is his income?

In 6 years his expenses amount to *Rs.* $750\times6=$ *Rs.* 4500.

In 4 years... ... ... *Rs.* $540\times4=$ *Rs.* 2160.

$\therefore$ in 10 years his income amounts to *Rs.* 6660,

$\because$ his debts of the first 6 years are paid off by the savings of the last 4 years,

$\therefore$ his yearly income $=$ *Rs.* $6660\div10=$ *Rs.* 666. *Ans.*

Ex. 3. Two persons started at the same time from A and B. One left A for B travelling 5 miles an hour, and the other from B for A travelling 7 miles an hour. The distance between A and B is 108 miles. When and where did they meet?

While the first walks 5 miles, the second walks 7 miles, and the distance to be travelled by both before they meet is 108 miles.

Now, $5+7=12$; and $108\div12=9$. $\therefore$ they meet after 9 hours.

Also, the distance from A where they meet $=5\times9$ or 45 miles.

Ex. 4. A man bought 75 cows at 50 rupees each, 94 cows at 43 rupees each and 106 cows at 48 rupees each; at what price per head must he sell the cows, so as to gain 595 rupees by his bargain?

The cost of 75 cows at *Rs.* 50 each $=$ *Rs.* $75\times50=$ *Rs.* 3750.

... 94 cows at *Rs.* 43 ... $=$ *Rs.* $94\times43=$ *Rs.* 4042.

... ... 106 cows at *Rs.* 48 ... $=$ *Rs.* $106\times48=$ *Rs.* 5088.

$\therefore$ the cost of 275 cows $=$ *Rs.* 12880.

gain $=$ *Rs.* 595.

$\therefore$ the selling price of 275 cows $=$ *Rs.* 13475.

$\therefore$ the selling price of a cow $=$ *Rs.* $13475\div275=$ *Rs.* 49.

Ex. 5. If 30 men can build a wall in 12 days, how many men can build it in 18 days?

In 12 days the work can be done by 30 men.
$\therefore$ in 2 days the work ... (30×6) or 180 men.
$\therefore$ in 18 days ... ... $(180\div9)$ or 20 men. *Ans.*

Ex. 6. Reduce 7 men, 12 women and 5 children to an equivalent number of children, supposing 2 women equivalent to a man, and 3 children equivalent to a woman.

1 woman $=$ 3 children ; $\therefore$ 12 women $=3\times12$ or 36 children.

Again, 1 man $=$ 2 women $=2\times3$ or 6 children.

$\therefore$ 7 men $=7\times6$ or 42 children.

Hence, 7 men $+$ 12 women $+$ 5 children $=(42+36+5)$ or 83 children.

Ex. 7. A tank has three pipes attached to it. By two of these 482 and 516 maunds of water respectively enter into it every hour, while by the third 322 maunds go out in the same time. When all the pipes are opened together the tank becomes full in 320 hours; how many maunds of water can the tank hold?

The quantity of water remaining in the tank per hour when all the pipes are opened together $=(482+516-322)$ or 676 maunds.

$\therefore$ in 320 hours, the water remg. $=676\times320$ or 216320 maunds.

Hence the tank can hold 216320 maunds of water.

Ex. 8. A man at his death directed in his will that his property should be divided among his four sons as follows:—The eldest to receive *Rs.*1032 more than the second; the second *Rs.*1023 less than what the third and fourth together receive; the third and the fourth together to receive *Rs.*3251; but the third to receive *Rs.*31 less than the fourth. Find the value of his whole property, and the share of each son.

Since the third and the fourth together receive *Rs.*3251, and the third *Rs.*31 less than the fourth, therefore the third's share $=(3251-31)\div2$ or *Rs.*$(3220\div2)$ *i.e.*, *Rs.*1610.

Therefore the fourth's share $=$ *Rs.*$(1610+31)$ or *Rs.*1641.
... second's ... $=$ *Rs.*$(3251-1023)$ or *Rs.*2228.
... eldest's ... $=$ *Rs.*$(2228+1032)$ or *Rs.*3260.

Hence, the whole estate $=$ *Rs.*$(1610+1641+2228+3260)$
$=$ *Rs.*8739. *Ans.*

Ex. 9. Prove that the sum of the six numbers that can be formed by different arrangements of the three digits 2, 5 and 7 taken all together can be represented by the expression $2\times(2+5+7)(10^2+10+1)$.

The six numbers that can be formed by different arrangements of the digits 2, 5, 7 are the following:—

257, 275, 527, 572, 725, 752.

In the above numbers, we see that

(1) 2 occurs twice in the hundreds' place, twice in the tens' and twice in the units,' and their sum $=2\times(200+20+2)=2\times2\times(100+10+1)$ $=2\times2\times(10^2+10+1)$.

(2) 5 occurs twice in the hundreds' place, twice in the tens' and twice in the units', and their sum $=2\times(500+50+5)=2\times5\times(10^2+10+1)$.

(3) 7 occurs twice in the hundreds' place, twice in the tens' and twice in the units', and their sum $=2\times(700+70+7)=2\times7\times(10^2+10+1)$.

Hence the sum of all the six numbers
$=2\times2\times(10^2+10+1)+2\times5\times(10^2+10+1)+2\times7\times(10^2+10+1)$
$=2\times(2+5+7)\times(10^2+10+1)$.

Miscellaneous Examples I.

1. What number must be added to 7965499 to give 541850036?

2. How much is the difference between 628716 and 79019 greater than the sum of 56095, 2800, 10009, 7097, 159, 3000 and 90829?

3. The sum of two numbers is 125678, and their difference is 1422; find the numbers.

4. The sum of two numbers is 15678, and the larger number exceeds the smaller by 1234; find the numbers.

5. What number multiplied by 1256 will give the same product as (i) 314 by 476; (ii) 7536 by 378?

6. A man having bought an estate, sold it again for *Rs.*21128 losing thereby *Rs.*1878; what did the estate cost him?

7. The population of a certain village is 1254 and one out of 33 dies annually. How many die in a year?

8. The product of two numbers is 2225808, and one of them is 936; what is the other number?

9. The difference of two numbers is 6782, and the greater is 178962; what is the smaller number?

10. What number is that which being divided by 4, the quotient increased by 6, the sum multiplied by 4, the product increased by 16, and the sum divided by 44, the quotient will be 10?

11. What number must be multiplied by 327 to produce 1203033?

12. How much greater is the product of 17 and 15 than the product of their sum and difference?

13. Of a town containing 434611 inhabitants, 57569 more are females than males. Find the numbers of males and females.

14. Express 6789×1231 as the difference of two square numbers.

15. The divisor is 712, the quotient 31 and the remainder 699. What is the dividend?

16. What least number must be added to 58667, that the sum may be divisible by 2564?

17. Subtract the value of the second and fourth digits fro that of the third and fifth digits in the number 123456.

18. The quotient arising from the division of 256329 by a certain number is 354, and the remainder is 387. Find the divisor.

19. A person, who was born in 1779, died at the age of 46 years; his son died 27 years afterwards, and his daughter died 13 years after his son; in what year did the daughter die?

20. Of what number is 7036 both divisor and quotient?

21. What number is that, which being divided by 24, the quotient increased by 26, the sum diminished by the difference between 40 and 27, the remainder multiplied by 4, and the product divided by 11 will give 12 for a quotient?

22. The quotient being $=5$ times divisor $=7$ times remainder $=$ 105; find the dividend.

23. The quotient being 958 and the divisor 607, find the dividend. What would the dividend be, had there been a remainder 44?

24. What least number must be subtracted from 2346, that the remainder may be divisible by 135? By what least number must the same be multiplied that the product may be divisible by 36?

25. A house and its furniture cost *Rs.*570600; the house is 8 times the furniture. What is the cost of the house?

26. Find the number, which if I multiply by 7, then subtract 31, then divide the result by 3, then add 5, and then multiply by 4, the result is the square of 10.

27. A merchant has three sorts of sugar; the first and second together weigh 12356 maunds; the third 7152 maunds less than the sum of the first and second; also the second weighs 1647 maunds less than the third. Find the quantity of each sort.

28. The product of two numbers is 1270374, and half of one of them is 3129; what is the other number?

29. There were 2244 pears on a tree. The owner gathered 46 daily for 14 days; he divided the remainder between his son and daughter, giving the former 5 for every 3 that he gave the latter; how many pears did the son receive more than the daughter?

30. The Duke of Wellington died in the year 1852, aged 83; Napoleon was born in the same year as the Duke, and died in 1821; what was Napoleon's age at the time of his death?

31. A speculator gained *Rs.*3560, and afterwards lost *Rs.*3479; he then gained *Rs.*6283, and then lost first *Rs.*1089, and then *Rs.*2361; by how much did his gains exceed his losses?

32. What least number must be subtracted from $72347 \div 11 \times 7$, that the remainder may be divisible by $17 \times 9 \div 3 \times 6$?

33. A merchant bought 122 maunds of oats at *Rs.*2 per maund,

and 256 maunds of an inferior sort at *Re.*1 per maund and mixing the two sorts sold the whole for *Rs.*525. How much did he gain or lose?

34. A man dies worth *Rs.*2427498 to be divided among his three sons. He directed in his will that the eldest and second together shall get *Rs.*1937734, and the second and third together *Rs.*1196570. How many does each receive?

35. How many words are there in a book of 347 pages, if there are 13 words in each line, and 40 lines in each page?

36. A water-tub has two pipes attached to it. The first discharges 14 seers and the second 15 seers of water per minute. When the tub is full, both the pipes are opened at once, and the tub becomes empty in 15 minutes. Find the content of the tub.

37. *A* is 27 years older than *B*, and 15 years younger than *C* who is 54 years of age; *D* is as old as the sum of *A*'s and *B*'s ages. Is *C* older or younger than *D*? how much?

38. *A* has 74 marbles, *B* has 34 more than *A*, and *C* has 16 more than *B*; *A* gives *B* and *C* each 19, *B* gives *A* and *C* each 34, and *C* gives *A* and *B* each 10. How many marbles have *A*, *B* and *C* respectively after these exchanges?

39. A person bought 68 bales of cloth containing 67048 yards; each bale contained 34 pieces, and each piece contained the same number of yards; find the number of yards in each piece.

40. The nuts in a bag were divided among 59 boys and 27 girls; each boy had 3 times as many as each girl; there were just nuts enough and one over to give the girls 7 nuts apiece. How many nuts did the bag contain?

41. A man's annual income is *Rs.*7836. His expenditure in January is *Rs.*632, in February and March *Rs.*1146, in April, May and June *Rs.*1698, and in each of the remaining 6 months *Rs.*595 on an average. How much does he save in the year?

42. A man divided his property worth *Rs.*12547 among his 4 sons, in such a manner that the eldest received *Rs.*126 more than the second, the second *Rs.*131 more than the third, and the third *Rs.*121 more than the fourth. How much did each receive?

43. Three pipes are attached to a water-tub. By two of these 36 and 24 maunds of water respectively enter into it every hour, while by the third 33 maunds go out in the same time. If the tub can hold 2673 maunds of water, when will it be full, if all the pipes are opened together?

44. Express 19191×1225 as the difference of two square numbers.

45. If 256512 be divided by 105, using its factors 3, 5 and 7, find the true quotient and the true remainder.

46. A gentleman left *Rs.*123600 to be divided among his two

sons, four daughters and one sister, in such a way that each daughter would receive twice as much as the sister, and each son one-half of what the three daughters would receive. What did the sister receive ?

47. A man worth 30 lacs of rupees, having no heirs, divides his whole property among his four faithful servants *A*, *B*, *C* and *D*. He gives to *B* twice as much as he gives to *A* and *Rs.*1234 more ; to *C* twice as much as *A* less *Rs.*2284, and to *D* *Rs.*32000. Find his bequest to *A*.

48. *A* and *B* walk at the rates of 10 and 13 miles per hour respectively. If they are walking towards each other, and if the distance between them be 207 miles, find when they will meet.

49. *A* says to *B* and *C*, I have *Rs.*1650 ; *B* replies, if I had *Rs.*753 more than I have, I should have as much as you have ; *C* adds, if I had *Rs.*105 more than I have, I should have as much as both of you. How many more rupees has *C* than *B* ?

50. To what number must 28 be added that the sum being multiplied by 25, the product will be 125625 ?

51. From what number must 302 be subtracted that the remainder being multiplied by 125, the product will be 321000625 ?

52. Divide *Rs.*40 between *A* and *B* in such a way that if *A* gets *Rs.*5, *B* shall get *Rs.*3.

53. Divide *Rs.*30 among *A*, *B* and *C* in such a way that if *A* gets *Re.*1, *B* shall get *Rs.*2 and *C* *Rs.*3.

54. If the sum of 250 and 173 be multiplied by their difference, and the product be divided by 33, find the result.

55. Add together the six numbers you can form with the three figures 3, 4 and 5, taken all together, and multiply the sum by 597.

56. Add together all the numbers that you can form with the four digits 1, 2, 3 and 0 taken all together.

57. Arrange the nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9, in three lines with three digits in each line, so that the sum of these digits may, taken in every possible direction, be 15.

58. Find the sum of all the numbers that you can form with the digits 1, 5, 7, only two digits being taken at a time.

59. By what number must 123456 be divided that if 15328 be added to the quotient and the sum divided again by 8, the quotient will be 7060 ?

60. *A*, *B* and *C* have between them 1467 marbles. *B* has three times as many as *A*, and *C* 131 marbles more than the sum of *A* and *B*. How many has each ?

61. Divide *Rs.*5000, among *A*, *B*, *C* and *D* in such a manner, that if *A* gets *Rs.*2, *B* shall get *Rs.*3, *C* *Rs.*4 and *D* *Rs.*11.

62. Simplify—

(1) $920 \div 23 \times 720 \div (42 \div 7) \times (78 \div 13) \div (5 \times 4)$.

(2) $1250 \times (72 \div 4) \div (20 \times 5) \times (64 \div 16) \div (111 \div 37)$.

63. What least number must be added to $3243 \div 3 \times 9$ that the sum may be divisible by $15 \div 5 \times 8 \times 7$?

64. What number less than 365 added to 730320 will make the number exactly divisible by 365?

65. A man spends *Rs.*1485 annually for 6 years and runs into debt. He then reduces his expenses to *Rs.*1109 a year, and in 10 years just manages to clear off his debts. What is his yearly income?

66. Multiply 765389 by 64164, and by 189279, and by 83256, making in each case only three partial multiplications.

67. A volume of a work contains 6 parts of 128 pages each, and there are 46 lines in each page and 58 letters in each line. How many letters are there in 9 volumes?

68. A man spends *Rs.*600 a year for 5 years and saves some money; he then raises his expenditure during the next 7 years to *Rs.*720 a year, and finds all his savings spent. What does he earn each year?

69. The sum of the product of two numbers and 1420 is 349612; one of the numbers is 864. Find the other number.

70. Find the number which being divided by 24 gives a quotient which if increased by 36 and the sum multiplied by 24 gives a product that will be greater than 876 by 300.

71. If in dividing a number by 336, the operation be performed by short division by employing the factors 6, 7 and 8 in succession and the several remainders be 1, 2 and 3; find the complete remainder.

72. If two men start from the same place and travel in opposite directions, the one at the rate of 42 miles and the other 45 miles a day, how far apart will they be at the end of 12 days?

73. If two men start from the same place and travel in the same direction, the one at the rate of 512 miles and the other 540 miles a week, how far apart will they be at the end of 8 weeks?

74. A dividend is 4637064283, the quotient is 80496 and the remainder is 11707; what is the divisor?

75. If 20 men can do a piece of work in 11 days, how many days will it take 22 men to do it?

76. *A*, *B*, *C* and *D* have among them *Rs.*69; *A*, *B* and *C* have among them *Rs.*48; *B* and *C Rs.*31, *B* having *Rs.*15 more than *C*; how many more rupees have *A* and *B* than *C* and *D*?

77. The product of three numbers is 535500; one of the numbers is 75, another is 68. What is the third?

78. The product of three numbers is 8937992; the third number is double the second, and the sum of the second and third is 906. Find the first number.

79. Divide *Rs*.3975 among A, B, C and D so that B may have *Rs*.23 more than A, C *Rs*.45 more than A and B together, and D *Rs*.29 less than B and C together.

80. A grazier bought a certain number of bullocks for *Rs*.4900, and sold a part of them for *Rs*. 3840 at *Rs*. 32 a head, and gained on those he sold *Rs*. 480. How much did he gain a head, and how many did he buy at first?

CHAPTER III.

Compound Quantities.

120. If one quantity contains another of the same kind an exact number of times, the first is said to be a **multiple** of the second, and the second a **submultiple** or **aliquot part** of the first.

121. We have already seen that in considering quantities of the same kind, we take an arbitrary but well-defined quantity of that kind as our **unit**, and finding *how many* times it is contained in each of them, we express them as whole numbers. But in this way very large quantities will be expressed by very high numbers, which give by inspection little idea of their relative values; to obviate this inconvenience we take such multiples of the *unit* as will enable us to avoid very high numbers. Thus, of length, we take a **yard** as our unit, but to measure long lengths we use the **mile**, a high multiple of the yard. Hence has arisen the custom of using large units for large quantities and small units for small quantities. Thus, we say that the price of a chair is 8 **rupees**; that of a book is 14 **annas**, and that of a pencil is 2 **pice**.

122. Since it is the custom to use more than one unit for things of the same kind, it would be convenient to select one quantity as the principal or **standard** unit, and thence derive the various minor or **auxiliary** units, either by dividing this unit into a number of equal parts or by multiplying it a number of times. The *standard* unit of any quantity and its *auxiliary* units are called its **denominations**.

123. In the preceding Chapter we have considered only such *abstract* numbers, or such *concrete* numbers of one denomination as are formed by figures whose local values are always regulated by the same fixed number *ten*; but the rules given can easily be extended to *concrete* numbers of different denominations, wherein the local values of the figures are connected by more numbers than one; as, for instance, to **rupees**, **annas** and **pies**, where twelve pies are equivalent to *one* anna, which is the next higher denomination; *sixteen* annas to one rupee, which is the next denomination in order; the *different* numbers 12 and 16 connecting the denominations, in the same manner, as the *fixed* number 10 was supposed to connect the denominations of Integers.

Here, the standard unit **rupee** is divided into 16 equal parts to obtain the auxiliary unit *anna*, and into 16×12 or 192 equal parts to obtain the auxiliary unit *pie*. Thus, the rupee, anna and pie are the various denominations of money.

124. The processes employed in cases of this nature are **Reduction**, and the fundamental operations are then called **Compound Addition, Compound Subtraction, Compound Multiplication** and **Compound Division**, each of which will be exemplified in order; and the various **Tables**, which furnish us with a list of the relative magnitudes of the different *auxiliary* units, and by means of which the above operations are conducted, are given below in order.

TABLE I. MONEY.

British Indian Money.

125.

3 Pies (*p.*) or 2 half-pice	make	1 Pice (*ps.*)
2 Pice	„	1 Half-anna.
4 Pice or 12 pies	„	1 Anna (1*a.*)
16 Annas	„	1 **Rupee** (*Re.*1 or 1/.)
15 Rupees	„	1 Sovereign.

126. Accounts in Bengali are kept by the following Table.

4 Cowries	make	1 Ganda	4 Pans	make	1 Chouk
5 Gandas	„	1 Buri (Paisa)	4 Chouks	„	1 Kahan or Rupee
4 Buris	„	1 Pan (Anna)			

Also 1 *Cowry* or *Bat*=3 *Krantis*=4 *Kags*=5 *Tals*=7 *Dwips*=9 *Dantis*=27 *Jabs*=80 *Tils*=320 *Ranus*=1280 *Bahars* or *Ghuns*=25600 *Bindus*.

Therefore 1 *Cowry*=4 *Kags*; *1 Kag*=20 *Tils*; 1 *Til*=16 *Ghuns*; 1 *Ghun*=20 *Bindus*.

127. The following Tables are in use in different parts of India.

IN BEHAR, N.-W. P. AND PUNJAB.

5 Cowries	make	1 Adhi
2 Adhis	„	1 Damri
2 Damris	„	1 Chhadam
2 Chhadams	„	1 Adhela
2 Adhelas	„	1 Paisa
2 Paisas	„	1 Taka
2 Takas	„	1 Anna.

IN BOMBAY.

100 Raes	make	1 Quarter
4 Quarters	„	1 Rupee

IN MADRAS.

1 Pagoda=*Rs.*3. 8*a.*

IN CEYLON.

100 Cents make 1 Rupee.

In British India the common medium of exchange is *silver*. The principal coin made of it is called a **Rupee**. The Rupee weighs 1 tola or 180 grains, and consists of 11 parts of silver and 1 of alloy. The weight of a gold *Mohur* is the same as that of a Rupee and is 180 grs. It consists of 11 parts of gold and 1 of alloy. The values of gold coins are variable, and therefore they are not used in mercantile transactions except the Sovereign, whose value is 15 rupees. The *Cowry*

is a shell brought from the Laccadive and Maldive Islands, and is used for very small payments. They vary in value according to supply in market but they are generally reckoned at 80 to a pice.

N. B.—The cowries as shells are now going out of use, but cowries (called karas) are in use in keeping accounts.

Of the copper coins, a half-anna weighs 200 grains; a pice weighs 100 grains, and a half-pice 50 grains.

15 *Sicca* Rupees = 16 Rupees. The *Doctor's* Gold Mohur = 16 Rupees; the *Lawyer's* Gold Mohur = 17 Rupees.

Gold coins (*obsolete*): Five-rupee piece; Ten-rupee piece; Gold Mohur; Double Gold Mohur.

Silver coins (*current*): Two-anna piece; Four-anna piece or Quarter-rupee; Eight-anna piece or Half-rupee; Rupee.

Copper coins (*current*): Pie; Half-pice; Pice or Paisa; Double paisa or Half-anna.

Note. *Re.* 1 = 2 half-rupees = 4 quarter-rupees or four-anna pieces = 8 two-anna pieces, and 1 anna = 2 double paisas. Also *Re.* 1 = 64 pice = 192 pies.

English Money.

128.

2 Farthings (*q.*)	make	1	Half-penny ($\frac{1}{2}d.$)
2 Half-pence	...	1	Penny (*d.*)
12 Pence	...	1	Shilling (1*s.* or 1/.)
20 Shillings	...	1	**Pound** (£1.)

[1, 2, 3 farthings are usually denoted by $\frac{1}{4}d.$, $\frac{1}{2}d.$, $\frac{3}{4}d.$, respectively.

Money as expressed by means of these denominations is called **Sterling** money, in order to distinguish it from **stocks, shares,** &c. The **Standard** gold coin of England is made of a metal consisting of 22 parts of *pure gold*, and 2 parts of *copper*. Each of these 24 parts is called a *Carat*. Pure gold is said to be 24 carats *fine* and standard gold 22 carats *fine*. The *Pound sterling* is represented by a gold coin called a **sovereign,** and from 40 pounds Troy of standard gold are coined 1869 sovereigns; and the value of gold of the *Mint-Fineness* called 22 carat gold, is £3. 17*s.* $10\frac{1}{2}d.$ per ounce.

The **standard** silver coin consists of 37 parts of *pure silver*, and 3 parts of *copper*. A pound Troy of this metal furnishes 66 *shillings*, and the *Mint-Price* of standard silver is 5*s.* 6*d.* per ounce. The silver coinage is not a *legal tender* for more than 40*s.*, the gold coinage being the *general* standard of value.

In the **copper coinage,** 24 pennies are made from an Avoirdupois pound of copper. This coin is not a legal tender for more than 12*d.* The coins now *current* in England are the following:—

Copper coins: Farthing; Half-penny; Penny.

Silver coins: Three-penny piece; Four-penny piece; Six-penny piece; Shilling; Florin (2*s.*); Half-crown (2*s.* 6*d.*); Crown (5*s.*).

Gold coins; Half-sovereign; Sovereign.

The following coins were formerly in use, but now they are *obsolete*.

Silver coins: Groat (4*d.*); Tester (6*d.*).

Gold coins : Noble (6*s.* 8*d.*) ; Angel (10*s.*) ; Half-Guinea (10*s.* 6*d.*). Mark or Merk (13*s.* 4*d.*) ; Guinea (21*s.*) ; Carolus (23*s.*) ; Jacobus (25*s.*) ; Moidore (27*s.*).

Note. 1 shilling = 2 six-pences = 3 four-penny pieces = 4 three-penny pieces. Also 1 half-crown = 5 six-pences ; 1 half-guinea = 21 six-pences ; 1 guinea = 42 six-pences.

Also £1 = 4 crowns = 8 half-crowns = 10 florins = 40 six-pences = 80 three-pences = 240*d.* = 960*q.*

I. REDUCTION.

129. When a quantity is expressed in one denomination only, it is called a **simple** quantity ; as 7 rupees ; 5 yards.

When a quantity is expressed in several denominations, it is called a **compound** quantity ; as *Rs.*8. 2*a.* 3*p.* ; 5 yards 2 feet 3 inches.

130. Reduction is the process by which we convert or change (1) a simple or a compound quantity into terms of its lower denominations, or (2) a simple quantity into terms of its higher denominations, so that the *real* or *absolute* values remain unaltered.

131. *To express a quantity in terms of its lower denominations.* (**Descending Reduction**).

RULE. Multiply the number in the highest denomination by the number of units of the next inferior denomination contained in one unit of the highest, and to the product add the number (if any) of the inferior denomination in the quantity proposed ; and repeat this for each succeeding denomination till the required one is obtained.

Ex. 1. Reduce *Rs.*315 to *pice.*

*Rs.*315	*Re.*1 = 16*a.*
16	∴ *Rs.*315 = (315 × 16)*a.* = 5040*a.*
5040*a.*	Again, 1*a.* = 4*ps.*
4	∴ 5040*a.* = (5040 × 4)*ps.* = 20160*ps.*
20160*ps.*	∴ *Rs.*315 = 20160*ps.*

Ex. 2. Reduce *Rs.*5. 14*a.* 6*p.* to *pies.*

Rs. 5. 14*a.* 6*p.*	*Rs.*5 = 5 × 16*a.* = 80*a.*
16	∴ *Rs.*5. 14*a.* = 94*a.*
94*a.* (5 × 16 + 14)	Again, 94*a.* = 94 × 12*p.* = 1128*p.*
12	∴ *Rs.*5. 14*a.* 6*p.* = 94*a.* 6*p.* = 1128*p.* + 6*p.*
1134*p.* (94 × 12 + 6)	= 1134*p.*

Ex. 3. Reduce £25. 13*s.* 6¾*d.* to *farthings.*

£ 25. 13*s.* 6¾*d.*	£1 = 20*s.* ; ∴ £25 = 25 × 20*s.* = 500*s.*
20	∴ £25. 13*s.* = 500*s.* + 13*s.* = 513*s.*
513*s.*	Again, 1*s.* = 12*d.* ; ∴ 513*s.* = 513 × 12*d.* = 6156*d.*
12	∴ 513*s.* 6*d.* = 6156*d.* + 6*d.* = 6162*d.*
6162*d.*	Again, 1*d.* = 4*q.* ; ∴ 6162*d.* = 6162 × 4*q.*
4	= 24648*q.*
24651*q.*	∴ £25. 13*s.* 6¾*d.* = 24648*q.* + 3*q.* = 24651*q.*

Here, the *denominations* are separated by a point as (.); and this is necessary to distinguish them from *ordinary* numbers, which do not require it, because their local values are all fixed and certain.

Examples XXIV.

1. Reduce to *annas* :—

(1) *Rs.*17 ; *Rs.*19 ; *Rs.*42 ; *Rs.*45 ; *Rs.*69 ; *Rs.*84 ; *Rs.*95.
(2) *Rs.*87 ; *Rs.*120 ; *Rs.*245 ; *Rs.*460 ; *Rs.*9. 12*a.* ; *Rs.*20. 14*a.*
(3) *Rs.*36. 6*a.* ; *Rs.*53. 13*a.* ; *Rs.*87. 11*a.* ; *Rs.*79. 15*a.* ; *Rs.*234. 11*a.*

2. Reduce to *pies* :—

(1) *Rs.*34 ; *Rs.*56 ; *Rs.*97 ; *Rs.*146 ; *Rs.*342 ; *Rs.*496.
(2) *Rs.*84. 5*a.* ; *Rs.*76. 12*a.* ; *Rs.*265. 9*a.* ; *Rs.*804. 13*a.* ; *Rs.*945. 6*a.*
(3) *Rs.*15. 8*a.* 3*p.* ; *Rs.*7. 13*a.* 11*p.* ; *Rs.*8. 0*a.* 5*p.* ; *Rs* 9. 10*a.* 9*p.*
(4) *Rs.*425. 7*a.* 9*p.* ; *Rs* 550. 3*a.* 11*p.* ; *Rs.*1250. 5*a.* 7*p.* ; *Rs.*5050. 14*a.* 1*p.* ; *Rs.*456. 14*a.* 11*p.* ; *Rs.*31. 10*a.* 1*p.* ; *Rs.*343. 8*a.* 7*p.*

3. Reduce (i) to *pice* and (ii) to *pies* :—

(1) *Rs.*52 ; *Rs.*19 ; *Rs.*112. 6*a.* ; *Rs.*36. 11*a.* 2*ps.* ; *Rs.*20. 8*a.* 3*ps.*
(2) *Rs.*87. 10*a.* 1*ps.* ; *Rs.*172. 5*a.* 3*ps.* ; *Rs.*225. 9*a.* 2*ps.* ; *Rs.*476. 12*a.* 1*ps.* ; *Rs.*782. 0*a.* 3*ps.* ; *Rs.*13. 10*a.* 3*ps.* ; *Rs.*215. 7*a.* 3*ps.*

4. Reduce (i) to *gandas* and (ii) to *cowries* (karas) :—

(1) *Rs.*19 ; *Rs.*34 ; *Rs.*56 ; *Rs.*78 ; *Rs.*105 ; *Rs.*84. 7*a.*
(2) *Rs.*102. 15*a.* 1*ps.* ; *Rs.*24. 14*a.* 3*ps.* ; *Rs.*405. 13*a.* ; *Rs.*75. 7*a.* 5 *gan.*
(3) *Rs.*48. 9*a.* 10 *gan.* ; *Rs.*53. 13*a.* 17 *gan.* ; *Rs.*9570. 14*a.* 16 *gan.*

5. Reduce to *cowries* (karas) :—

*Rs.*53. 13*a.* 17 *gan.* 2 *cow.* ; *Rs.*68. 9*a.* 11 *gan.* 1 *cow.* ; *Rs.*18. 6*a.* 12 *gan.* 2 *cow.* ; *Rs.*5942. 0*a.* 17 *gan.* 3 *cow.*

6. Reduce (i) to *pice* and (ii) to *pies* :—

(1) 175 half-rupees ; 370 quarter-rupees ; 845 two-anna pieces.
(2) 425 double-paisas ; 3116 two-anna pieces ; 2415 half-rupees.
(3) 34212 quarter-rupees ; 20157 double-paisás ; 67950 four-anna pieces ; 827 eight-anna pieces.

7. Reduce (i) to *half-rupees* ; (ii) to *quarter-rupees* and (iii) to *two-anna pieces.*

(1) *Rs.*729 ; *Rs.*925 ; *Rs.*1228 ; *Rs.*1427 ; *Rs.*4243 ; *Rs.*97403.
(2) *Rs.*858. 8*a.* ; *Rs.*9726. 8*a.* ; *Rs.*73246 ; *Rs.*57509.

8. Reduce (i) to *half-annas* and (ii) to *half-pice* :—

*Rs.*75. 6*a.* ; *Rs.*132. 9*a.* ; *Rs.*150. 0*a.* 2*ps.* ; *Rs.*3005. 10*a.* 2*ps.*

9. Reduce :—

(1) A lac of rupees to *paisás* ; *Rs.*7125. 4*a.* to *four-anna pieces* ; *Rs.*6075. 8*a.* to *two-anna pieces* ; *Rs.*1250. 7*a.* 2*ps.* to *double-paisás* ; *Rs.*9864. 8*a.* to *eight-anna pieces.*
(2) *Rs.*1325. 9*a.* 1*ps.* to *half-paisas* ; *Rs.*3116. 14*a.* 6*p.* to *double-paisas* ; *Rs.*2415. 10*a.* 9*p.* to *half-pice.*

10. Reduce to *shillings* :—

(1) £345 ; £498 ; £795 ; £1402 ; £9086 ; £8092.

(2) £71. 1*s.* ; £490. 18*s.* ; £790. 13*s.* ; £3456. 17*s.* ; £6403. 7*s.*

11. Reduce to *pence* :—

(1) £65 ; £98 ; £156 ; £405 ; £1849 ; £5043 ; £9236.

(2) £134. 15*s.* ; £198. 13*s.* ; £416. 11*s.* ; £526. 5*s.* ; £926. 7*s.*

(3) £2. 6*s.* 8*d.* ; £40. 10*s.* 6*d.* ; £11. 7*s* 9*d.* ; £374. 11*s.* 8*d.*

(4) £655. 13*s.* 6*d.* ; £71. 13*s.* 5*d.* ; £343. 13*s.* 5*d.* ; £1274. 19*s.* 9*d.*

12. Reduce to *farthings* :—

(1) £4. 8*s.* $4\frac{3}{4}$*d.* ; £7. 13*s.* $11\frac{3}{4}$*d.* ; £13. 19*s.* $0\frac{1}{4}$*d.* ; £29. 10*s.* 11*d.*

(2) £101 9*s.* $2\frac{3}{4}$*d.* ; £153. 3*s.* $4\frac{1}{2}$*d.* ; £600. 6*s.* 3*d.* ; £83920. 16*s.* $2\frac{3}{4}$*d.*

13. Reduce (i) to *half-pence* and (ii) to *farthings* :—

(1) 15*s.* 6*d.* ; 18*s.* 9*d.* ; 13*s.* 11*d.* ; 19*s.* 6*d.* ; 8*s.* 10*d.* ; 17*s.* 5*d.*

(2) £4080 ; £8608 ; £8734 ; £726. 18*s* ; £517. 13*s.* ; £2125. 6*s.*

(3) £79 14*s.* 8*d* ; £47. 19*s* $9\frac{1}{2}$*d.* ; £389. 12*s.* $8\frac{1}{2}$*d.* ; £879. 18*s.* $0\frac{1}{2}$*d.*

(4) £1560. 10*s.* $4\frac{1}{2}$*d.* ; £2145. 18*s.* $7\frac{1}{2}$*d.* ; £9136. 15*s.* $9\frac{1}{2}$*d.*

(5) 3899 half-sovereigns ; 4807 crowns ; 8608 half-crowns; 6530 florins; 5869 six-pences ; 6958 groats ; 8009 three-penny pieces ; 9076 guineas ; 3089 half-guineas ; 7632 four-penny pieces ; 1445 moidores ; 2047 nobles ; 3286 florins ; 1983 six-pences.

14. Reduce (i) to *three-penny pieces*, (ii) to *four-penny pieces*, and (iii) to *six-pences* :—

(1) £95 ; £128 ; £8076 ; £1857 ; £9083 ; £9072.

(2) £11. 14*s.* ; £144. 17*s.* ; £2145. 11*s.* ; £4265. 15*s.* ; £3264. 17*s.*

15. Reduce :—

(1) 95 guineas 17*s.* $9\frac{3}{4}$*d.* to *farthings* ; £450. 16*s.* 6*d.* to *six-pences.*

(2) £570. 12*s.* to *florins* ; £382. 7*s.* 6*d.* to *half-crowns* ; £589. 15*s.* to *crowns* ; £3500 17*s.* 6*d.* to *half-crowns.*

(3) £99. 9*s.* 9*d.* to *three-pences* ; 5573 half-crowns to *pence.*

(4) 9571 half-crowns to *six-pences* ; 9100 half-crowns to *three-pences.*

16. Reduce to *farthings* :—

(1) 71 *gui.* 16*s.* $2\frac{1}{2}$*d.* ; 937 *flor.* 1*s.* $2\frac{1}{4}$*d.* ; 2902 *cr.* 1*s.* $3\frac{3}{4}$*d.*

(2) 150 *half-sov.* 7*s.* $2\frac{1}{2}$*d.* ; 79924 *gui.* 12*s.* $2\frac{3}{4}$*d.* ; 7255 *flor.* 1*s.* $3\frac{3}{4}$*d.*

17. For how many children can a treat be provided with *Rs.*32. 8*a.* at 2 annas a head ?

18. How many two-pice stamps can I buy for *Rs.*5. 6*a.* 2*ps.* ?

19. If the cost of a telegram is 3*d.* a word, how many words can be sent for £1. 3*s.* 3*d.* ?

20. A poor woman had only *Rs.*2. 1*a.* 8*p.* to live upon. She spent daily 4 pies for her food. How many days did she live upon ?

132. *To express a simple quantity in terms of its higher denominations.* **(Ascending Reduction.)**

RULE. Divide the number by the number of units which make one unit of the next higher denomination, setting down the remainder (if any) as of the same denomination as its dividend; and continue this process till we come to the required denomination.

Ex. 1. Reduce 1560*ps.* to *rupees, &c.*

4 | 1560*ps.*
16 | 390*a.*
Rs. 24-6*a.*

∵ 4 pice = 1 anna.
∵ 16 annas = 1 rupee.
∴ the result is *Rs.* 24. 6*a.*

Ex. 2. Reduce 30857*p.* to *rupees, annas*, and *pies*.

12 | 30857*p.*
16 | 2571 − 5*p.*
Rs. 160 − 11*a.*

∵ 12 pies = 1 anna.
∵ 16 annas = 1 rupee.
∴ the result is *Rs.* 160. 11*a.* 5*p.*

Ex. 3. Reduce 97403*q.* to *pounds*.

4 | 97403*q.*
12 | 24350 − 3*q.*
20 | 2029 − 2*d.*
£101 − 9*s.*

∵ 4*q.* = 1*d.*
∵ 12*d.* = 1*s.*
∵ 20*s.* = £1.
∴ the result is £101. 9*s.* $2\frac{3}{4}$*d.*

Ex. 4. Reduce 36173 half-pence to *guineas*.

2 | 36173 half-pence
12 | 18086 − 1 *half-penny.*
21 { 3 | 1507 − 2*d.*
21 { 7 | 502 − 1 } 16*s.*
gui. 71 − 5 }

∵ 2 half-penny = 1*d.*
∵ 12*d.* = 1*s.*
∵ 21*s.* = 1 *gui.*
∴ the result is 71 *gui.* 16*s.* $2\frac{1}{2}$*d.*

Examples XXV

1. Reduce to *rupees, annas* and *pies* :—

(1) 25325*p.* ; 57509*p.* ; 51039*p.* ; 679298*p.* ; 37921*p.* ; 456786*p.*
(2) 643294*p.* ; 732394*p.* ; 1982345*p.* ; 967573*p.* ; 1043324*p.*

2. Reduce to *rupees, annas* and *pies* :—

987945*ps.* ; 1234567*ps.* ; 547321*ps.* ; 894956*ps.* ; 5537792*ps.*

3. Reduce to *rupees, annas, &c* :—

(1) 8320 gandas ; 7680 cowries (karas) ; 379498 gandas ; 40768 buris.
(2) 1045673 double-paisás ; 2067544 half-paisás ; 1077760 cowries (karas).
(3) 342876 buris ; 596824 paisás ; 23679 double-paisás ; 103678 half-paisás ; 1155440 cowries (karas).

4. Reduce to *rupees* :—

(1) 1648 half-rupees ; 1892 quarter-rupees ; 2530 two-anna pieces.
(2) 2896 annas ; 5952 paisás ; 920320 gandas ; 24320 cowries (karas).

5. Reduce to *pounds*, *shillings* and *pence* :—

(1) 69132*d*.; 60948*d*.; 90231*d*.; 2733*d*.; 89900*d*.; 157362*d*.
(2) 147040*q*.; 284061*q*.; 123290*q*.; 350000*q*.; 805639799*q*.
(3) 3456799*q*.; 1300137*q*.; 10000019*q*.; 2840799*q*.; 4157399*q*.; 36503249*q*.

6. Reduce to £. *s*. *d*. :—

(1) 899045 half-sovereigns; 971112 crowns; 48073 florins; 886101 half-crowns; 85730 half-pence; 13029 three-pences.
(2) 15137 four-penny pieces; 82556 florins; 28892 half-crowns; 2857 four-penny pieces; 987653 half-pence; 47285 guineas.
(3) 23645 moidores; 49726 half-guineas; 183491 six-pences; 281062 three-pences; 40340 farthings.

7. How much money will be required to buy 37528 penny stamps?

8. A dealer bought 438 cocoanuts at 9 pies each; how many rupees, &c. had he to pay for them?

9. I distributed among 1682 beggars a sum of money, giving them two pice a head; what sum did I spend?

10. If during a festival 2250 people on an average cross the Hugli Bridge daily, each paying 2 pice, what is the collection of the ferry farmer, if the festival lasts for 16 days?

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133. There are some cases in Reduction where we cannot pass directly step by step from the given denomination to the one proposed. We must in such cases pass through an intermediate denomination common to both, and it will be advisable to keep such common denomination as high as possible. Then, find by division what quantity of the proposed denomination is equivalent to the given quantity.

Ex. 1. Reduce £253. 9*s*. 10*d*. to *half-crowns*.

$$\begin{array}{r|l} & £253.\ 9s.\ 10d. \\ & \underline{20} \\ & \underline{5069s.} \\ & \underline{12} \\ 3,0 & \underline{608\,3,8d.} \\ & 2027-28d. \end{array}$$

1 half-crown = 2*s*. 6*d*. = 30*d*.
∴ the result is 2027 half-crowns and 28*d*. or 2*s*. 4*d*. over.

Ex. 2. Reduce *Rs*.31. 10*a*. 2*p*. to £. *s*. *d*., when 1*q*. = 2 *pies*.

$$\begin{array}{l} Rs.31.\ 10a.\ 2p. \\ \underline{16} \\ \underline{506a.} \\ \underline{12} \\ 6074p. \end{array} \qquad \begin{array}{r|l} 2 & \underline{6074p.} \\ & 3037q. \end{array} \qquad \begin{array}{r|l} 4 & \underline{3037q.} \\ 12 & \underline{759-1q.} \\ 2,0 & \underline{6,3-3d.} \\ & £3-3s. \end{array}$$

∴ *Rs*.31. 10*a*. 2*p*. = £3. 3*s*. $3\frac{1}{4}$*d*.

134. Proof. Descending and ascending Reductions are inverse processes; if therefore we perform one process on a given

quantity, and on the result the other process, we ought to get the original quantity.

Thus, if by the descending process we find that £25. 13*s.* $6\frac{3}{4}$*d.* = 24651*q.*, we ought by the ascending process to find that 24651*q.* = £25. 13*s.* $6\frac{3}{4}$*d.*

Examples XXVI.

1. Reduce (i) to *guineas* and (ii) to *half-guineas* :—

£63; £105; £96. 16*s.*; £876. 15*s.*; £538; £10728.

2. Reduce (i) to *crowns* and (ii) to *half-crowns* :—

£265. 10*s.*; £589. 15*s.*; £437. 10*s.*; £620. 5*s.*; £5189. 15*s.*

3. Reduce to *crowns* :—

10987 guineas; £89000; £36. 17*s.* 6*d.*; 18756 four-penny pieces.

4. Reduce to *half-crowns* :—

£48. 17*s.* 6*d.*; £382. 7*s.* 6*d.*; £583. 2*s.* 6*d.*; 670 half-guineas.

5. Reduce to *guineas* :—

28906 florins; 107284 half-crowns; 23810 crowns; 760 half-crowns; £647. 0*s.* 11*d.*; £375. 16*s.* $0\frac{3}{4}$*d.*

6. Reduce to *half-guineas* :—

325 crowns; 10867 half-sovereigns; 3150 four-penny pieces; £3240. 10*s.* 6*d.*; 147 half-crowns.

7. Reduce to £. *s.* *d.*, (1*d.* = 11*p.*) :—

*Rs.*35. 9*a.* 3*p.*; *Rs.*707. 11*a.* 7*p.*; *Rs.*2510. 8*a.* 4*p.*

8. Reduce to *Rs.* *a.* *p.*, (1*q.* = 2 *pies*) :—

£32. 14*s.* 7*d.*; £96. 17*s.* 6*d.*; £903. 17*s.* $6\frac{1}{4}$*d.*; 54 half-guineas; 107 florins; 17 half-crowns.

9. If a guinea be equal to *Rs.*10. 8*a.*; find the number of two-anna pieces contained in 1760 guineas.

10. Reduce 7500 Sicca rupees to *current rupees* and 6432 rupees to *Sicca rupees.*

II. COMPOUND ADDITION.

135. Keeping in mind what was said in Art. 123, we need no additional inquiry to inform us that the fundamental operations on *Compound Quantities* must be performed as in *Integers*, with this difference, that instead of carrying and borrowing *tens*, we must do the same with the *different numbers* which connect their parts together; and we shall therefore merely enunciate the rule for each at the beginning of the portion of the work appropriated to it.

136. Compound Addition is the method of finding a single quantity which is equal to two or more quantities of the same kind. This single quantity is called the **sum** of the given quantities.

RULE. Arrange the quantities under one another according to their denominations, so that units of the same denomination may be in the same vertical column, and draw a line below them. Add together the numbers of the lowest denomination; reduce the sum to the next higher denomination; set down the remainder, if any, under the column, and carry the quotient to the first figure of the next column. Repeat the process with all the columns.

Ex. 1. Add together *Rs.*14. 15*a.* 10*p.*, *Rs.*54. 14*a.* 9*p.*, *Rs.*156. 11*a.* 2*p.*, and *Rs.*34. 14*a.* 10*p.*

Rs.	*a.*	*p.*	
			10*p.*+9*p.*+2*p.*+10*p.*=31*p.*=2*a.* **7***p.*
14	15	10	*Carry* 2*a.*; 2*a.*+15*a.*+14*a.*+11*a.*+14*a.*
54	14	9	=56*a.*=*Rs.*3. **8***a.*
156	11	2	*Carry Rs.*3; *Rs.*3+*Rs.*14+*Rs.*54+*Rs.*156+*Rs.*34
34	14	10	=*Rs.***261**.
*Rs.*261	8	7	*Ans.*

Ex. 2. Add together £156. 8*s.* $9\frac{3}{4}d.$, £33. 15*s.* $11\frac{3}{4}d.$, £204. 0*s.* $1\frac{1}{2}d.$, £5275. 17*s.* 8*d.* and £105. 18*s.* $6\frac{1}{2}d.$

£.	*s.*	*d.*	
156	8	$9\frac{3}{4}$	3*q.*+3*q.*+2*q.*+2*q.*=10*q.*=$2\frac{1}{2}d.$ *Carry* 2*d.*;
33	15	$11\frac{3}{4}$	2*d.*+9*d.*+11*d.*+1*d.*+8*d.*+6*d.*=37*d.*=3*s.* **1***d.*
204	0	$1\frac{1}{2}$	*Carry* 3*s.*; 3*s.*+8*s.*+15*s.*+17*s.*+18*s.*=61*s.*=£3. **1***s.*
5275	17	8	*Carry* £3; £3+£156+£33+£204+£5275+£105
105	18	$6\frac{1}{2}$	=£**5776**.
£5776	1	$1\frac{1}{2}$	*Ans.*

Examples XXVII.

1. Add together :—

(1)		(2)		(3)		(4)		(5)		(6)	
As.	*p.*	*As.*	*p.*	*As.*	*p.*	*As.*	*p.*	*As.*	*p.*	*As.*	*p.*
9	7	12	3	9	8	13	4	12	3	15	4
12	3	13	7	11	2	7	8	14	4	11	10
9	4	4	9	13	4	9	10	3	7	4	5
11	10	7	10	3	7	13	8	2	6	8	11

(7)			(8)			(9)			(10)		
Rs.	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
2	2	2	8	11	9	42	10	9	67	10	6
3	4	3	10	5	9	54	12	6	71	12	9
5	7	9	9	12	5	67	9	3	62	14	9
8	10	6	12	11	6	75	11	6	73	13	8
9	12	7	15	6	8	72	6	7	85	7	5

(11)			(12)			(13)			(14)		
Rs.	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
5	11	3	12	12	3	6	14	9	47	5	2
9	10	10	19	4	10	14	0	3	1	15	9
2	14	9	4	15	8	15	15	5	65	6	0
3	5	11	7	9	5	27	12	11	88	15	3
1	6	7	23	7	6	7	14	4	14	15	10
11	13	6	25	0	2	29	0	5	34	14	10
15	7	7	8	14	3	104	13	1	54	14	9

(15)			(16)			(17)			(18)		
Rs.	*a.*	*ps.*	*Rs.*	*a.*	*ps.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
7	11	2	27	11	2	378	9	10	98	0	9
8	14	3	9	14	3	4	7	4	448	6	5
13	12	1	4	10	1	56	8	8	3839	4	0
315	10	2	156	8	2	464	0	3	97	3	2
23	7	2	215	13	2	368	6	8	136	3	7
625	15	3	18	7	1	535	7	1	4837	4	9
24	0	1	106	14	0	97	3	2	28	10	9
129	13	3	315	0	2	893	15	9	234	11	6
56	8	1	57	14	3	14	10	7	536	12	11

(19)			(20)			(21)			(22)		
Rs.	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
1135	4	3	1325	10	9	3004	7	6	74037	9	4
1243	6	9	7602	11	3	907	5	2	80668	12	0
1575	8	8	3006	7	7	1235	10	7	50087	13	4
2007	7	7	4040	8	6	2727	11	5	136	7	4
3445	9	10	3050	12	5	3647	12	9	3270	2	8
4002	10	11	2225	13	8	7532	9	8	5971	14	8
997	11	10	110	6	6	2121	13	10	58065	9	4
1005	9	9	965	14	11	3333	15	8	360	2	8
2220	13	7	1097	13	4	2025	7	6	943	5	4
997	15	3	2110	6	9	1605	0	10	72459	4	0

2. Add together :—

(1)		(2)		(3)		(4)			(5)			(6)		
s.	*d.*	*s.*	*d.*	*s.*	*d.*	*£.*	*s.*	*d.*	*£.*	*s.*	*d.*	*£.*	*s.*	*d.*
3	$7\frac{3}{4}$	19	$8\frac{1}{4}$	19	$10\frac{1}{4}$	37	13	6	3	7	$6\frac{1}{2}$	18	15	$7\frac{1}{4}$
14	$6\frac{1}{2}$	1	$9\frac{3}{4}$	18	$4\frac{1}{2}$	29	12	4	69	11	10	76	14	2
2	11	15	9	9	5	6	3	9	13	0	$4\frac{1}{4}$	25	10	$2\frac{3}{4}$
15	$8\frac{3}{4}$	13	$3\frac{1}{2}$	15	$8\frac{3}{4}$	55	17	2	37	13	$2\frac{3}{4}$	13	13	$3\frac{1}{2}$
13	4	10	$9\frac{1}{2}$	14	$9\frac{3}{4}$	7	10	10	26	15	7	66	4	$7\frac{1}{4}$

(7)

£.	s.	d.
456	14	$8\frac{3}{4}$
9	16	$4\frac{1}{2}$
83	18	$10\frac{1}{2}$
17	19	7
686	7	$9\frac{1}{4}$
8	15	$6\frac{1}{2}$
3548	19	$9\frac{1}{2}$
95	8	$8\frac{3}{4}$

(8)

£.	s.	d.
8	19	$10\frac{3}{4}$
1379	17	$6\frac{3}{4}$
897	16	$9\frac{1}{2}$
89	18	11
4357	8	$11\frac{3}{4}$
52765	15	$8\frac{3}{4}$
99	19	$11\frac{1}{4}$
67	5	10

(9)

£.	s.	d.
7	19	$3\frac{1}{2}$
46	12	4
276	4	$7\frac{1}{2}$
77	7	$9\frac{3}{4}$
8760	10	6
795	15	$3\frac{1}{4}$
20	4	4
813	11	$7\frac{3}{4}$

(10)

£.	s.	d.
2769	10	$8\frac{1}{2}$
36	11	$2\frac{3}{4}$
472	13	10
4792	18	$4\frac{1}{2}$
3279	15	$8\frac{1}{4}$
24	8	11
429	17	$5\frac{3}{4}$
4198	15	$4\frac{1}{4}$

3. Add together :—

(1)

Rs.	a.	p.
3672	6	9
4278	13	6
236	4	1
5982	14	6
3716	8	4
410	7	10
6759	0	5
4917	0	0
427	12	6
218	8	5
29	15	8
6374	8	11
7109	15	7
492	7	5

(2)

Rs.	a.	p.
8274	5	7
329	8	6
415	2	9
42	5	10
2736	7	4
9	15	7
8138	14	4
725	4	6
87	9	11
234	15	4
9027	5	9
4378	9	3
274	2	5
42	9	7

(3)

Rs.	a.	p.
527	9	8
8436	10	2
4167	9	8
429	8	3
927	7	7
8	1	2
72	7	9
429	0	5
7283	8	6
5432	12	3
710	10	6
636	8	2
42	3	7
9245	8	6

(4)

£.	s.	d.
7214	18	$7\frac{1}{4}$
829	2	$1\frac{1}{2}$
3484	19	11
151	3	$9\frac{1}{2}$
40	14	$3\frac{3}{4}$
2607	17	$10\frac{1}{2}$
263	6	6
90	18	$8\frac{1}{4}$
485	13	7
7324	7	$4\frac{3}{4}$
934	16	$1\frac{1}{4}$
78	15	$10\frac{1}{2}$

(5)

£.	s.	d.
4614	13	$3\frac{1}{2}$
12	4	$5\frac{1}{4}$
6078	11	3
85	7	$3\frac{3}{4}$
843	19	10
7913	5	$8\frac{1}{2}$
24	6	8
1012	14	$6\frac{1}{2}$
820	12	4
537	9	$11\frac{3}{4}$
125	16	8
8416	15	$4\frac{1}{4}$

(6)

£.	s.	d.
9241	12	$5\frac{1}{2}$
159	3	$9\frac{1}{2}$
63	17	$10\frac{1}{4}$
4375	19	4
88	6	$7\frac{3}{4}$
797	15	9
972	13	3
2356	11	$6\frac{1}{2}$
38	5	$8\frac{1}{4}$
125	18	5
6316	4	$2\frac{3}{4}$
244	3	7

4. A cash-box contained 89 sovereigns, 35 half-sovereigns, 19 half-crowns, 25 florins, 31 shillings and 15 six-penny bits ; find the value of the coins in £. *s. d.*

5. A tradesman bought goods to the value of £1368. 12*s*. 6*d*.; he paid for carriage, £25. 16*s*. 9*d*., and other charges, £2. 15*s*. $8\frac{1}{2}$*d*.; he gained by the sale of the goods £269. 15*s*. $3\frac{1}{4}$*d*.; how much did he sell the goods for?

6. A stationer bought some books for *Rs* 79. 12*a*. 6*p*., some paper for *Rs*.161. 4*a*. 3*p*., some pens for *Rs*.14. 10*a*. and some envelopes for *Rs* 12. 8*a*. 6*p*. How much must he charge for all these articles, so as to gain exactly *Rs*.100 by his bargain?

7. A collection was once made in a district for a charitable purpose. The following coins were obtained: 99 gold mohurs, 1875 rupees, 990 eight-anna pieces, 5891 four-anna pieces, 1276 two-anna pieces, 90617 half-anna pieces and 81516 pice. What did the collection amount to in *Rs. a. pies*?

8. Add together 53 guineas, 107 sovereigns, 161 half-guineas, 55 half-sovereigns, 223 half-crowns, 505 four-penny pieces, and 603 farthings.

III. COMPOUND SUBTRACTION.

137. **Compound Subtraction** is the method of finding what quantity is left when a smaller quantity is taken from a greater of the same kind. The quantity thus left is called the **difference** of the given quantities.

RULE. Write the less number below the greater, so that units of the same denomination may be under one another, and draw a line below. Begin at the right hand and subtract (if possible) each number in the lower line from the corresponding one in the upper and place the remainder underneath. But if, in any case, the number in the lower line be greater than the one above it, add to the upper one as many units of the same denomination as make one unit of the next higher denomination, and then subtract, taking care to add 1 to the next number in the lower line. Proceed thus through all the columns.

Ex. 1. Subtract *Rs*.47. 12*a*. 9*p*. from *Rs*.72. 15*a*. 3*p*.

Rs.	*a.*	*p.*
72	15	3
47	12	9
Rs.25	2	6

3*p*. is less than 9*p*., so add 12*p*., to 3*p*. and 1*a*. to 12*a*.; 15*p*. − 9*p*. = **6***p*.
15*a*. − 13*a*. = **2***a*., and *Rs*.72 − *Rs*.47 = *Rs*.**25**.

Ex. 2. Subtract £207. 13*s*. $8\frac{3}{4}$*d*. from £304. 2*s*. $10\frac{1}{2}$*d*.

£.	*s.*	*d.*
304	2	$10\frac{1}{2}$
207	13	$8\frac{3}{4}$
£ 96	9	$1\frac{3}{4}$

2*q*. is less than 3*q*.; so add 4*q*. to 2*q*. and 1*d*. to 8*d*.; 6*q*. − 3*q*. = **3***q*. or $\frac{3}{4}$*d*.
10*d*. − 9*d*. = **1***d*.
2*s*. is less than 13*s*., so add 20*s*. to 2*s*. and £1 to £207; 22*s*. − 13*s*. = **9***s*.; £304 − £208 = £**96**.

Examples XXVIII.

1. Perform the operation of subtraction in the following :—

(1)

Rs.	a.	p.
55	15	10
47	8	11

(2)

Rs.	a.	p.
106	12	9
77	15	10

(3)

Rs.	a.	p.
57	6	3
46	9	10

(4)

Rs.	a.	ps.
75	11	2
49	12	3

(5)

Rs.	a.	ps.
126	3	1
82	8	3

(6)

Rs.	a.	p.
150	4	10
24	5	9

(7)

Rs.	a.	p.
1000	8	4
488	15	6

(8)

Rs.	a.	p.
269	5	11
189	13	10

(9)

Rs.	a.	p.
4172	8	5
2008	14	9

(10)

Rs.	a.	p.
772	13	9
347	15	11

(11)

Rs.	a.	p.
5400	14	7
3216	15	10

(12)

Rs.	a.	p.
3406	4	7
2958	13	9

(13)

Rs.	a.	p.
4658	7	6
4139	9	8

(14)

Rs.	a.	p.
50		
48	14	11

(15)

Rs.	a.	p.
575	10	6
89	11	9

(16)

Rs.	a.	p.
1000		
101	10	8

(17)

Rs.	a.	p.
7071	15	10
5707	10	11

2. Perform the operation of subtraction in the following :—

(1)

s.	d.
17	9
11	$8\frac{1}{4}$

(2)

s.	d.
17	$5\frac{3}{4}$
5	$9\frac{1}{2}$

(3)

s.	d.
19	$0\frac{1}{2}$
14	$11\frac{3}{4}$

(4)

s.	d.
18	$3\frac{1}{4}$
11	$7\frac{1}{2}$

(5)

s.	d.
10	$3\frac{1}{4}$
4	$7\frac{3}{4}$

(6)

£.	s.	d.
58	15	$3\frac{1}{2}$
19	4	$7\frac{3}{4}$

(7)

£.	s.	d.
95	14	2
37	6	$3\frac{3}{4}$

(8)

£.	s.	d.
586	17	$1\frac{1}{2}$
298	13	$1\frac{3}{4}$

(9)

£.	s.	d.
100	14	7
50	14	$7\frac{1}{2}$

(10)

£.	s.	d.
98	6	$2\frac{1}{4}$
67	11	$4\frac{3}{4}$

(11)

£.	s.	d.
100	3	3
95	15	$6\frac{1}{4}$

(12)

£.	s.	d.
611	17	$2\frac{1}{2}$
492	18	$8\frac{3}{4}$

(13)

£.	s.	d.
743	0	$4\frac{3}{4}$
275	15	$5\frac{1}{4}$

(14)

£.	s.	d.
525	14	$7\frac{1}{2}$
345	17	$8\frac{3}{4}$

(15)

£.	s.	d.
536	8	$7\frac{1}{4}$
89	13	$9\frac{3}{4}$

(16)

£.	s.	d.
837	14	$2\frac{1}{2}$
358	18	$6\frac{3}{4}$

(17)

£.	s.	d.
86	15	$9\frac{1}{4}$
9	18	$11\frac{1}{2}$

3. Subtract :—

(1) *Rs.*979. 15*a.* 9*p.* from *Rs.*5707. 15*a.* 7*p.*
(2) *Rs.*2102. 13*a.* 11*p.* from *Rs.*4365. 10*a.* 9*p.*
(3) *Rs.*6779. 14*a.* 8*p.* from *Rs.*7865. 12*a.* 6*p.*
(4) £554. 12*s.* $7\frac{3}{4}$*d.* from £1739. 7*s.* $6\frac{1}{4}$*d.*
(5) £1975. 13*s.* $9\frac{1}{2}$*d.* from £3003. 10*s.* 4*d.*
(6) *Rs.*55734. 12*a.* 4*p.* from *Rs.*88659. 8*a.* 3*p.*
(7) The sum of *Rs.* 14. 3*a.* 5*p.* and *Rs.*9. 8*a.* 7*p* from *Rs.*53. 11*a.* 6*p.*
(8) The sum of £5. 6*s.* $4\frac{1}{2}$*d.*, £31. 15*s.* $10\frac{1}{4}$*d.*, £43. 18*s.* $5\frac{3}{4}$*d.*, and £25. 16*s.* $4\frac{1}{4}$*d.* from £371. 14*s.* $6\frac{3}{4}$*d.*

4. What must be added to £157. 16*s.* $9\frac{1}{2}$*d.* to make £355. 13*s.* 4*d.* ?

5. After spending *Rs.*237. 14*a.* 3*ps.*, how much has a man left out of *Rs.*532. 10*a.* ?

6. A man has 50 guineas in his purse ; what would he have left after paying bills amounting to £49. 8*s.* $11\frac{1}{4}$*d.* ?

7. A tradesman, in making out a bill, copied 16*s.* 3*d.* for £16. 3*s.* and £10. 8*s.* for 10*s.* 8*d.* By what amount was the bill wrong ?

8. By how much is *Rs.*803. 11*a.* 3*p.* greater than *Rs.*213. 8*a.* 4*p.* ?

9. *A* borrowed from *B* *Rs.*387. 5*a.* 8*p.* and then *Rs.*39. 9*a.* 1*p.* ; repaid him *Rs.* 28. 7*a.* and again borrowed *Rs.*625. 13*a.* 11*p.* ; find what will be the amount of his debt still due if he makes payment of *Rs.*967. 3*a.* 7*p.*

10. Find the value of *Rs.*20. 15*a.* 11*p.* + *Rs.*28 11*a.* 3*p.* − *Rs.*17. 12*a.* 5*p.* + *Rs.*59. 13*a.* 6*p.* − *Rs.* 13. 10*a.* 4*p.* + *Rs.*18. 3*a.* 7*p.* − *Rs.*28. 12*a.* 9*p.* − *Rs.*10. 14*a.* 3*p.*

11. A man has *Rs.*5000 in the bank ; he draws *Rs.*2500 on Monday, *Rs.*1175. 4*a.* on Wednesday, and *Rs.*959. 6*a.* on Saturday. What has he left in the bank ?

12. A boy took the sum of 19*s.* $11\frac{1}{4}$*d.* three times out of a bag containing £5. What was left ?

13. A house and furniture are worth *Rs.* 1001. 11*a.* 10*p.* The house costs *Rs* 750 14*a.* 11*p* What is the value of the furniture ?

14 *A*, *B* and *C* together owe £107 11*s.* 8*d.* ; the sum of the debts of *A* and *B* is £70. 5*s.* 5*d.*, and of *B* and *C* £80. 16*s.* 1*d.* How much does each owe ?

15 *A*, who has *Rs.*5. 4*a.*, gives *B* *Rs.*3. 7*a.* 6*p.* and *C* *Rs.*2. 9*a.* ; but he receives from *D* *Rs.*10. 10*a.* 8*p.*, and from *E* *Rs* 3. 11*a.* 6*p* less than he received from *D* ; how much has he after these payments ?

16. A tradesman's cash in hand on Monday morning was £5. 13*s.* 6*d.* His cash receipts on Monday amounted to £2. 15*s.* $6\frac{1}{2}$*d.*

and on the following days of the week were, respectively, £4. 18s. 4d., £3. 13s. 6½d., £5. 10s. 10½d., £4. 12s. 11d., and £16. 9s. 8¾d. His cash outlay during the week amounted to £24. 17s. 5½d. What cash had he remaining at the end of the week?

IV. COMPOUND MULTIPLICATION.

138. Compound Multiplication is the method by which we find the sum of a compound quantity repeated as many times as there are units in a given number. The sum found is called the **product.**

139. When the Multiplier is not greater than 20.

RULE. Place the Multiplier under the lowest denomination of the multiplicand and draw a line below. Beginning with the lowest denomination multiply by the given multiplier, and find the number of the next higher denomination contained in the product; put down the remainder (if any), and carry the quotient to the next product, and repeat the process till all the denominations are multiplied.

Ex. 1. Multiply *Rs.*72. 11*a.* 9*p.* by 7.

Rs.	*a.*	*p.*
72	11	9
		7
Rs. 509	2	3

9*p.* × 7 = 63*p.* = 5*a.* **3***p.*; *carry* 5*a.*
11*a.* × 7 = 77*a.*, with 5*a.* = 82*a.* = *Rs.*5. **2***a.*; *carry Rs.*5.
*Rs.*72 × 7 = *Rs.*504, with *Rs.*5 = *Rs.***509.**

Ex. 2. Multiply £9. 19*s.* 7¾*d.* by 17.

£	*s.*	*d.*
9	19	7¾
		17
£169	13	11¾

3*q.* × 17 = 51*q.* = 12*d.* 3*q* = 12¾*d.*; *carry* 12*d.*
7*d.* × 17 = 119*d.*; 119*d.* + 12*d.* = 131*d.* = 10*s.* **11***d.*; *carry* 10*s.*
19*s.* × 17 = 323*s.*; 323*s.* + 10*s.* = 333*s.* = £16. **13***s.*; *carry* £16.
£9 × 17 = £153; £153 + £16 = £**169.**

140. When the Multiplier is a number greater than 20, and can be resolved into two or more factors none of which is greater than 20, multiply by each of the factors in succession, and the last result will be the product required. (Art. 81.)

Ex. Multiply *Rs.*99. 13*a.* 9*p.* by 28 and £9. 19*s.* 7¾*d.* by 42.

28 = 4 × 7.

Rs.	*a.*	*p.*
99	13	9
		4
*Rs.*399	7	0
		7
*Rs.*2796	1	0

42 = 6 × 7.

£	*s.*	*d.*
9	19	7¾
		6
£59	17	10½
		7
£419	5	1½

141. When the multiplier exceeds or falls short of a pr oduct by a small number, multiply by such product and then by this number and add or subtract for the required product.

Ex. Multiply *Rs.*240. 7*a.* 10*p.* by 29, and £17. 8*s.* 5¼*d.* by 139.

$29=28+1=4\times7+1.$ $139=144-5=12\times12-5.$

Rs.	*a.*	*p.*	
240	7	10	
		4	
Rs 961	15	4	
		7	
*Rs.*6733	11	4	product by 28
240	7	10	 1
*Rs.*6974	3	2	 29

£.	*s.*	*d.*	
17	8	5¼	
		12	
£209	1	3	
		12	
£2508	15	0	product by 144
87	2	2¼	 5
£2421	12	9¾	 139

142. When the Multiplier is a very large number.

RULE. Multiply by 10 as many times in succession as there are figures in the multiplier less 1; then multiply the given quantity by the units' figure of the multiplier, the first product by the tens' figure, the second product by the hundreds' figure and so on. The sum of these partial products will give the required product.

Ex. Multiply £16. 12*s.* 9¼*d.* by 7249.

£.	*s.*	*d.*	£.	*s.*	*d.*		
16	12	9¼ × 9 =	149	14	11¼	product by	9
		10					
£166	7	8½ × 4 =	665	10	10	....	40
		10					
£1663	17	1 × 2 =	3327	14	2	....	200
		10					
£16638	10	10 × 7 =	116469	15	10	....	7000
			£120612	15	9¼	....	7249

143. When the multiplier is a large number, as in the above example, and we are told to proceed by Compound Multiplication, the following is the simplest method.

£.	*s.*	*d.*
16	12	9¼
		7249
£120612	15	9¼

4)	7249*q.*	$=1q.\times7249.$
	1812 ...**1***q.*	
	65241	$=9d.\times7249.$
12)	67053	
	5587 ...**9***d.*	
	86988	$=12s.\times7249.$
2,0)	9257.5	
	4628 ...**15***s.*	
	115984	$=£16\times7249.$
	£120612	

144. In compound multiplication we may reduce the multiplicand to the lowest denomination contained in it, then multiply this result by the multiplier, and then reduce the product back again. This method is generally tedious.

Ex. Multiply £5045. 6*s.* $2\frac{1}{4}$*d.* by 4342.
£5045. 6*s.* $2\frac{1}{4}$*d.* = 48434979*q.*
and 48434979*q.* × 4342 = 210304639749*q.*
and 210304639749*q.* = £21906733 6*s.* $1\frac{1}{4}$*d.* *Ans.*

Examples XXIX.

1. Multiply :—

(1) *Rs.*18. 8*a.* 4*p.* by 2 ; *Rs.*42. 10*a.* 6*p.* by 3 ; *Rs.*67. 11*a.* 6*p.* by 8.
(2) *Rs.*51. 11*a.* 7*p.* by 4 ; *Rs.*67. 13*a.* 9*p.* by 7 ; *Rs.*58. 2*a.* 7*p.* by 6.
(3) *Rs.*65. 12*a.* 8*p.* by 5 ; *Rs.*84. 11*a.* 5*p.* by 11.
(4) *Rs.*48. 14*a.* 10*p.* by 9 ; *Rs.*66. 3*a.* 4*p.* by 18.
(5) £19. 18*s.* $7\frac{1}{4}$*d.* by 8 ; £3. 9*s.* $7\frac{3}{4}$*d.* by 12 ; £87. 8*s.* $11\frac{3}{4}$*d.* by 10.
(6) £37. 19*s.* $9\frac{1}{2}$*d.* by 9 ; £374. 12*s.* $10\frac{1}{4}$*d.* by 7.
(7) £549. 13*s.* $7\frac{3}{4}$*d.* by 11 ; £49. 13*s.* $0\frac{1}{4}$*d.* by 19.
(8) £497. 19*s.* $7\frac{1}{4}$*d.* separately by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.
(9) *Rs.*666. 10*a.* 9*p.* ... ... 2, 3, 4, 5 6, 7, 8, 9, 10, 11 and 12.
(10) £6. 12*s.* 5*d.* ... 13, 14, 16, 18, and 20.
(11) *Rs.*104. 12*a.* 5*p.* ... ... 13, 14, 15, 17, 19 and 20.
(12) £86. 10*s.* $7\frac{3}{4}$*d.* ... ... 15, 16, 17, 18 and 19.

2. Multiply (by factors) :—

(1) *Rs.*194. 8*a.* 7*p.* by 24 ; *Rs.*320. 14*a.* 10*p.* by 21.
(2) *Rs.*586. 13*a.* 10*p.* by 64 ; *Rs.*142. 0*a.* 9*p.* by 132.
(3) *Rs.*1005. 12*a.* 3*p.* by 72 ; *Rs.*133 6*a.* 6*p.* by 75.
(4) *Rs.*205. 4*a.* 3*p.* by 108 ; *Rs.*140. 2*a.* 6*p.* by 144.
(5) *Rs.*249. 15*a.* 5*p.* by 198 ; *Rs* 8967. 8*a.* 6*p.* by 351.
(6) £98. 18*s.* 3*d.* by 96 ; £68. 7*s.* $4\frac{3}{4}$*d.* by 35 ; £13. 7*s.* $4\frac{3}{4}$*d.* by 275.
(7) £99. 0*s.* $7\frac{3}{4}$*d.* by 77 ; £4. 8*s.* $9\frac{1}{4}$*d.* by 121 ; £13. 15*s.* $6\frac{1}{2}$*d.* by 132.
(8) *Rs.*277. 5*a.* 2*p.* by 216 ; *Rs.*1230. 10*a.* 1*p.* by 224.

3. Multiply (by factors and parts) :—

(1) *Rs.*77. 2*a.* 4*p.* by 23 ; *Rs.*13. 15*a.* 4*p.* by 62 ; £9. 19*s.* $7\frac{3}{4}$*d.* by 31.
(2) *Rs.*3. 15*a.* 10*p.* separately by 67, 71 and 79.
(3) *Rs.*398. 15*a.* 2*p.* ... ... 69, 59 and 41.
(4) £130. 18*s.* $9\frac{1}{2}$*d.* ... 89, 93 and 113.
(5) £808. 12*s.* $7\frac{1}{4}$*d.* 79, 83 and 131.
(6) *Rs.*19. 11*a.* 6*p.* ... ... 379 and 845.
(7) *Rs.*8. 14*a.* 2*p.* ... ... 1234 and 5678.
(8) *Rs.*37. 15*a.* 6*p.* ... ... 9803 and 5840.
(9) £5045. 6*s.* $2\frac{1}{4}$*d.* ... ... 923, 956 and 2765.

4. Multiply :—

(1) £324. 12*s.* $6\frac{1}{2}$*d.* by 394 ; £2. 16*s.* $9\frac{1}{2}$*d.* by 702.

(2) *Rs.*19. 4*a.* 6*p.* by 3210 ; *Rs.*23. 6*a.* 2*p.* by 3684.
(3) £9. 15*s.* 10$\frac{3}{4}$*d.* by 4508 ; £3. 18*s.* 11$\frac{1}{2}$*d.* by 57089.
(4) *Rs.*42. 4*a.* 4*p.* separately by 3005 and 7082.
(5) £567. 13*s.* 8$\frac{3}{4}$*d.* ... ... 8736 and 98736.

5. Find the values of :—

(1) 19 things at 3*a.* 2*p.* each. (2) 156 things at 11*a.* 6*p.* each.
(3) 96 ... 9*a.* 5*p.* ... (4) 315 ... ... 15*a.* 8*p.* ...
(5) 428 ...16*s.* 10$\frac{1}{2}$*d.* (6) 728 ... ... 7*s.* 7$\frac{3}{4}$*d.* ...
(7) 943 ... ... *Rs.*4. 2*a.* ... (8) 625 ... ... £1. 13*s.* 6*d.*...
(9) 729 ... ... *Rs.*7. 5*a.* 3*p.* ... (10) 829 ... ... *Rs.*8. 11*a.* 5*p.*...
(11) 1502 ... ... 19*s.* 5$\frac{3}{4}$*d.* ... (12) 2014 ... ... 17*s.* 6*d.* ...

6. Make out the following bills :—

(1) 17 yards of calico at 6*a.* 6*p.* per yard ; 143 yards of long cloth at 12*a.* 10*p.* per yard ; 14 yards of merino at *Rs.*2. 3*a.* 6*p.* per yard ; 204 yards of flannel at 14*a.* 9*p.* per yard ; 456 yards of linen at *Re.*1. 12*a.* per yard ; and 755 yards of silk at *Rs.*3. 5*a.* 4*p.* per yard.

(2) 40 seers of Assam Tea at *Rs.*3. 3*a.* 4*p.* per seer ; 65 lbs. of China Tea at *Rs.*2. 5*a.* 4*p.* per lb ; 35 seers of coffee at *Re.*1. 12*a.* 4*p.* per seer ; 145 seers of sugar at 7*a.* 4*p.* per seer ; and 122 seers of best sugar at 10*a.* 4*p.* per seer.

(3) 23 yards of silk at 5*s.* 4$\frac{1}{2}$*d.* per yard ; 5 yards of velvet at 13*s.* 6*d.* per yard ; 8 yards of velveteen at 3*s.* 11$\frac{1}{2}$*d.* per yard ; 13 yards of linen at 3*s.* 2*d.* yer yard ; 19 yards of flannel at 1*s.* 9*d.* per yard ; and 26 yards of calico at 11$\frac{3}{4}$*d.* per yard.

7. A man distributed a certain sum of money to 79 poor persons and gave £17. 12*s.* 9$\frac{1}{4}$*d.* to each ; find the sum of money distributed.

8. A bankrupt's estate can pay 14*a.* 10$\frac{1}{2}$*p.* in the rupee, what will a creditor receive who has lent 3125 rupees, and how much will he lose ?

9. How much money must be added to £1000 that each of 33 people may receive £35. 3*s.* 4*d.* ?

10. A gowala exchanges 59 calves each worth *Rs.*15. 10*a.* for 37 cows each worth *Rs.*26. 4*a.* ; ought he to receive, or to pay any money ? how much ?

11. If I spend £2. 7*s.* 1$\frac{1}{2}$*d.* a day, how much is that in a year of 365 days ?

12. There are 53 chests of drawers ; in each chest there are 4 drawers ; in each drawer there are 10 compartments ; and in each compartment there are deposited £32. 5*s.* 6*d.* How much money is deposited in the chests ?

V. COMPOUND DIVISION.

145. Compound Division is the method by which (1) we break up a compound quantity into as many *equal parts* as there are

units in a given number, and thus find the value of one of these parts, (2) we find how many times one compound quantity is contained in another of the same kind. The first method is called **Partition** and the second **Quotition.**

146. In the first case the divisor is an abstract number, and the quotient telling *the value of each part* is a compound quantity of the same kind as the dividend. In the second case the divisor is a compound quantity of the same kind as the dividend, and the quotient telling *how many times* is an *abstract* number.

147. *When the divisor is an abstract number.*

RULE. Place the dividend and divisor as in Simple Division. Find how often the divisor is contained in the highest denomination of the dividend, put down the quotient, and reduce the remainder (if any), to the next inferior denomination. Add to it the number of that denomination in the dividend, and repeat the division. Continue the process step by step through all the denominations.

(1) When the divisor does not exceed 20, the division can be performed *mentally* thus :—

Ex. Divide *Rs.*436. 5*a.* 4*p.* by 11.

Rs.	*a.*	*p.*	
11)436	5	4	*Rs.*436 ÷ 11 is *Rs.***39** with *Rs.*7 over.
Rs. 39	10	8	*Rs.*7 = 112*a*, with 5*a.* = 117*a.* ;
			117*a.* ÷ 11 is **10***a.* and 7*a.* over.
			7*a* = 84*p.*, with 4*p.* = 88*p.*, which ÷ 11 is **8***p.*

(2) When the divisor is a number larger than 20.

Proceed as in the following Examples.

Ex. Divide £52. 10*s.* $7\frac{1}{2}d.$ by 41, and *Rs.*3441. 5*a.* 9*p.* by 129.

(1)

£.	*s.*	*d.*	
41)52	10	$7\frac{1}{2}$	(1£
41			
11			
20			
230			(5*s.*
205			
25			
12			
307			(7*d.*
287			
20			
4			
82			(2*q.*
82			

(2)

Rs.	*a.*	*p.*	
129)3441	5	9	(26*Rs.*
258			
861			
774			
87			
16			
1397			(10*a.*
129			
107			
12			
1293			(10*p.*
129			
3*p.*			

∴ the required quotient is *Rs.*26. 10*a.* 10*p.* and 3*p.* over.

∴ the required quotient is £1. 5*s.* $7\frac{1}{2}d.$

148. When the divisor is the product of two or more factors, divide by each of them successively, and five the remainder as in Simple Division.

Ex. Divide £1478. 13*s.* 8¾*d.* by 77.

		£.	*s.*	*d.*		
77 {	7)	1478	13	8¾		The final remainder is $6 \times 7 + 2$
	11)	211	4	9¾	. . . 2*q.*	or 44*q.*
		£19	4	0¾	. . . 44*q.*	∴ the quotient is £19. 4*s.* 0¾*d.* and 11*d.* over.

149. When there is a **remainder** after division, we can always find a quotient which is correct **to the nearest pie** or **farthing** by the following Rule.

RULE. Neglect the remainder, if it is *less* than the divisor divided by 2; but *otherwise* add 1*p.* or 1*q.* to the quotient.

Ex. Find to the *nearest pie* the result of dividing *Rs.*727. 15*a.* 10*p.* by 67, and to the *nearest farthing* £333. 19*s.* 4¾*d.* by 29.

(1)				(2)			
	Rs.	*a.*	*p.*		£.	*s.*	*d.*
67)	727	15	10 (10*Rs.*	29)	333	19	4¾ (11£.
	67				29		
	57				43		
	16				29		
	927 (13*a.*				14		
	67				20		
	257				299 (10*s.*		
	201				29		
	56				9		
	12				12		
	682 (10*p.*				112 (3*d.*		
	67				87		
	12				25	Here $2 \times 16 = 32$,	
Here $2 \times 12 = 24$, which					4	which is greater	
is less than 67.					103 (3*q.*	than 29.	
∴ Quotient is					87	∴ Quotient is	
*Rs.*10. 13*a.* 10*p.*					16	£11. 10*s.* 4*d.*	

150. *When the divisor is 10, 100, 1000, &c.*

RULE. Cut off from the right of each succeeding dividend as many figures as there are ciphers in the divisor; the figures to the left will at each step give the quotient and the figures to the right the remainder.

Ex. Divide *Rs.*1179. 2*a.* 8*p.* by 100, and £9797. 5*s.* 6*d.* by 900.

(1)

```
        Rs.    a.  p.
100 ) 11,79    2   8
         16
      a.12,66
          12
      p.8,00
```

∴ Quotient = *Rs.*11. 12*a.* 8*p.*

(2)

```
          £.      s.   d.
       9)9797     5    6
   100)£10,88    11    8½ .... 6q.
           20
       s.17,71
           12
        d.8,60
            4
        q.2,42
```

The final remainder is $42 \times 9 + 6$ or 384*q.* or 8*s.*

∴ Quotient = £10. 17*s.* 8½*d.* and 8*s.* over.

Examples XXX.

1. Divide :—

(1) *Rs.*11. 13*a.* 8*p.* by 2 ; *Rs.*393. 14*a.* 4*p.* by 7 ; *Rs.*328. 15*a.* 4*p.* by 5.
(2) *Rs.*5161. 9*a.* 4*p.* by 3 ; *Rs.*440. 5*a.* 6*p.* by 9 ; *Rs.*436. 5*a.* 4*p.* by 11.
(3) *Rs.*5392. 1*a.* 4*p.* by 8 ; *Rs.*576. 8*a.* by 12 ; *Rs.*1721. 7*a.* 10*p.* by 14.
(4) £26. 15*s.* 3½*d.* by 2 ; £87. 16*s.* 8¼*d.* by 9 ; £614. 2*s.* 6¼*d.* by 7.
(5) £79. 13*s.* 9*d.* by 12 ; £147. 11*s.* 6¾*d.* by 15 ; £95. 2*s.* 3¾*d.* by 11.
(6) £241. 8*s.* 8¼*d.* by 63 ; £1990. 10*s.* 9*d.* by 42 ; £75. 1*s.* 10½*d.* by 45.
(7) *Rs.*8370. 15*a* separately by 17, 51 and 126.
(8) *Rs.*12342. 12*a.* 2*p.* ... ... 19, 59 and 325.
(9) *Rs.*3253. 15*a.* ... ... 23, 87 and 712.
(10) £1302. 18*s.* by 144 ; £890. 12*s.* 6*d.* by 125.
(11) £75. 6*s.* 4½*d.* by 103 ; £4718. 14*s.* 8*d.* by 132.
(12) £7549. 17*s.* 6*d.* by 859 ; £77573. 18*s.* 9½*d.* by 4578.

2. Divide by the *short* method :—

(1) £239. 14*s.* 4½*d.* separately by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.
(2) *Rs.*1088. 12*a.* separately by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.
(3) *Rs.*1877. 7*a.* 4*p.* by 14 ; *Rs.*2757. 9*a.* by 18 ; *Rs.*7023. 2*a.* by 20.
(4) £623. 5*s.* 4½*d.* by 13 ; £318. 10*s.* 7*d.* by 14 ; £139. 13*s.* 8*d.* by 16.

3. Divide by using factors :—

(1) *Rs.*517. 11*a.* 4*p.* by 35 ; *Rs.*34. 11*a.* by 45 ; *Rs.*3868. 3*a.* 6*p.* by 99.
(2) *Rs.*3639. 1*a.* 6*p.* by 81 ; *Rs.*3191. 6*a.* by 132 ; *Rs.*5761. 8*a.* by 144.
(3) £579. 18*s.* by 45 ; £1328. 13*s.* 6*d.* by 56 ; £453. 11*s.* 6¾*d.* by 77.
(4) £374. 10*s.* 3*d.* by 108 ; £576. 3*s.* by 144 ; £386. 16*s.* 5¼*d.* by 99.

4. Divide :—

(1) *Rs.*2625. 1*a.* 8*p.* by 10.
(2) £176. 16*s.* 8*d.* by 10.
(3) *Rs.*3395. 13*a.* 4*p.* by 100.
(4) £73. 12*s.* 11*d.* by 100.

7

(5) £876. 2s. 11d. by 100. (6) Rs.1151. 9a. 2p. by 1000
(7) £9658. 17s. $3\frac{1}{4}$d. by 1000. (8) Rs.4579. 2a. 8p. by 400.
(9) Rs.6925 by 800; Rs.3625 by 6000; Rs.11375 by 2400.
(10) £1556. 5s. by 3600; £513. 8s. 9d. by 3100; £2559. 7s. 6d. by 18900.

5. Divide :—

(1) Rs.73298. 3a. 8p. separately by 842, 912 and 8317.
(2) Rs.84566. 2a. 8p. ... ... 392, 573 and 7856.
(3) Rs.56789. 15a 8p. by 9357. (4) Rs.98767. 5a. 2p. by 10048.
(5) £6011656. 5s. $8\frac{1}{4}$d. by 2331. (6) £467325. 10s. $1\frac{1}{2}$d. by 2803.
(7) £530866. 17s. 6d. by 2772. (8) £4420895. 0s. $3\frac{3}{4}$d. by 3001.

6. Find, to the *nearest pie* or *farthing*, the result of dividing :—

(1) Rs.33. 9a. 4p. by 9. (6) Rs.2684. 2a. 9p. by 241.
(2) Rs.511. 8a. 5p. by 97. (7) Rs.523. 6a. 8p. by 100.
(3) Rs.29. 10a. 3p. by 31. (8) £1867. 16s. $8\frac{3}{4}$d. by 407.
(4) £150. 4s. 9d. by 12. (9) £15104. 19s. 2d. by 100.
(5) £74. 6s. $10\frac{3}{4}$d. by 23. (10) £2160. 18s. 11d. by 1000.

7. If Rs.2757. 9a. be equally divided among 18 people; how much will each receive?

8. A man spends Rs.5611. 14a. in a year of 365 days; how much does he spend in a week of 7 days?

9. After buying 15 books I have £2. 15s. $7\frac{1}{2}$d. left out of £7. What was the price of each book?

10 The cost of 720 goats is Rs.712. 8a.; what is the cost of each goat?

11. 205 sovereigns, all equally light, are worth £201. 15s. $11\frac{1}{4}$d.; find the worth of each.

12. A cattle-dealer bought 11 cows at Rs.8. 4a. each; after spending Rs.26. 4a. in feeding them, he sells 3 of them for Rs.11. 4a. each; at what price must he sell each of the others to gain Rs.23 by the bargain?

---

151. *When the divisor is a compound quantity of the same kind as the dividend.*

RULE. Reduce the dividend and the divisor to the same denomination, and then proceed as in Simple Division.

Ex. 1. Divide Rs.113. 14a. 6p. by Rs.12. 10a. 6p.

Rs.113. 14a. 6p. = 21870p.; Rs.12. 10a. 6p. = 2430p.

∴ the quotient required = 21870 ÷ 2430 = 9. *Ans.*

Ex. 2. How many cricket balls each worth 5s. $7\frac{1}{2}$d. can I buy with £134. 14s. $4\frac{1}{2}$d.?

£134. 14s. $4\frac{1}{2}$d. = 129330q.; 5s. $7\frac{1}{2}$d. = 270q.

∴ the number of balls = 129330 ÷ 270 = 479. *Ans.*

Examples XXXI.

1. Divide :—

(1) *Rs.*175. 9*a.* 4*p.* by *Re.*1. 12*a.* 8*p.* ; *Rs.*854. 2*a.* 8*p.* by *Rs.*20. 13*a.* 4*p.*
(2) *Rs.*438. 7*a.* by *Rs.*6. 5*a.* 8*p.* ; *Rs.*4012. 2*a.* by *Rs.*25. 11*a.* 6*p.*
(3) £28 2*s.* 6*d.* by 12*s.* 6*d.* ; £150. 7*s.* 5*d.* by 6*s.* $3\frac{1}{2}d.$
(4) £286. 3*s.* 2*d.* by £1. 11*s.* $1\frac{1}{4}d.$; £144. 13*s.* $11\frac{1}{2}d.$ by 9*s.* $11\frac{3}{4}d.$
(5) *Rs.*22831. 1*a.* 6*p.* by *Rs.*66. 2*a.* 10*p.* ; £4808. 14*s.* by £7. 8*s.* 5*d.*
(6) *Rs.*200157. 8*a.* 10*p.* by *Rs.*576. 13*a.* 2*p.* ; £131. 4*s.* $4\frac{1}{2}d.$ by 10*s.* $7\frac{1}{2}d.$

2. How often is

(1) *Rs.*760. 6*a.* 8*p.* contained in *Rs.*6843. 12*a.* ?
(2) *Rs.*3. 12*a.* 10*p.* ... ... *Rs.*2771. 11*a.* 6*p.* ?
(3) *Rs* 2. 15*a.* 4*p.* ... ... *Rs.*2366. 10*a.* 8*p.* ?
(4) £35. 16*s.* $7\frac{3}{4}d.$ ... ... £9961. 7*s.* $6\frac{1}{2}d.$?
(5) £2579. 0*s.* $0\frac{3}{4}d.$ ... ... £399745. 9*s.* $8\frac{1}{4}d.$?

3. Find the quotient and the remainder in the division of :—

(1) *Rs.*9607. 15*a.* 10*p.* by *Rs.*26. 5*a.* 2*p.*
(2) *Rs.*1225. 11*a.* 9*p.* by *Rs.*55. 10*a.* 8*p.*
(3) £568. 13*s.* 8*d.* by £1. 8*s.* 6*d.*
(4) £339 14*s.* $7\frac{1}{2}d.$ by £4. 11*s.* $9\frac{1}{4}d.$

4. How many dollars worth 4*s.* $1\frac{1}{2}d.$ each must be given in exchange for £235. 10*s.* 9*d.* ?

5. To how many persons may *Rs.*607. 12*a.* be distributed giving *Rs.*46. 12*a.* to each ?

6. How many hats each costing £1. 2*s.* $3\frac{1}{2}d.$ can be bought for £134. 17*s.* $3\frac{1}{2}d.$?

7. How many cows at *Rs.*108. 12*a.* each can I buy with the proceeds of selling 87 horses at *Rs.*1151. 4*a.* each ?

8. How many days must a labourer work at 2*s.* $1\frac{1}{2}d.$ a day to earn £51 ?

9. I buy a number of books at 2*s.* $9\frac{1}{2}d.$ each and sell them at 3*s.* 3*d.* each If I thereby make a profit of £2. 4*s.*, how many books must I buy ?

10. I buy 60 gallons of wine at £1. 3*s.* 6*d.* a gallon and £1. 10*s.* is gained by selling it at £1. 2*s.* 6*d.* a gallon. How much water is added ?

II. MEASURES OF WEIGHT.

152. Indian Bazar Weight.

4 Sikis	make 1 **Tola**
5 Sikis	„ 1 Kancha

4 Kanchas or 5 Tolas	make	1 Chhatak (*ch.*)
4 Chhataks	,,	1 Poa
4 Poas or 16 Chhataks	,,	1 **Seer** (*sr.*)
5 Seers	,,	1 Pasari
8 Pasaris or 40 Seers	,,	1 **Maund** (*md.*)

The weight of a rupee is called a *tola*. A seer = 80 tolas.
49 Bazar maunds = 54 Factory maunds.

Bombay Local Weight.			Madras Local Weight.		
4 Dhans	make	1 Ratika	180 Grains	make	1 Tola
8 Ratikas	,,	1 Masha	3 Tolas	,,	1 Palam
4 Mashas	,,	1 Tank	8 Palams	,,	1 Seer
72 Tanks	,,	1 Seer	5 Seers	,,	1 Vis
40 Seers	,,	1 Maund	8 Vis	,,	1 Maund
20 Maunds	,,	1 Kandi.	20 Maunds	,,	1 Kandi.

49 Bazar maunds = 144 Bombay maunds. 175 Bazar maunds = 576 Madras maunds. 25 Bombay maunds = 28 Madras maunds.

153. English Standard Weight. (*Avoirdupois*).

16 Drams (dr.)	make	1 Ounce (oz.)
16 Ounces	,,	1 **Pound** (lb.)
28 Pounds	,,	1 Quarter (qr.)
4 Quarters or 112 lbs	,,	1 Hundred-weight (cwt.)
20 Hundred-weights	,,	1 Ton.

1 Stone = 14 lbs. ; 1 Cental = 100lbs.

A stone of butcher's meat	= 8lbs.	A sack of flour	= 280lbs.
A sack of Coal	= 2 cwt.	A barrel of ,,	= 196lbs.
A barrel of Gunpowder	= 100lbs.	A peck of ,,	= 14lbs.
A pack of wool	= 240lbs.	A quartern loaf	= 4lbs.
A Firkin of Butter	= 56lbs.	A pocket of Hops	= 168lbs.
A Great Pound of Silk	= 24 oz.	Two Fodders of Lead	= 39 cwt.

A pound (Avoirdupois) = 7000 grains (Troy) ; 7 Bazar maunds = 576lbs. (Avoir.) ; 1 Bombay maund = 28lbs. (Avoir.) ; 1 Madras maund = 25lbs. (Avoir.) ; 3 Factory maunds = 2 cwt. ; 35 seers = 72 lbs. (Avoir.).

The Jeweller's Tables.

Indian Jeweller's Weight.			English Troy Weight.		
4 Dhans	make	1 Rati (*ra.*)	24 Grains (gr.)	make	1 Penny-weight (dwt.)
6 Ratis	,,	1 Anna (*a.*)	20 Penny-weights	,,	1 Ounce (oz. Tr.)
8 Ratis	,,	1 Masha (*ma.*)	12 Ounces or 5760 grains	,,	1 Pound (lb. Tr.)
12 Mashas or 16 annas	,,	1 **Tola** or Bhari			

1 Tola = 180 grs. Troy ; 1 Bazar maund = 100lbs. Troy ; 1 pound = 32 tolas ; a *Carat* = $3\frac{1}{6}$ grs. (for weighing diamonds.)

[Gold, silver, jewels and precious stones are weighed by the Troy weight].

Measures of Weight for Medicines.

BENGAL PHYSICIANS' WEIGHT.	ENGLISH APOTHECARIES' WEIGHT.
4 Dhans make 1 Rati	20 Grains make 1 Scruple (℈)
10 Ratis " 1 Masha	3 Scruples " 1 Dram (ʒ)
8 Mashas " 1 **Tola**	8 Drams " 1 Ounce (℥)
	12 Ounces " 1 Pound (lb.)

[The Apothecaries' weight is now out of use, except in selling drugs by retail].

144 lbs. (Avoir.) = 175 lbs. (Troy or Apoth.); 1 lb. (Troy or Apoth.) = 5760 grains; 1 lb. (Avoir.) = 7000 grs Troy; 1 lb (Avoir.) + the weight of a double-pice (200 grs.) = half-a-seer.

Note. The term 'carat' applied to gold has a relative meaning only; any quantity of pure gold, or of gold alloyed with some other metal, being supposed to be divided into 24 equal parts, called *carats*; if the gold be pure, it is said to be 24 carats fine; if 22 parts be pure gold and 2 parts alloy, it is said to be 22 carats fine.

Standard gold is 22 carats fine; *Jeweller's* gold is 18 carats fine.

Ex. 1. Reduce 14cwt. 3qrs. 24lbs. to *ounces*, and 32250 kanchas to *maunds*.

(1)
cwt. qrs. lbs.
14 3 24
4
59 qrs.
28
1676 lbs.
16
26816 oz. *Ans.*

(2)
4)32250 kan.
16) 8062 ch.........2 kan.
4,0) 50,3 sr...........14 ch.
12 md..........23 sr.

∴ the result = 12 mds. 23 sr. 14 ch. 2 kan.

Ex. 2. Reduce 425095 grs. of gold to lbs. &c., and 11ka. 13mds. 3 vis 5 palams 2 tolas to *tolas* (Mad.).

(1)
24 { 4)425095 grs.
6)106273...3 } 7 grs.
2,0) 17712 dwts. 1.
12) 885 oz...12 dwts.
73 lbs....9 oz.

∴ the result = 73 lbs. 9 oz. 12dwts. 7grs.

(2)
ka. md. vis pa. tolas.
11 13 3 5 2
20
233 mds.
8
1867 vis
40
74685 palams
3
224057 tolas. *Ans.*

Examples XXXII.

(Indian Bazar and Avoirdupois Weights).

1. Reduce (i) to *kanchas* and (ii) to *tolas* :—

(1) 20 mds. 13 sr. 7 ch.; 12 mds. 15 sr. 10 ch.; 75 mds. 32 sr. 15 ch.
(2) 46 mds. 25 sr. 12 ch.; 25 mds.; 45 mds. 12 sr. 8 ch.

2. Reduce to *kanchas* :—

(1) 30 mds. 27 sr. 12 ch. 2 kan.; 45 mds. 30 sr. 8 ch. 1 kan.
(2) 210 mds. 15 sr. 2 ch. 3 kan.; 220 mds. 17 sr. 3 kan.

3. Reduce to *maunds, &c.* :—

4123000 kan.; 30205676 kan.; 15025276 tolas; 4876235 poas; 4320578 ch.; 4362508 tolas; 782504 poas.

4. Reduce to *tolas* :—

2mds. 5vis 4sr.; 5kan. 15mds. 4vis; 8kan. 14mds. 7vis 5 palams.

5. Reduce to *dhans* :—

2 mds. 14 sr. 57 ta.; 8 kan. 16 mds. 25 sr. 55 ta. 3 m.; 10 kan. 10 mds.; 39 sr. 16 ta. 3 m. 2 rat.

6. Reduce 156728306 tolas to *kandis*; 460879025 dhans to *kandis*; 786250 tanks to *kandis*; 4586 seers to *kandis.*

7. Reduce :—

(1) 11 cwt. 2 qrs. 17 lbs. 15 oz. to *ounces*; 3 cwt. 13 lbs. to *ounces.*
(2) 6 tons 5 st. to *ounces*; 4 tons 15 cwt. 2 qrs. 12 lbs. to *pounds.*

8. Reduce to *drams* :—

(1) 2 tons 10 cwt. 1 qr. 2 lbs. 3 oz. 3 drs.; 3 tons 14 cwt. 3 qrs. 25 lbs. 11 oz. 9 drs.; 3 tons 3 qrs. 3 oz.; 27 lbs.
(2) 8 st. 11 lbs. 9 drs.; 16 lbs. 12 oz. 13 drs.; 18 cwt. 73 lbs. 9 drs.

9. Reduce to *tons, cwt., &c.*

(1) 87654 lbs.; 378539 oz.; 1693539 drs.; 65437 drs.
(2) 2345820 drs.; 1008001 oz.; 237023 oz.; 59653007 st.
(3) 21633 lbs.; 17739853 oz.; 5390054 drs.; 713969416 drs.

10. Add together :—

(1)

mds.	sr.	ch.
25	10	5
110	12	3
115	20	12
97	27	15
102	15	7

(2)

mds.	sr.	ch.	kan.
65	10	10	2
72	15	8	2
102	14	10	3
125	30	13	2
207	32	15	1

(3)

mds.	sr.	ch.	kan.
115	30	7	1
202	27	10	2
323	15	12	3
222	9	5	2
313	32	14	3

(4)

tons	cwt.	qrs.	lbs.	oz.
16	0	3	5	15
8	16	0	0	14
28	8	1	27	6
210	6	3	14	11
17	17	0	15	12
412	15	3	18	13

(5)

cwt.	qrs.	lbs.	oz.
32	2	15	12
47	0	25	7
5	3	17	10
23	1	19	15
1	2	10	8
9	3	0	14

(6)

tons	cwt.	qrs.	lbs.
32	12	2	25
18	15	0	20
23	10	1	16
14	18	1	27
25	4	0	3
35	12	2	19

11. Perform the operation of subtraction in the following :—

(1)

mds.	sr.	ch.
530	10	12
396	27	15

(2)

mds.	sr.	ch.	kan.
672	12	10	0
127	24	14	3

(3)

mds.	sr.	ch.	kan.
427	10	10	1
212	25	14	3

(4)

cwt.	qrs.	lbs.
47	0	12
32	3	22

(5)

tons	cwt.	qrs.	lbs.
75	7	1	16
41	14	2	19

(6)

cwt.	qrs.	lbs.	oz.
112	2	23	8
59	0	27	10

12. Multiply :—

(1) 110 mds. 20 sr. 12 ch. separately by 24, 36 and 72.
(2) 225 mds. 22 sr. 13 ch. 2 kan. ... ... 144, 126 and 360.
(3) 20 tons 3 qrs. 12 lbs. ... ... 132 and 143.
(4) 25 tons 18 cwt. 2 qrs. 15 lbs. ... ... 144 and 1728.
(5) 8 tons 87 lbs. 13 drs. ... ... 18, 29, 47 and 133.

13. Divide :—

(1) 252 mds. 10 sr. 12 ch. separately by 63 and 84.
(2) 1230 mds. 22 sr. 15 ch. ... ... 112 and 336.
(3) 3125 mds. 10 sr. 10 ch. 2 kan ... ... 167 and 4008.
(4) 48 tons 17 cwt. 3 qrs. 27 lbs. 1 oz. ... 9, 17 and 500.
(5) 30 tons 15 cwt. 2 qrs. 15 lbs. ... ... 144 and 864.
(6) 1061 cwt. 2 qrs. by 37 cwt. 3 qrs. 18 lbs. ; 89 cwt. 22 lbs. by 3 cwt. 1 qr. 6 lbs.; 404 mds. 35 sr. 2 ch. 2 kan. by 23 mds. 32 sr. 10 ch. 2kan.

14. If 41 cwt. cost £52. 10*s.* $7\frac{1}{2}$*d.*, what is the price of a cwt. ?

15. A chest of tea weighing 1 cwt. 1 qr. 15 lbs. cost £22. 8*s.* $10\frac{1}{4}$*d.* what is the cost of 1 lb. ?

16. At a school feast the children on the average ate 9 oz. of cake a piece, and 84 lbs. 6 oz. of cake were eaten ; how many children were there in the school ?

(*Indian Jeweller's and Troy Weights.*)

1. Reduce to *dhans* :—

25 tolas 10 m. 4 r. 3 dh. ; 150 tolas 14 a. 5 r. ; 162 tolas 13a. 3r. 2 dh.

2. Reduce to *tolas* :—

56430 dhans ; 53426 ratis ; 37484 dhans ; 32458 ratis.

3. Reduce to *grains* :—

(1) 12 lbs. 10 oz. 15 dwts. 14 grs ; 15 lbs. 11 oz. 17 grs. 9lbs. 18dwts.
(2) 16 lbs. Troy ; 9 oz. 17 dwts. 22 grs. ; 165 oz. 280 grs.

4. Reduce to *lbs., etc.* (Troy) :—

13600 grs. ; 146320 grs. ; 400903 dwts. ; 6739 oz. ; 873521 grs.

5. Add together :—

(1)

tolas	m.	r.	dh.
45	10	7	3
63	8	6	2
58	9	5	1
62	11	7	3
39	8	4	2
112	6	3	1

(2)

tolas	a.	r.	dh.
47	10	3	2
52	9	2	1
65	8	5	3
77	13	4	0
82	14	5	3
75	12	4	2

(3)

lbs.	oz.	dwts.	grs.
64	11	16	14
21	10	12	13
2	0	1	16
12	10	0	18
24	11	12	0
14	1	0	1

6. Perform the operation of subtraction in the following :—

(1)

tolas	m.	r.	dh.
530	8	4	2
327	12	6	3

(2)

tolas	a.	r.	dh.
579	11	3	2
380	14	5	3

(3)

lbs.	oz.	dwts.	grs.
81	10	9	18
14	11	12	19

(4)

lbs.	oz.	dwts.	grs.
225	8	14	15
167	11	18	19

(5)

lbs.	oz.	dwts.	grs.
13	0	7	18
11	11	15	23

(6)

tolas	a.	r.	dh.
467	10	3	0
279	13	5	2

7. Multiply :—

(1) 115 tolas 7 m. 5 r. 1 dh. separately by 72 and 80.
(2) 210 tolas 10 a. 2 r. 2 dh. ... ... 132 and 143.
(3) 22 lbs. 7 oz. 12 dwts. 20 grs. ... ... 64 and 96.
(4) 83 lbs. 17 dwts. 5 grs. ... ... 26, 131 and 257.

8. Divide :—

(1) 1125 tolas 8 m. 6 r. 3 dh. separately by 132 and 144.
(2) 1020 tolas 12 a. 4 r. 2 dh. ... ... 172 and 516.
(3) 606 lbs. 4 oz. 15 dwts. 20 grs. ... ... 131 and 500.
(4) 110 lbs. 10 oz. 14 dwts. 16 grs.... ... 136 and 272.
(5) 2025 lbs. 2 oz. 18 dwts. 8 grs. by 5 lbs. 6 oz. 280 grs.

9. If 28 lbs. 9 oz. of gold be worth £1343. 6*s.* 10½*d.*, what is the worth of 1 ounce?

10. A certain number of forks, each weighing 3 oz. 5 dwts. and double that number of spoons, each weighing 3 oz. 10 dwts. are made out of 10 bars of silver, each weighing 3 lbs. 5 oz.; find the number of spoons.

(Native Physician's and Apothecaries' Weights.)

1. Reduce to *dhans* :—
25 tolas 6 m. 8 r. 3 dh.; 32 tolas 5 m. 9 r. 2 dh.; 8 tolas 7 m. 5 r. 2 dh.

2. Reduce to *tolas* :—
1224 ratis; 13200 dhans; 426507 dhans; 2406 ratis.

3. Reduce to *grains* :—
3 lbs. 5 ʒ 19 grs.; 2 lbs. 4 drs. 2 scr.; 18 lbs. 2 oz. 4 drs. 2 scr. 12 grs.

4. Reduce to *pounds*, &c. :—
270083 grs.; 26984℈; 92200 grs.; 51960 grs.; 17599 grs.

5. Add together :—

25	7	8	3
32	5	7	2
49	7	5	2
55	3	6	1
60	6	5	3
79	4	6	2

(2) oz.	drs.	scrs.	grs.
11	4	2	11
10	3	0	4
16	0	1	14
10	0	1	16
6	2	2	18
14	5	1	0

(3) lbs.	℥	ʒ	℈	grs.
15	3	5	1	17
18	10	6	2	5
20	9	1	2	12
25	7	3	0	18
36	5	4	1	14
26	8	6	2	15

6. Perform the operation of subtraction in the following :—

(1) tolas	m.	r.	dh.
125	6	3	0
58	7	6	2

(2) lbs.	oz.	drs.	scrs.	grs.
28	7	1	2	4
12	8	2	1	12

(3) lbs.	℥	ʒ	℈
75	7	3	0
49	10	6	2

7. Multiply :—

(1) 32 tolas 6 m. 8 r. 3 dh. separately by 132 and 143.
(2) 45 lbs. 7 oz. 3 drs. 2 scrs. 8 grs. ... ... 16 and 64.
(3) 7 lbs. 6 ʒ 14 grs. ... ... 53, 71 and 2500.

8. Divide :—

(1) 65 tolas 7 m. 6 r. 2 dh. separately by 72 and 81.
(2) 120 lbs. 9 oz. 5 drs. 2 scrs. 12 grs. ... ... 120 and 132.
(3) 270 lbs. 5 ℥ 6 ʒ 2 scrs. ... ... 46, 53 and 1000.
(4) 7 lbs. by 2 ʒ 2 ℈ and 1234 lbs. 68 ℈ by 44 lbs. 23 ℈.

9. How many pills, each containing 6 ℈ 2 grs. can be made out of 2 lbs. 11℥ 6ʒ of rhubarb?

154. *To convert from one system of weights into another.*

(1) To convert Indian weight into Troy, multiply the weight in tolas by 3 and divide by 8; the result will be the weight in *oz.* Troy. Or multiply the weight in tolas by 180; the result will be the weight in *grains* Troy.

Conversely, to convert Troy weight into Indian weight, multiply the weight in *oz.* Troy by 8 and divide by 3; the result will be the weight in *tolas*. Or divide the weight in grs. Troy by 180; the result will be the weight in *tolas*.

(2) To convert Indian weight into Avoir., multiply the weight in chhataks by 9 and divide by 70; the result will be the weight in lbs. Avoir. Or multiply the weight in seers by 72 and divide by 35; the result will be the weight in lbs. Avoir. Or multiply the weight in maunds by 36 and divide by 49; the result will be the weight in *cwts.* Avoir.

Conversely, to convert Avoir. weight into Indian weight, multiply the weight in lbs. Avoir. by 70 and divide by 9; the result will be the weight in *chhataks*. Or multiply the weight in lbs. Avoir. by 35 and divide by 72; the result will be the weight in *seers*. Or multiply the weight in cwts. by 49 and divide by 36; the result will be the weight in *maunds*.

(3) To convert lbs. Avoir. into Troy, multiply the weight in lbs. Avoir. by 7000; the result will be the weight in *grains* Troy. Conversely to convert Troy weight into Avoir., multiply the weight in lbs. Troy by 144 and divide by 175; the result will be the weight in lbs. Avoir.

(4) As the weight in grains of both Apoth. and Troy weights is the same, therefore the one may be taken for the other.

Ex. 1. Convert 9 cwt. 3 qrs. 6 lbs. into *Indian weight.*

9 cwt. 3 qrs. 6 lbs. = 1098 lbs. = $1098 \times 70 \div 9$ ch. = 8540 ch.
= 13 mds. 13 sr. 12 ch. *Ans.*

Ex. 2. Convert 6 mds. 26 sr. 14 ch. into *cwt, etc.* (Avoir.)

6 mds. 26 sr. 14 ch. = 4270 ch. = $4270 \times 9 \div 70$ lbs. (Avoir.)
= 549 lbs. = 4 cwt. 3 qrs. 17 lbs. *Ans.*

Ex. 3. Reduce 1 cwt. 2 lbs. (Avoir.) to *Troy weight.*

1 cwt. 2 lbs. = 114 lbs = 114×7000 grs. Troy = 798000 grs.
= 138 lbs. 6 oz. 10 dwts. *Ans.*

Examples XXXIII.

1. Reduce to *tolas* :—

1440 grs.; 7 lbs 7 oz. 17 dwts. 12 grs.; 16 lbs. 6 oz.; 2 oz. 5 dwts.

2. Reduce to *grains* (Troy) :—

(1) 16 sr. 8 ch.; 25 sr. 14 ch. 3 tolas; 1 md. 5 sr. 14 ch.; 4 vis 15 palams 2 tolas.

(2) 8 tons 8 cwt. 98 lbs. 3045 grs.; 425 tons 19 cwt. 100 lbs. 15 oz. 200 grs.; 1 cwt. 1 qr. 25 lbs.

3. Reduce 20 lbs. Avoir. to *Troy weight*; 16 dwts. to *Apoth. weight*; 5 drs. Apoth. to *Troy weight*; 525 lbs. Troy to *mds., sr., &c.*

4. Reduce 96 tolas to *oz. Troy*; 37400157 grains Troy to (Madras) *mds., vis, &c.*; 309432159 lbs. Avoir., to *mds., sr., &c.*

5. Reduce to *maunds, sr., &c.* :—

14 cwt. 1 qr. 24 lbs.; 10 cwt. 3 qrs. 20 lbs.; 3 tons 12 cwt. 1 qr. 8 lbs; 3 tons 19 cwt. 8 lbs.; 4 cwt. 3 qrs. 8 lbs.

6. Reduce to *tons, cwt., &c.* :—

7 mds.; 15 mds. 38 sr. 12 ch.; 9 mds. 7 sr. 8 ch.; 10 mds. 20 sr.; 53 mds. 15 sr.; 21 mds. 35 sr.

7. Reduce 1137 lbs. 6 oz. Troy to *lbs. Avoir*; 2 cwt. 3 qrs. 17 lbs. and 5 cwt. 18 lbs. 14 oz. to *Troy weight.*

8. Convert 6 tons 10 cwt. 65 lbs. into *Madras maunds*; 8 tons 2 cwt 9 lbs. into *Bombay maunds*; 368 Bombay maunds and 140 Madras maunds into *tons, cwt., &c.*

9. How many 2 lbs. packets of tea can be made from a chest weighing 7 cwt. 3 qrs. 16 lbs.?

10. Each ton of ore obtained from a gold mine yields on an average 2 oz. 1 dwt. 15 grs. of fine gold. How much fine gold will be obtained from 293 tons?

11. How many coins each weighing 1 oz. 8 dwt. can be made of 770 lbs. of metal?

12. A truck is loaded with 120 sacks; each sack weighs 7 sr. 10 ch., and contains 84 seers of grain. What is the weight of the whole in maunds and seers?

13. How many pounds Avoir. are equal to 175 lbs. Troy?

14. Multiply 88 ka. 12 mds. 16 sr. (Bombay) separately by 99 66 and 144; and 4 ka. 5 mds. 15 sr. by 3268.

15. A train consists of 29 trucks of equal weight; 9 of them weigh 53 tons 1 cwt. 1 qr. 3 lbs. What do the rest of them weigh?

16. Convert 2 qrs. 16 lbs. into *seers*, 10 cwt. 1 qr. 13 lbs. into *maunds*, and 15 lbs. 2 oz. 5 dwts. 20 grs. into lbs. *Avoir.*

17. Express 576 lbs. Avoir. as lbs. *Troy*, 58 lbs. 4 oz. Troy as lbs. *Avoir.*, and 16 dwts. 16 grs. in *Apoth-weight.*

18. Reduce 9720 grs. Troy to *tolas* and find how many *lbs.* are there in 12288 tolas?

19. How many times is a weight of 6 tons 7 cwt. 27 lbs. 5 oz. contained in 159 tons 1 cwt. 10 lbs. 13 oz.?

20. What is the whole weight of 217 waggon loads, each containing 3 tons 13 cwt. 3 qrs. 13 lbs.?

21. 797 tons 19 cwt. 2 qrs. 14 lbs. is divided among a certain number of people, so that each receives 5 tons 3 cwt. 2 qrs. 15 lbs. How many of them were there?

22. 84 poor men have distributed equally among them 252 mds. 10 sr. 12 ch. of rice; what share will each receive?

23. If 5 ka. 15 mds. 30 sr. of a certain article can be bought for a rupee, what quantity can be bought for 2384 rupees?

24. 21 tons 3 cwt. 1 qr. 17 lbs. 5 oz. 8 drs. of rice are to be packed in bags of equal size. How many bags will be required if each hold 24 lbs. 6 oz. 8 drs.?

25. Reduce 2457600 dhans to *maunds.*

26. Multiply 109 ka. 13 mds. 6 sr. (Madras) separately by 72, 35 and 750; and 5 ka. 15 mds. 30 sr. by 4503.

27. Divide :—

(1) 6 mds. 6 sr. 27 ta. (Bombay) by 73.

(2) 311 ka. 10 mds. 36 sr. 4 palams (Madras) by 503.

28. Divide 64 ka. 7 mds. 12 sr. by 15 mds. 13 sr. (Madras).

29. Divide 160 ka. 10 mds. 39 sr. by 15 mds. 3 sr. (Bombay).

30. If standard gold contained 12 parts of pure gold to 1 part of copper, and 247 oz. Troy were coined into 960 sovereigns; wha would be the weight of pure gold in a sovereign?

31. How many bars of gold each weighing 5 oz. 13 dwts. 21 grs. can be made out of a bar weighing 88 lbs. 8 oz. 14 dwts. 15 grs.?

32. Find the weight of 73 iron bars, each weighing 17 cwt. 2 qrs. 19 lbs. 5 oz.

33. How many bars of iron each weighing 11 lbs. 10 oz. 11 drs. must be taken to make up a weight of 4 tons 8 cwt. 3 lbs. 6 oz. 15drs.?

34. Express in *Troy-weight* the weight of a silver dish weighing 3 sr. 2 poas, and of 6 scruples of soda.

35. Which is the heavier, a pound of gold or a pound of feathers? and by how much?

III. MEASURES OF LENGTH.

155. **Indian Lineal Measure.**

3 Yabs	make	1 Anguli
4 Angulis	"	1 Mushti
3 Mushtis	"	1 Bighat (*span*)
2 Bighats or 24 angulis	"	1 Háth or **Cubit**
4 Háths	"	1 Danda or Dhanu
2000 Dandas or 8000 háths	"	1 Kros or Kos
4 Kros	"	1 Yo-jan.

156. **English Lineal Measure.**

3 Barley-corns (in length)	make	1 Inch (*in.* or 1′)
12 Inches	"	1 Foot (*ft.*)
3 Feet	"	1 **Yard** (*yd.*)
$5\frac{1}{2}$ Yards	"	1 Rod, Pole (*po.*) or Perch.
40 Poles, or 220 yds.	"	1 Furlong (*fur.*)
8 Furlongs, or 1760 yds.	"	1 Mile (*mi.*)
3 Miles	"	1 League (*lea.*)

1 yard = 2 cubits; 1 Ilahi Gaj (N.-W.P.) = 33in.; 1 Kros = 4000yds.; 1 Karam (Madras) = 3 cubits; 1 Kathi (Bombay) = 9·4ft.; 1 half-yard = 1 ft. 6 in.

Cloth Measure.

IN BENGAL.			ENGLISH.		
3 Angulis	make	1 Girah	$2\frac{1}{4}$ Inches	make	1 Nail (*nl.*)
8 Girahs	"	1 Hath	4 Nails	"	1 Quarter (*qr.*)
2 Haths or 16 girahs	"	1 Gaj	4 Quarters	"	1 Yard
IN BOMBAY.			3 Quarters	"	1 Flemish ell
2 Angulis	make	1 Tasu	5 Quarters	"	1 English ell
24 Tasus	"	1 Gaj	6 Quarters	"	1 French ell

1 Nail = 1 Girah ; 1 Bombay gaj (cloth-measure) = 27 in. ; 1 Bengal gaj = 36 in. = 1 yard.

Land Measure.

IN BENGAL.			ENGLISH.		
4 Haths	make 1	Katha	25 Links	make 1	Pole or Rod
20 Kathas	„ 1	Bigha	100 Links	„ 1	Chain
80 Haths	„ 1	Rasi	10 Chains	„ 1	Furlong

In the N.-W.P., 3 Ilahi Gaj = 1 Bans or Ganteh and 20 Bans = 1 Jarib

The following measures are sometimes used :—

1 Inch = 72 points = 12 lines ; 1 Palm = 3 in. ; 1 Hand = 4 in. (for measuring horses) ; 1 Span = 9 in. ; 1 Cubit = 18 in. ; 1 Pace = $2\frac{1}{2}$ft. (military) = 5 ft. (geometrical) ; 1 Fathom = 6 ft. ; 1 Cable's length = 120 fathoms ; 1 Knot (nautical) = 6080 ft. : 1 Degree of Latitude = 60 Knots ; 1 Chain = 4 poles = 22 yds. ; 80 Chains = 1 mile.

157. To reduce poles to yards, we have to multiply by $5\frac{1}{2}$; but since $5\frac{1}{2}$ yds. is 11 half-yards, we multiply the poles by 11, and divide the product by 2. In the converse operation, to divide by $5\frac{1}{2}$, we multiply the yards by 2, and divide the product by 11. The remainder in each case is half-yard, and note that 1 half-yd. is $1\frac{1}{2}$ ft. = 1 ft. 6 in. Also, in reducing miles and furlongs to yards, multiply by 1760 and 220 respectively, unless prevented by the form of the question. To reduce yards to miles, divide by 1760.

Note. 1 half-yd. = 1 ft. 6 in. Also 1 po. = 5 yds. 1 ft. 6 in.

Ex. 1. Reduce 9 mi. 4 fur. 23 po. 4 yds. 2 ft. 9 in. to *inches*.

```
mi.  fur.  po.  yds.  ft.  in.
 9    4    23    4     2    9
 8
 76 fur.
 40
3063 po.
  11
2)33693 half-yds.
  16846 yds. + 1 half-yd.
```

```
or 16846 yds.  1 ft.  6 in.
       4 yds.  2 ft.  9 in.
   16851 yds.  1 ft.  3 in.
           3
   50554 ft.
      12
   606651 in.  Ans.
```

Ex. 2. Reduce 3126749 inches to *miles, &c.*

```
12)3126749 in.
3) 260562 ft...5 in.    ∴ the result
   86854 yds.          = 49 mi.  2 fur.  31 po.  7 half-yds.  5 in.
       2               = 49 mi.  2 fur.  31 po.  3 yds. 1 ft. 11 in.
11) 173708 half-yards.
40)  15791 po ..7 half-yds. [for 7half-yds. = 3½ yds. = 3 yds. 1ft. 6in.]
8)     394 fur ..31 po.
        49 mi....2 fur.
```

Examples XXXIV.

1. Reduce (i) to *haths* or *cubits* and (ii) to *angulis* :—

15 kros 1008 dandas ; 6 yojan 2 kros 1780 dandas ; 20 bi. 4 kat. ; 25 bi. 15 kat. 3 cubits ; 10 kros 875 dandas 3 haths.

2. Reduce to *gaj*, *&c.* :—

34256 angulis ; 94605 girahs ; 420367 angulis ; 7035 girahs.

3. Reduce to *inches* :—

(1) 3 fur. 135 yds. 4 in. ; 5 mi. 200 yds. 3 in. ; 512 yds. 2 ft. 9 in. ; 4 lea.
(2) 2 mi. 7 fur. 15 po. 1 yd. 1 ft. 6 in. ; 13 lea. 1 mi. 4 fur. 37 po. 1 ft. 8 in.
(3) 31 mi. 4 fur. 115 yds. 1 ft. 8 in. ; 25 mi. 6 fur. 17 po. 4 yds. 3 in.
(4) 25 mi. 459 yds. 31 in. ; 25 fur. 39 po. 3 yds. 2 ft. 8 in.

4. Reduce 7 mi. 5 fur. 32 po. 4 yds. to *yards* ; 2 lea. 2 mi. 7 fur. to *yards* ; 5 mi. 3 fur. 208 yds. 1 ft. to *feet* ; 15 mi. 5 fur. 31 po. to *poles*.

5. Reduce to *miles*, *&c.* :—

(1) 57383 yds. ; 1847638 ft. ; 268543 in. ; 304935 ft. ; 53628 ft.
(2) 1081080 in. ; 231031 yds. ; 517900 in. ; 36090 ft. ; 2000000 in.

6. Reduce 183810 ft. to *leagues* ; 152017634 in. to *miles*.

7. Reduce :—

(1) 20 yds. 3 qrs. 1 nl. to *nails*.
(2) 5 miles to *fathoms*.
(3) 35 ells 4 qrs. to *nails*.
(4) 16 ells 1 qr. 3 nls. 1 in. to *in.*
(5) 500 fathoms to *yards*.
(6) 5 furlongs to *fathoms*.
(7) 35 kros to *cubits*.
(8) 5 miles to *links*.
(9) 1 gaj 1 hath 1 girah to *angulis*.
(10) 16 haths 9 in. to *feet*.

8. Reduce :—

(1) 2897 in. of cloth to *yards*.
(2) 567912 cubits to *bighas*, *&c.*
(3) 201494 jabs to *dandas*.
(4) 74310 tasu to *gaj*, *&c.*
(5) 25 kros to *miles* and *yards*.
(6) 76 miles to *kros* and *haths*.
(7) 1 kros 1999 dandas 1 gaj 1 hath 7 girahs 2 angulis to *angulis*.

9. Add together :—

(1)

yds.	ft.	in.
22	2	7
54	1	9
67	2	10
85	0	11
92	1	3

(2)

po.	yds.	ft.	in.
7	3	1	11
12	$2\frac{1}{2}$	2	4
9	4	0	7
2	$3\frac{1}{2}$	1	9
10	1	2	8

(3)

mi.	fur.	po.	yds.
14	3	17	$2\frac{1}{2}$
23	5	33	4
37	1	24	5
43	7	31	$1\frac{1}{2}$
75	6	36	$2\frac{1}{2}$

(4)

mi.	po.	yds.	in.
3	84	$2\frac{1}{2}$	7
12	113	0	9
6	0	$4\frac{1}{4}$	11
25	44	3	8

(5)

yds.	qrs.	nls.
25	3	2
37	0	3
54	1	1
49	2	3

(6)

ells	qrs.	nls.
35	2	3
42	4	5
37	2	2
25	4	3

10. Perform the operation of subtraction in the following :—

(1) mi.	fur.	po.	(2) fur.	po.	yds.	(3) mi.	po.	yds.	(4) yds.	qrs.	nls.
24	0	7	6	37	4	6	0	0	18	2	0
11	5	18	5	18	$4\frac{1}{2}$	3	37	$2\frac{3}{4}$	8	3	3

11. Multiply :—

(1) 5 fur. 78 yds. 2 ft. 7 in. by 56 ; 42 yds. 2 qrs. 2 nls. by 40.
(2) 13 lea. 2 mi. 6 fur. 25 po. separately by 42 and 97.
(3) 25 mi. 6 fur. 23 po. 3 yds. 2 ft. 8 in. ... ... 56 and 83.
(4) 20 dandas 1 hath 7 girahs ... ... 4, 5 and 12.

12. Divide :—

(1) 3179 lea. 1 mi. 5 fur. 16 po. by 46 ; 485 yds. 3 qrs. 3 nls. 2 in. by 11.
(2) 478 mi. 6 fur. 19 po. 2 yds. 1 ft. 10 in. separately by 96 and 4397.
(3) 679 mi. 7 fur. 125 yds. 2 ft. 6 in. ... ... 11 and 120.
(4) 275 dandas 1 gaj 4 girahs ... ... 3, 5, 10 & 12.
(5) 1027 mi. 1 fur. 6 po. by 17 mi. 5 fur. 27 po.

13. If 67 pieces of cloth measure 2335 yds. 2 qrs. 7 in., what is the length of 1 piece ?

14. If a person complete a journey of 422 mi. 3 fur. 38 po. in 37 days ; what distance does he travel per day ?

15. Find the aggregate of 4 kros 1 gaj 1 ha. 7 gir. ; 5 kros 1978 dan. 4 gir. ; 2 kros 150 dan. 1 gaj. 1 ha. 2 gir. ; 11 kros 25 dan. 1 ha. 5 gir. and 6 kros 1 gaj 3 gir.

16. How many lengths each equai to 9 po. 3 yds. 1 ft. 3 in. will make up 1 mile 6 fur. 26 po. 4 yds. 2 ft. 9 in. ?

IV. MEASURES OF SURFACE.

158. **Land Measure in Bengal.**

20 Square cubits or Gandas make 1 Chhatak.
16 Chhataks " 1 Katha.
20 Kathas " 1 **Bigha**.

IN N.-W. P.

20 Kachvansi make 1 Bisvansi
20 Bisvansi ,, 1 Bisva
20 Bisvas ,, 1 Bigha

IN BOMBAY.

39¼ Square cubits make 1 Kathi
20 Kathis ,, 1 Pand
20 Pands ,, 1 Bigha
6 Bighas ,, 1 Rukeh
20 Rukehs ,, 1 Chahur

IN PUNJAB.

9 Sarsi make 1 Marla
20 Marlas ,, 1 Kanal
4 Kanals ,, 1 Bigha
2 Bighas ,, 1 Ghuma

IN MADRAS.

144 Sq. Inches make 1 Sq. ft.
2400 Sq. feet ,, 1 Ground or Manai
24 Grounds ,, 1 Cawny
484 Cawnies ,, 1 Sq. mile.

1 Bengal Bigha = 1600 sq. yds.; 1 N.-W. P. Bigha = 3025 sq. yds.; 1 Punjab Bigha = 1620 sq. yds.; 1 Bombay Bigha = 3927 sq. yds. Also 1 Madras Cawny = 6400 sq. yds. = 4 Bengal Bighas.

159. English Square Measure.

144 Square Inches (*sq. in.*)	make	1 Square Foot (*sq. ft.*)
9 Square Feet	"	1 **Square Yard** (*sq. yd.*)
$30\frac{1}{4}$ Square Yards	"	1 Square Pole (*sq. po.*)
40 Square Poles	"	1 Rood (*ro.*)
4 Roods or 4840 sq. yds.	"	1 **Acre** (*ac.*)
640 Acres	"	1 Square Mile (*sq. mi.*)

10,000 Sq. links make 1 Sq. chain	484 Sq. yds. make 1 Sq. chain.
10 Sq. chains " 1 Acre	100,000 Sq. links...1 Acre.

A Rod of Brickwork = $272\frac{1}{4}$ sq. ft. A Rod of Building = 36 sq. yds. A Square of Flooring, Roofing, &c. = 100 sq. ft. A Yard of Land = 30ac. A Hide of Land = 100 ac. One sq. chain = 10,000 sq. links.

40 ac. = 121 Bengal Bighas; 5 ac. = 8 N.-W.P. Bighas; 81 ac. = 242 Punjab Bighas; 160 ac. = 121 Madras Cawnies. Also 1 sq. mi. = 1936 Bengal Bighas = 1024 N.-W.P. Bighas = 484 Madras Cawnies.

160. To reduce square poles to square yards, we have to multiply by $30\frac{1}{4}$; but since $30\frac{1}{4}$ sq. yds. is 121 qr.-sq. yds., we multiply the sq. poles by 121 and divide by 4. In the converse operation, to divide by $30\frac{1}{4}$, we multiply the sq. yds. by 4 and divide by 121. The remainder in each case is qr.-sq. yds. and note that 1 qr.-sq. yd. is $2\frac{1}{4}$ sq. ft. = 2 sq. ft. 36 sq. in.

Also in reducing acres and roods to sq. yards, multiply by 4840, and 1210 respectively, unless prevented by the form of the question. To reduce square yards to acres, divide by 4840.

Note. 1 qr.-sq. yd. = 2 sq. ft. 36 sq. in.; 2 qr.-sq. yds. = 4 sq. ft. 72 sq. in.; 3 qr.-sq. yds. = 6 sq. ft. 108 sq. in. Also 1 sq. po. = 30 sq. yds. 2 sq. ft. 36 sq. in.

Ex. 1, Reduce 3 ac. 2 ro. 23 sq. po. 10 sq. yds. 8 sq. ft. 18 sq. in. to *sq. inches*.

```
  3 ac.  2 ro.  23 sq. po.  10 sq. yds.  8 sq. ft.  18 sq. in.
  4
 14 ro.                          17646 sq. yds.  5 sq. ft.  126 sq. in.
 40                                  9
583 sq. po.                     158819 sq. ft.
121                                144 = 12 × 12.
4)70543                       22870062 sq. in.  Ans.
 17635 sq. yds. + 3 qr.-sq. yds.
=17635 sq. yds. 6 sq. ft. 108 sq. in.
    10          8          18
 17646 sq. yds. 5 sq. ft. 126 sq. in.
```

Ex. 2. Reduce 9532482 sq. inches to *acres*.

144 { 12)9532482 sq. in.
12) 794373...6
9) 66197 sq. ft. 9 } 114 sq. in. ∴ the result
7355 sq. yds...2 sq. ft. = 1 ac. 2ro. 3 sq. po. 17 qr.-sq.yds. 2 sq. ft. 114 sq. in.
4
121 { 11)29420 qr.-sq. yds. = 1 ac. 2 ro. 3 sq. po. 4 sq. yds. 2 sq. ft. 36 sq. in.
11) 2674......6
4,0) 24,3 sq. po. 1 } 17 qr.-sq. yds. + 2 sq. ft. 114 sq. in.
4) 6 ro...3 sq. po. = 1 ac. 2ro. 3sq. po 4sq. yds. 5sq. ft. 6sq.in.
1 ac...2 ro.

Examples XXXV.

1. Reduce to *gandas* or *square cubits* :—

5 bi. 3 kat. 6 ch.; 45 bi. 9 kat. 7 ch.; 25 bi. 15 kat. 4 ch. 15 ga.; 135 bi. 11 kat.; 425 bi. 17 kat. 13 ch. 17 ga.; 29 bi. 17 kat.

2. Reduce to *bighas* :—

357628 ch.; 10486 ga.; 8326675 sq. cubits; 4675900 ga.; 125720 ch.

3. Reduce to *kachvansi* :—

24 bi. 15 bisv.; 136 bi. 14 bisv. 17 bisvansi; 86 bi. 7 bisv.; 423 bi. 10 bisv. 12 bisvansi 15 kachv.

4. Reduce to *square inches* :—

8 sq. mi. 340 caw.; 15 sq. mi. 285 caw. 12 grounds; 25 sq. mi. 375 caw. 20 grounds 1452 sq. ft.; 3 caw. 13 manies 5 sq. ft.

5. Reduce to *sq. karam* or *sarsai* :—

26 ghm. 1 bi.; 42 ghm. 1 bi. 3 ka. 15 marlas; 42 bi. 2 ka. 4 sar.

6. Reduce to *kathis* :—

163 bi. 7 pands 3 ka.; 4 cha. 108 bi. 15 pands; 42 bi. 112 ka.

7. Reduce :—

(1) 246053 kachvansi to *bighas*.
(2) 34512876 kathis to *bighas*.
(3) 43276850 sq. in. to *cawnies*.
(4) 403207654 kathis to *chahurs*.
(5) 1130692 manies to *sq. miles*.
(6) 8740361 sq. sarsai to *ghumas*.

8. Reduce to *sq. inches* :—

(1) 17 sq. yds. 8sq. ft.; 3 sq. yds. 6 sq. ft. 75 sq. in.; 29 sq. yds.; 54 sq. yds. 8 sq. ft. 104 sq. in.; 3 ro. 17 po. 21 sq. yds. 8 sq. ft.
(2) 17 ac. 14 po.; 1 ac. 2 ro. 3 po. 4 sq. yds.; 3 ro. 22 po. 21 sq. yds. 8 sq. ft. 116 sq. in.; 56 ac. 2 ro. 25 po. 37 sq. yds. 5 sq. ft. 73 sq. in.
(3) 38 ac. 2 ro. 35 po.; 324 sq. po.; 3 sq. mi.; 4 ac. 26 po.; 42 ac.

9. Reduce to *acres* :—

(1) 16553 sq. po.; 13678 sq. yds.; 170184 sq. ft.; 82973 sq. po.; 895487 sq. yds.; 2709437 sq. ft.

(2) 123456789 sq. in.; 94501362 sq. in.; 455462764 sq. in.; 72013512032 sq. in.; 355433005 sq. in.

10. Reduce :—

(1) 14 ac. to *sq. links.* (2) 1803 ac. to *sq. miles.*
(3) 5200000 sq. yds. to *sq. miles.* (4) 428 sq. chains to *sq. inches.*
(5) 5621 sq. po. to *sq. chains.* (6) 535 sq. miles to *bighas.*

11. Reduce (*Bengal bighas*) :—

5445 bighas to *acres*; 2560 ac. to *bighas*; 9680 bi. to *acres*; 14400 ac. to *bighas*; 7260 bi. to *acres*; 92360 ac. to *bighas.*

12. Reduce : 629200 Bengal bighas to *N.-W. P. bighas*: 9720 Bengal bighas to *Punjab bighas*; 320780 Bengal bighas to *Madras Cawnies*; 768000 N.-W. P. bighas to *Bengal bighas* and 28800000 Punjab bighas to *Bengal bighas.*

13. Add together :—

(1)

bi.	ka.	ch.
30	15	10
19	17	12
25	18	13
31	12	15
28	8	9

(2)

sq. yds.	sq.ft.	sq. in.
32	2	98
12	8	120
19	7	47
23	6	135
45	7	85

(3)

ac.	ro.	po.
29	3	28
35	3	35
45	0	25
17	1	20
19	2	16

(4)

ro.	sq.po.	sq.yds.
74	19	15
6	34	$11\frac{1}{4}$
17	0	$27\frac{1}{2}$
23	39	$16\frac{3}{4}$

(5)

ac.	ro.	po.	sq.yds.
35	1	23	$12\frac{1}{2}$
9	2	15	$27\frac{1}{4}$
11	1	24	11
42	0	35	$2\frac{1}{4}$

(6)

ac.	po.	sq.yds.	sq.ft.	sq.in.
25	11	0	8	23
36	39	11	0	136
7	0	27	6	0
18	20	23	7	94

14. Perform the following subtractions :—

(1)

bi.	kat.	ch.
125	8	9
76	12	13

(2)

ac.	ro.	po.
96	1	19
29	3	30

(3)

ac.	ro.	po.	sq.yds.
45	1	29	$25\frac{1}{2}$
39	3	18	$27\frac{1}{2}$

15. Multiply :—

(1) 120 bi. 14 kat. 10 ch. by 49; 125 bi. 15 kat. 12 ch. by 154.
(2) 17 ac. 1 ro. 31 po. by 72; 2 ro. 27 po. 15 sq. yds. 8 sq. ft. by 6 and by 10.
(3) 37 ac. 3 ro. 19 po. 28 sq. yds. 4 sq. ft. 103 sq. in. by 8 and by 75.

16. Divide :—

(1) 112 bi. 18 kat. 14 ch. by 99; 1539 bi. 15 kat. 7 ch. by 102.
(2) 82 bi. 16 kat. 12 ch. by 72; 130 ac. 1 ro. 28 po. by 120.
(3) 854 ac. 3 ro. 27 po. 8 sq. yds. 8 sq. ft. 45 sq. in. by 9 and by 246.
(4) 166 ac. 2 ro. 6 po. 30 sq. yds. 5 sq. ft. by 7 ac. 38 po. 17 sq yds. 1 sq. ft.; 935 bi. 12 kat. 12 ch. by 55 bi. 12 ch.

17. How many allotments each equal to 2 ro. 5 po. 13 sq. yds. 6 sq. ft. 108 sq. in. can be formed out of 158 ac. 2 ro. 20 po.?

18. A certain district contains 514164 ac. and another 95805 ac. How many sq. miles does the one contain more than the other?

V. MEASURES OF SOLIDITY.

161. Bengal Measure of Solidity.

13824 Cubic Angulis	make 1 Cubic Cubit or C. hath.
8 Cubic Cubits	„ 1 Cubic yard.
8 Cubic yards or 64 cub. cubits	„ 1 Chouka.

162. English Measure of Solidity.

1728 Cubic Inches (*cub. in.*)	make 1 Cubic Foot (*cub. ft.*)
27 Cubic feet	„ 1 **Cubic yard** (*cub. yd.*)

1 Cub. hath = 5832 cub. in. A Load of rough Timber = 40 cub. ft. A Load of squared Timber = 50 cub. ft. A Ton of Shipping = 42 cub. ft. A Stack of wood = 108 cub. ft. A Cord of wood = 128 cub. ft.

Examples XXXVI.

1. Reduce to *cub. cubits* :—

42 choukas 54 cub. cubits ; 87 choukas 62 cub. cubits ; 146 choukas 32 cub cubits ; 144 choukas.

2. Reduce to *cub. in.* :—

24 cub. yds. 7 cub. ft. 144 cub. in. ; 18 cub. yds. 1274 cub. in. ; 12 cub. yds. 23 cub. ft. ; 23 cub. yds. 1000 cub. in.

3. Reduce to *cub. yds.* :—

200000 cub.in. ; 138297 cub. in. ; 141721 cub.in. ; 863005 cub.in.

4. Reduce to *choukas* :—

36248742 cub. cubits ; 4308756 cub. cubits ; 862097 cub. cubits.

5. Reduce 1053 choukas 28 cub. cubits to *cubic angulis*.

6. Add together :—

(1)

Chouka	cub. yds.	cub. hath.
18	6	4
27	5	7
134	4	5
49	3	2
234	3	6

(2)

c. yds.	c. ft.	c. in.
53	7	1249
27	23	472
29	16	1384
45	18	1186
33	9	1324

(3)

c. yds.	c. ft.	c. in.
328	15	323
237	19	484
785	10	1259
546	0	342
729	11	1075

7. Perform the following subtractions :—

(1)

c. yds.	c. ft.	c. in.
49	15	542
39	23	736

(2)

c. yds.	c. ft.	c. in.
150	0	0
59	25	1001

(3)

c. yds.	c. ft.	c. in.
527	0	1
279	1	259

8. Multiply :—

(1) 2 cub. yds. 5 cub. ft. 704 cub. in. by 11 and by 23.
(2) 275 cub. yds. 17 cub. ft. 125 cub. in. by 56.

9. Divide :—

(1) 372 cub. yds. 1236 cub. in. by 64.
(2) 6739 cub. yds. 2 cub. ft. 468 cub. in. by 19 and by 509.
(3) 18809 cub. yds. 1 cub. ft. 1156 cub. in. by 723 cub. yds. 11 c.ft. 84c.in.

10. A certain number of bins, each containing 8 cub. yds. 152 cub. in., contain 1512 cub. ft. 1064 cub. in. ; find the number.

VI. MEASURES OF CAPACITY.

163. 1st. Tables of Corn or Dry Measure.

Indian.

BENGAL MEASURE.			BOMBAY MEASURE.		
5 Chhataks	make	1 Kunka	36 Tanks	make	1 Tipari
2 Kunkas	,,	1 Khunchi	2 Tiparis	,,	1 Seer
2 Khunchis	,,	1 Rek	4 Seers	,,	1 Payli
2 Reks	,,	1 Pali	16 Paylis	,,	1 Phara
2 Palis	,,	1 Doan	8 Pharas	,,	1 Kandi
2 Doans	,,	1 Kati	25 Pharas	,,	1 Muda
8 Katis	,,	1 Arhi	MADRAS MEASURE.		
20 Arhis	,,	1 Bish	8 Ollaks	make	1 Paddi
16 Bishes	,,	1 Kahan	8 Paddis	,,	1 Markal
16 pa. or 8do.	,,	1 **Maund** (*md.*)	5 Markals	,,	1 Phara
20 Doans	,,	1 Sali	80 Pharas	,,	1 Garce

In Bengal, lime is measured thus : 1 Phara $= 27' \times 20' \times 9'$; 6 Pharas = 5 cub. hath ; 80 Pharas = 100 mds. ; 1 markal (Madras) = 750 cub. in.

English.

2 Quarts (*qt.*)	make	1 Pottle (*pot.*)
2 Pottles or 4 qts.	,,	1 **Gallon** (*gal.*)
2 Gallons	,,	1 Peck (*pk.*)
4 Pecks	,,	1 Bushel (*bus.*)
2 Bushels	,,	1 Strike (*str.*)
4 Bushels	,,	1 Coomb (*co.*)
2 Coombs or 8 bus.	,,	1 Quarter (*qr.*)
5 Quarters	,,	1 Load (*ld.*)
2 Loads or 10 qrs.	,,	1 Last.

COAL MEASURE.

4 Pecks	make	1 Bushel
3 Bushels	,,	1 Sack
12 Sacks or 36 bus.	,,	1 Chaldron

A gallon (*Imperial*) contains 277·274 cub. in. ; hence a bushel (*Imperial*) consisting of 8 gallons, contains $8 \times 277{\cdot}274$ or 2218·192 cub.in.

164. 2nd. Tables of Liquid Measure.

Indian.

4 Chhataks	make	1 Póa
4 Póas	,,	1 **Seer**
40 Seers	,,	1 Maund

The weight of a seer for this measure varies in different localities from 40 tolas to 112 tolas.

English.

WINE MEASURE.

4 Gills (*gil.*)	make	1 Pint (*pt.*)
2 Pints	,,	1 Quart (*qt.*)
4 Quarts	,,	1 **Gallon** (*gal.*)
63 Gallons	,,	1 Hogshead (*hhd.*)
2 Hogsheads or 126 gallons.	,,	1 Pipe (*pipe.*)
2 Pipes	,,	1 Tun

10 Gallons = 1 Anker
18 Gallons = 1 Runlet
42 Gallons = 1 Tierce
84 Gallons or 2 Tierces = 1 Puncheon

ALE AND BEER MEASURE.

2 Pints	make	1 Quart
4 Quarts	,,	1 **Gallon**
36 Gallons	,,	1 Barrel (*bar.*)
$1\frac{1}{2}$ Barrels or 54 gallons	,,	1 Hogshead
2 Hogsheads	,,	1 Butt
2 Butts	,,	1 Tun

9 Gallons = 1 Firkin
18 Gallons = 1 Kilderkin.

A pint of pure water weighs a pound and a quarter therefore a gallon of distilled water weighs 10lbs (Avoir.), when the barometer is at 30 in. and the air at a temperature of 62° Fah. thermometer. Hence the weight of a cubic foot of water is very nearly 1000 oz. (Avoir.)

165. English Apothecaries' Measure.

60 Minims (*m.*) or drops	make	1 Fluid Dram (*fl. dr.*)
8 Fluid Drams	,,	1 Fluid Ounce (*fl. oz*)
20 Fluid Ounces	,,	1 Fluid Pint (*O. Octarius.*)
8 Pints	,,	1 **Gallon** (*C; Congius*)

A tea-spoonful = 1 fluid dram. A desert-spoonful = $2\frac{1}{2}$ fluid drams. A table-spoonful = 4 fluid drams. 1 Fluid ounce = 1 ounce (Avoir.)

Examples XXXVII.

1. Reduce to *chhataks* : 2 mds. 3 do. 2 pa. 3 ch.; 1 md. 3 do. 1 khun.; 8 kah. 14 bis. 16 arh.; 125 mds. 6 do. 1 pa. 1 rek.; 14 kah. 10 do.; 17 salis 58 pa. 2 reks.

2. Reduce : 3842 ch. to *maunds*; 201372 kunikas to *maunds*; 48762035 ch. to *maunds*; 467032000 ch. to *kahans*; 246780 reks to *maunds*; 346780 khun. to *doans.*

3. Reduce : 125 pharas to *tanks*; 416 mudas to *tanks*: 1 ka. 3 ph. 5 paylis 1 tipari 26 tanks to *tanks*; 6932843 tiparis to *mudas*; 54038764 tanks to *kandis.*

4. Reduce : 205 pharas to *ollaks*; 1 garce 45 pharas 2 markals 3 paddis to *ollaks*; 28 pharas 4 markals 54 ollaks to *ollaks*; 256284 ollaks to *garces*; 123456 ollaks to *pharas*; 2368 paddis to *pharas*; 987600 ollaks to *markals.*

5. Reduce to *gallons* : 2 qrs. 7 bus. 2 pks.; 3 lds. 3 qrs. 3 pks.; 54 qrs. 7 bus. 6 gal.; 64 lasts 1 ld. 3 qrs. 7 bus. 1 pk.

6. Reduce to *quarts* : 25 qrs. 2 bus. 2 pks. ; 7 lds. 2 co. 3 pks. ; 17 lasts 1 qr. 7 pks. ; 356 qrs. 7 bus. 2 pks. 1 gal. ; 3 lds. 3 bus.

7. Reduce : 598712 gals. to *quarters* ; 800574 bus. to *lasts* ; 205634 qts. to *coombs* ; 986753 strikes to *quarters*.

8. Reduce to *loads* : 89765 pks. ; 56789 pts. ; 356187 qts. ; 1000000 pks. ; 97324 pts. ; 4357 gals

9. Reduce to *gills* : 1 hhd. 35 gals. ; 5 pipes ; 2 pipes 7 gals. 1 qt. ; 3 tuns 1 hhd. 57 gals. , 27 tuns 1 pipe 1 hhd. 54 gals. 1 qt. 1 pt.

10. Reduce to *pints* : 2 qrs. 1 gal. ; 2 qrs. 5 bus. 3 pks. 1 gal. ; 987 bar. 25 gals. 3 qts. 1 pt. ; 21 tuns 3 hhds. 54 gals. 2 qts.

11. Reduce : 8 gals 2 fl oz. to *fl. drams* ; 5 C. 7 O. 17 fl. oz. 5 fl. dr. 45 m. to *minims* ; 3 O. 2 fl. oz. 40 m. to *minims*.

12. Reduce : 56321 pts. to *pipes* ; 1000000 qts. to *tuns* ; 5279 pts. to *gallons* ; 62741 gills to *gallons* ; 3720812 gills to *quarters*.

13. Reduce : 84381 pts. to *tuns* ; 24357 gills to *pipes* ; 9000 gals. to *butts* ; 58428092 gills to *lasts* , 5849206 qts. to *hogsheads*.

14. Reduce to *gallons* ; 882743 minims ; 58428092 minims.

15 What is the weight of :4 gals. 3 pts. of water in Avoir. ?

16. What is the weight in *kan'tis* of 256 pharas of lime ?

17. What is the weight of 12 cub. yds. 12 cub. ft. of water in lbs. Avoir. ? In 250 packs of wool, how many tons ?

18. Add together :—

(1)

mds.	do.	recks.
145	6	3
47	5	2
258	4	1
96	7	2
74	0	1

(2)

gals.	qts.	pts.	gils.
57	3	1	3
38	1	1	2
45	2	0	3
26	3	0	3
18	2	1	0

(3)

qrs.	bus.	pks.	gals.
19	6	3	1
38	7	1	1
11	4	3	0
4	7	3	1
32	5	2	0

(4)

gals.	qts.	pts.
49	3	1
34	1	0
25	0	1
51	3	1
30	1	0

(5)

lds.	qrs.	bus.
13	4	7
24	3	4
37	4	0
43	2	1
58	3	6

(6)

C.	O.	fl.oz.	fl.dr.	m.
3	5	18	7	10
	7	13	1	45
1	4	9	3	15
2	0	19	5	20
	3	5	6	30

19 Perform the following subtractions—

(1)

gals.	qts.	pt.	gils.
57	2	1	2
26	3	1	3

(2)

gals.	qts.	pt.
240	0	0
140	3	1

(3)

tuns	hhds.	gals.	pts.
2	2	0	0
1	3	32	4

(4)

lds.	qrs.	bus.	pks.	gal.
7	3	5	2	0
3	4	7	3	1

(5)

bus.	pks.	gal.
57	1	0
39	3	1

(6)

C.	O.	fl.oz.	fl.dr.	m.
6	3	12	1	15
2	6	17	5	40

20. Multiply :—

(1) 15 qrs. 6 bus. 3 pks. 1 gal. separately by 54 and 111.
(2) 27 gal. 3 qts. 1 pt. 3 gils. ... ... 36 and 236.

21. Divide :—

(1) 5863 gals. 3 qts. 1 pt. 3 gils. separately by 8 and 75.
(2) 6564 lds. 1 qr. 4 bus. 2 pks. 1 gal. ... ...5 and 67.
(3) 739 qrs. 4 bus. 2 pks. 1 gal. by 11 ; 244 qrs. 3 bus. 1 pk. by 3 qrs. 3 pks. ; 7 O. 11 fl.oz. 6 fl.dr. 20 m. by 10.

22. How many sacks of corn can be filled out of a bin containing 52 qrs., if each sack hold 3 bus. 1 pk. ?

23. How long will a butt of beer last a man who drinks 2 qts. 1 pt. daily ?

24. A dishonest inn-keeper buys 2 pipes of wine, and mixes 1 qt. 1 pt. of water with every 3 gallons of wine. How many gallons will he have to sell ?

25. How many jars, each containing 2 gals. 3 qts. 1 pt. 3 gils. can be filled out of a cask containing 285 gallons ?

VII. MEASURES OF TIME.

166. Indian Measure of Time.

60 Anupals (*anu.*)	make	1 Bipal (*bip.*)
60 Bipals	„	1 Pal (*pal.*)
60 Pals	„	1 Danda (*dan.*)
7½ Dandas or 3 hours	„	1 Prahar (*pr.*)
8 Prahars or 60 dandas	„	1 **Din** or day (*da.*)
7 Dins	„	1 Saptaha (*sap.*)
15 Dins	„	1 Paksha (*pak.*)
30 Dins or 2 pakshas	„	1 Mas or month (*ma.*)
12 Masas	„	1 Batsar or year (*ba.*)
12 Batsars	„	1 Yuga.

2½ Dandas = 1 Ghanta ; 1 Danda = 24 minutes. A chandra mas (lunar month) = 29½ days, nearly.

167. English Measure of Time.

60 Seconds (*sec.* or 1^s)	make	1 Minute (*min.* or 1^m)
60 Minutes	„	1 Hour (*hr.*)
24 Hours	„	1 **Day** (*da.*)
7 Days	„	1 Week (*wk.*)
365 Days	„	1 **Year** (*yr.*)
100 Years	„	1 Century.

A month = 30 days. A year = 4 quarters = 12 calendar months = 52 weeks.

A fortnight = 2 weeks. A month = 4 weeks. A Leap-year = 366 days. Each day is considered to commence at midnight.

168. The number of days in the *Calendar* Months are recollected by means of the following lines :—

Thirty days hath September,
April, June and November ;
February has twenty-eight alone,
And all the rest have thirty-one ;
But leap-year coming once in four,
February then has one day more.

Bengali Months.			*English Months.*		
1.	Baisakh	(বৈশাখ)	1.	January	= 31 days.
2.	Jaistha	(জ্যৈষ্ঠ)	2.	February	= 28 ,,
3.	Ashárh	(আষাঢ়)	3.	March	= 31 ,,
4.	Srávan	(শ্রাবণ)	4.	April	= 30 ,,
5.	Bhádra	(ভাদ্র)	5.	May	= 31 ,,
6.	Aswin	(আশ্বিন)	6.	June	= 30 ,,
7.	Kártick	(কার্ত্তিক)	7.	July	= 31 ,,
8.	Agraháyan	(অগ্রহায়ণ)	8.	August	= 31 ,,
9.	Pous	(পৌষ)	9.	September	= 30 ,,
10.	Magh	(মাঘ)	10.	October	= 31 ,,
11.	Falgoon	(ফাল্গুন)	11.	November	= 30 ,,
12.	Chaitra	(চৈত্র)	12.	December	= 31 ,,

Mahomedan Names : Maharam (মহরম), Safar (শফর), Raviulayal (রবিয়ল আউয়ল), Raviassani (রবিয়সসানি), Jamadiyal-auyal (জমাদিয়ল আউয়ল), Jamadiyassani (জমাদিয়সসানি), Rajab (রজব), Saban (শাবন), Ramjan (রমজান), Saoyal (শওয়াল), Jelkad (জেলকদ্), and Jelhajja (জেলহজ্জ).

A Bengali month is generally supposed to consist of 30 days ; but this is not strictly correct. Some months are 29 days, some 30, some 31 and some 32.

THE HINDU CALENDAR.

169. The Hindu **Chandra Batsar** (Lunar year) consists of 354 days 8 hrs. 48 min. 57 sec. It is therefore shorter than the **Saur Batsar** (Solar year) by 10 days 21 hrs. 23 min. 12 sec. After a period of $32\frac{1}{2}$ months the difference amounts to a month ; consequently to make the Lunar-year system correspond with the Solar-year system, a month is intercalated on the occurrence of two conjunctions of the Sun and Moon in the same sign of the Zodiac. The intercalated month and the month preceding it go by the same name. The intercalated month is called **Mala** or **Intercalary Mas.** This is done in those parts of India where the lunar year and lunar month are reckoned. In Bengal, where solar year and solar month are reckoned, a month is rejected in a period of every $32\frac{1}{2}$ lunar months as unfit for any religious festival, in order to make the religious festivals of particular months recur in those months. The rejected month is called *Mala Mas.*

THE ENGLISH CALENDAR.

170. The interval of time between two passages of the Sun across the meridian of any place when taken at its *mean magnitude*, is termed a *day* or a *mean solar day*, which is supposed to be divided into 24 equal portions called *mean solar hours*. It appears from the observations and calculations of Astronomers that the time between the Sun's leaving a certain point (First point of Aries) in his path called the *Ecliptic* and returning to it again, consists of 365·242218 such days or of 365 days, 5 hours, 48 minutes, 47½ seconds, very nearly, which is therefore termed a *Solar Year*.

For the purposes of civil life it would be exceedingly inconvenient that one year should commence at one time of the day and another at a different time; and this circumstance gave rise to the invention of the *civil year*, which will be explained in the next Articles.

171. When the Science of Astronomy was much less perfect than it is at present, the length of the solar year was much less accurately known; and accordingly we find that in the time of *Julius Cæsar* it was supposed to consist of 365 days 6 hours, or of 365¼ days, *exactly*. On this supposition, it is evident that if out of *four* years in succession, any *three* consisted of 365 days each and the remaining one of 366 days, the Sun would have returned at the end of those *four* years to the place in the *Ecliptic* which it occupied at their commencement.

The scheme was called the *Julian Calendar*; and if the hypothesis had been correct, it would have been attended with much convenience; the additional *day* was called **Intercalary**, and the *year* in which it was added or inserted was termed **Bissextile**.

The regulation, applied to the years of the *Christian Era*, was so managed that whenever the number of years was divisible by 4, the corresponding year consisted of 366 days and was called **Leap-year**, the month of *February* having 29 days in that year, and each of the remaining three years 28 days, without interfering at all with their order.

Hence also, the remainder after the division of any other number of years by 4, was the number of years since a leap-year occurred up to that year. Thus, in the year 1893 this remainder is 1; and accordingly it is 1 year since the last leap-year happened and it is 3 years before the next will occur, according to this scheme.

172. Since the true solar year is 365·242218 days and not 365·25 days, it is evident that the reckoning of time according to the Julian Calendar would place the end of the year *after* the time when the Sun had returned to the point of the Ecliptic occupied by it at the beginning of the year and consequently in *advance* of the course of the *Seasons*; but, the error in one year is 365·25 − 365·242218 = ·007782 of a day. Therefore in 400 years the error would amount to ·007782 × 400 or 3·1128 days.

Now, according to the Julian Calendar 400 years would comprise 100 Leap-years ; and since we find that this reckoning falls nearly 3 days *after* the true time, if there were only 97 Leap-years in 400 years ; the Julian year would very nearly agree with the true solar year ; and it is accordingly ordained that whenever the *numbers* expressing the **Centuries** as 16, 17, 18, 19, &c., denoting 1600, 1700, 1800, 1900, &c., are *not* divisible by 4, the corresponding year shall *not* be a Leap-year although according to the Julian Computation it would ; as, 1600 would be a Leap-year, but 1700, 1800, 1900 would not.

The Calendar thus corrected, though not absolutely accurate, is well adapted to every *practical* purpose, as the error in 5000 *years* will not amount to much more than *twenty-eight* hours. The correction was first promulgated in Europe by *Pope Gregory* in the year 1582 and the calendar has since been called the *Gregorian Calendar* ; but it was not introduced into *Protestant Countries* till a much later period. In *England* it was adopted on the *second* day of September 1752 when the error amounted to 11 days ; and it is called the **New Style** to distinguish it from the Julian Calendar which is now termed the **Old Style.**

The New Style has not yet been adopted in Russia, so that since 1752 they have had one more leap-year (1800) than we have, and they are now 12 days behind us. Thus Old Michaelmas and Old Christmas taking place 12 days after New Michaelmas and New Christmas.

173. The **Civil year** thus fixed and determined is then subdivided into twelve Calendar Months, as described in the Table. The word *Month* however is used in different senses ; sometimes to denote a *twelfth* part of a year ; sometimes as equivalent to 4 weeks or 28 days ; and accordingly a year is equivalent to 13 months and 1 day, or to 52 weeks and 1 day, with the addition of another day when it happens to be Leap-year.

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174. To reduce prahars to *dandas*, multiply by 15, and divide the product by 2 ; the remainder (if any) is a half-danda or 30 pals. Conversely, to reduce dandas to *prahars*, multiply by 2 and divide the product by 15 ; the remainder (if any) is equal to so many half-dandas.

Ex. Reduce 8 sap. 5da. 3pr. 4dan. 45pals. to *bipals* ; 266330 sec. to *days*, and 2 yrs. 15 da. 6 hrs. to *minutes*.

(1) 8 sap. 5 da. 3 pr. 4 dan. 45 pals.	
7	= 3682 dan. 30 pals.
61 da.	4 45
8	3687 dan. 15 pals.
491 pr.	60
15	221235 pals.
2)7365	60
3682 dan. + 1 half-dan.	13274100 bipals. *Ans.*

(2) 6,0)26633,0 sec.
6,0) 443,8 ... 50 sec.
24 { 3)73 ... 58 min.
24 { 8)24 ... 1 } 1 hr.
3da.

∴ the result
= 3da. 1hr. 58m. 50sec.

(3) 2yrs. 15da. 6hrs.
365
745 da.
24
17886 hrs.
60
1073160 min. *Ans.*

Examples XXXVIII.

1. Reduce to *anupals* :—
(1) 5dan. 30pa. (2) 12pr. 6dan. 40bip. (3) 8sap. 5da. 5pr.
(4) 3ba. 6m. 5da. 5dan. (5) 4sap. 6da. 6pr. 50pa. 40bip.
(6) 6 pr. (7) 13 sap. (8) 12 days. (9) 10da. 5pr. 45anu.
(10) 46 ba. 267 da. 57 dan. 43 pa. 51 bip.

2. Reduce to *seconds* :—
(1) 27 wks. 5 da. 15 hrs. ; 6 hrs. 25 min. 32 sec. ; 5 wks. 3 da.
(2) 3 yrs. 147 da. 15 hrs. ; 76 da. 19 hrs. 43 min. 57 sec.
(3) 2 da. 4 hrs. 51 min. 50 sec ; 4 mo. 2 wks. 23 hrs. ; 3 leap-years.

3. Reduce : 15 yrs. 26da. 2hrs. 27min. to *minutes* ; 19yrs. 153da. 8hrs. to *hours* ; 3 yrs. 315da. to *minutes*.

4. Reduce :—
(1) 563472 pals. to *dins.* ; 59018732 anupals to *dins.*
(2) 8463045 bipals to *prahars* ; 74632508 anupals to *dandos.*
(3) 673985643 anupals to *days* ; 36438005 dan. to *batsars.*

5. Reduce :—
(1) 72015 hours to *weeks* ; 2706359sec. to weeks ; 38567min. to *days.*
(2) 123456 sec. to *hours* ; 3456794 sec. to *days* ; 579574 min. to *years.*

6. Reduce to *years* :—
71871900 sec. ; 1301416510 sec. ; 713969410 sec. ; 413419020 sec.

7. Add together :—

(1)

sap.	da.	pr.
36	5	6
24	4	2
48	6	5
2	3	4
18	6	7

(2)

dan.	pal.	bip.	anu.
41	36	57	51
39	48	39	47
49	55	13	58
59	26	49	38
21	50	28	19

(3)

din.	dan.	pal.	bip.	anu.
60	57	19	21	27
73	40	23	17	13
9	55	19	18	29
37	20	40	19	24
47	30	59	29	34

(4)

hrs.	min.	sec.
15	42	45
57	36	40
32	12	14
16	37	45
5	51	41
24	19	40

(5)

da.	hrs.	min.	sec.
35	14	32	30
47	16	25	27
54	18	52	57
43	21	37	29
62	22	58	57
40	15	20	32

(6)

wks.	da.	hrs.	min.
10	5	14	31
18	4	12	38
25	0	10	14
75	6	23	59
53	4	19	23
40	0	17	25

8. Perform the following subtractions :—

(1)

sap.	da.	pr.
527	5	5
418	6	7

(2)

dins	dan.	pal.	bip.
80	50	40	20
50	55	50	36

(3)

da.	hrs.	min.	sec.
17	1	0	17
7	17	31	22

(4)

da.	hrs.	min.	sec.
24	14	46	31
4	21	18	52

(5)

wks.	da.	hrs.
7	3	18
4	6	20

(6)

yrs.	da.	hrs.	min.	sec.
7	129	13	26	17
3	273	18	34	29

9. Multiply :—

(1) 7 dins 5 dan. 30 pal. 15 bip. by 74, and by 140.
(2) 9 ba. 8 ma. 27 da. 45 dan. 56 pal. 38 bip. 52 anu. by 43, 67.
(3) 43 days 18 hrs. 45 min. by 77, and by 147.
(4) 17 wks. 4 da. 13 hrs. 27 min. 36 sec. by 9, and by 79.
(5) 17 years 110 da. 17 hrs. 57 sec. by 144.

10. Divide :—

(1) 694 dins 7 pr. 3 dan. 30 pal. by 32.
(2) 2056 ba. 5 ma. 27 da. 44 dan. 15 pal. by 87.
(3) 17 wks. 5 da. 18 hrs. 25 min. by 49.
(4) 878 wks. 4 da. 15 hrs. 37 min. 36 sec. by 9, and by 56.

11. How many days are there (the last day mentioned in each case being excluded) from
(1) April 5, 1863 to Nov. 3, 1863? (2) Dec. 31, 1863 to Dec. 31, 1864? (3) Sep. 21, 1863 to March 1, 1864? (4) Nov. 16, 1882 to Sep. 5, 1884?

12. How many bipals are there in a year of 365 days 6 hours?

13. A solar year = 365 days 5 hrs. 48 min. $47\frac{1}{2}$ sec.: (1) how many more seconds are there in a solar year than in a common year? (2) how many seconds less than in a leap-year?

14. How many portions of time each equal to 1 day 7 hrs. 45 min. 56 sec. are contained in 346 days 18 hrs. 34 min. 32 sec.?

15. If the 1st of April is a Monday, on what day of the week will Christmas fall that year?

VIII. MEASURES OF ANGLES.

175. **English Angular Measure.**

60 Seconds (60″) make 1 Minute (′)
60 Minutes „ 1 Degree (°)
90 Degrees „ 1 **Right Angle** (1*rt. gle.*)

IX. MEASURES OF NUMBERS.

176. BENGALI TABLE.

4 Units make 1 Ganda
5 Gandas „ 1 Buri
4 Buris „ 1 Pan
16 Pans „ 1 Kahan

ENGLISH TABLE

12 Units make 1 Dozen
12 Dozens „ 1 Gross
12 Gross „ 1 Great Gross
20 Units „ 1 Score (*Kuri*)
120 Units „ 1 Long Hundred

FOR PAPER.

24 Sheets = 1 Quire ; 20 Quires = 1 Ream ; 10 Reams = 1 Bale.

Examples XXXIX.

1. Reduce to *seconds* :—

(1) $172^\circ\ 8'\ 25.''$ (2) $275^\circ\ 30'\ 26.''$ (3) $144^\circ\ 12'\ 38.''$ (4) $57^\circ\ 7'\ 45.''$

2. Reduce to *right angles degrees &c.* :—

(1) $206265.''$ (2) $865408''$. (3) $718276''$. (4) $42861'$ (5) $78205'$.

3. Add together : $175^\circ\ 32'\ 45''$, $75^\circ\ 59'\ 27''$, $114^\circ\ 28'\ 47''$, $105^\circ\ 45''$, $144^\circ\ 12'\ 38''$, $160^\circ\ 52'\ 58''$, and $175^\circ\ 20'\ 46''$.

4. Subtract $149^\circ\ 53'\ 56''$ from $277^\circ\ 36'\ 47''$.

5. Multiply $24^\circ\ 12'\ 16''$ by 42 ; $19^\circ\ 14'\ 25''$ by 36.

6. Divide $25^\circ\ 25'\ 32''$ by 16 ; $144^\circ\ 44'\ 7''$ by 22.

7. In 56 reams of paper, how many sheets ?

8. Reduce 67835 kahans 11 pans 18 ga. 3 units to *units*.

9. Reduce 7297865 units to *kahans* ; 9 scores to *dozens*.

10. Multiply 9 kahans 2 pans 17 ga. 2 units by 82, and by 346.

Examples XL.

(Recapitulatory Exercises).

1. In 340 pistoles at 17*s*. 6*d*. each, how many pounds sterling ?

2. How many moidores of 27*s*. each, are equivalent to 198 guineas and to £500638. 1*s*. ?

3. In £453. 16*s*. 8*d*., how many pieces of coin valued at 3*s*. 4*d*. each ? how many at 11*s*. 8*d*. each ?

4. What number of weights of 14 oz. 13 drs. each, are equivalent to 25 cwt 2 qrs. 13 lbs. 14 oz. 12 drs. ?

5. If I spend £2. 7*s*. $1\frac{1}{2}$*d*. a day, how much is that in 28 weeks, and also in a year of 365 days ?

6. If each of 114 persons receive £1. 18*s*. $6\frac{1}{2}$*d*., what is received by them all ?

7. If the clothing of 754 soldiers come to £3178. 11*s*. $7\frac{1}{2}$*d*., how much is that for each man ?

8. If a person complete a journey of 422 mi. 3 fur. 38 po. in 37 days ; what distance does he travel each day ?

9. A year being equivalent to 365 days 6 hours, find the number of years, &c., in 295402374 seconds.

10. Multiply 4 dins 3 pr. 2 dan. 25 bip. 15 anu. by 401.

11. Reduce 9367875 angulis to *kros* ; 14978631 gandas to *bighas*.

12. Find how often a rod 2 ft. 10 in. in length, must be applied to measure 10 miles 140 yds.

13. Find the number of yards in 40 pieces of cloth, each containing 42 yds. 2 qrs. 2 nls.

14. If a soldier's pay for a year of 365 days be £9. 2*s.* 6*d.*; how much is that for a day?

15. If a person's yearly income be £65. 12*s.* 6*d.*, and he lay by £20 a year; how much does he spend each day?

16. How many pounds of silver are there in a half-dozen of dishes, each weighing 51 oz. 10 dwts. and a dozen of plates, each weighing 15 oz. 15 dwts. 22 grs.?

17. Express 452 dan. 48 pal. 45 bip. in *English measure.*

18. If 145 sheep cost £169. 3*s.* 4*d.*, what is the price of a score at the same rate?

19. If 8 packages of cloth, each consisting of 4 parcels, each parcel of 10 pieces, and each piece of 26 yards, cost *Rs.*66560; what is the price of a yard?

20. The sum of £263. 8*s.* 11½*d.* is distributed equally among a number of persons, so that the share of each is £37. 12*s.* 8½*d.*; find the number of persons.

21. A boy's school, to and from which he walks daily, is distant from his home 1 kros 250 dan. 1 gaj 1 hath 7 girahs. How many girahs does he walk every day?

22. Reduce 35 tons 19 cwt. 99 lbs. 12 oz. 135 grs. to *grains.*

23. Reduce 294322493 sq. in. to *acres, &c.*

24. Find the weight of copper coin required to pay a debt of £1000, when 3 pennies weigh 1 oz.

25. Which is the heavier, 1 lb. of gold or 1 lb. of sugar?

26. If 28 lbs. 9 oz. of gold be worth £1343. 6*s.* 10½*d.*, what is the worth of 1 ounce?

27. Among how many boys can I distribute £14. 9*s.* 9*d.*, giving to each boy a half-crown, a florin, a four-penny piece, and also a three-penny piece?

28. If a man's net income be £1785. 12*s.* 6*d.*, how much may he spend on an average per day to the nearest farthing, so as not to run into debt?

29. Reduce 5792685 inches to *miles, &c.*

30. Jadu was born at 6 o'clock A. M., 24th June, 1872; how old will he be at 3 o'clock P.M., 10th Jan., 1898?

31. Find the sum of 32 cwt. 2 qrs. 15 lbs. 12 oz.; 47 cwt. 25 lbs. 9 oz.; 5 cwt. 3 qrs. 17 lbs. 10 oz.; 23 cwt. 1 qr 19 lbs. 15 oz.; and 9 cwt. 3 qrs. 14 oz.; divide the sum by 4 cwt. 2 qrs. 18 lbs. 8 oz.

32. If 1*s.* 5¼*d.* be the unit of money, what will be the measure of £7. 17*s.* 6*d.* and of £20. 1*s.* 0¾*d.*?

33. If 2 ft. 6 in. be the unit of length, what number will represent (i) 10 miles, (ii) 25 miles 760 yds.?

34. If 6 hrs. 32 min. 10 sec. be the unit of time, what will be the measure of 74 days 1 hr. 49 min. 20 sec.?

35. If 2 lbs. 5 oz. be the unit of weight, what number will measure 5 cwt. 6 lbs. 9 oz. ?

36. If 5 sec. be the unit of time, what will be the measure of 3 hrs. 5 sec. and of 15 hrs. 20 min. ?

37. 22nd September 1897 was Wednesday. What day of the week was 22nd September 1797 and what day of the week will 22nd September 1997 be ?

38. 19th September 1897 was Sunday. What day of the week was 23rd January 1807 and what day of the week will 23rd January 1907 be ?

X. MISCELLANEOUS PROPOSITIONS.

(IN COMPOUND QUANTITIES.)

177. The Unitary Method. (*Simple Cases*).

If the value, weight, length, &c. of any number of units be given, we can by Compound Division find that of one unit of the same kind ; and the value, weight, length, &c. of one unit being found, we can by Compound Multiplication find that of any number of units of the same kind. The solution which combines these two processes is called **The Method of Reduction to the Unit or The Unitary Method.**

(1) The value, weight, length, &c. of **one** unit being given, we can by *Compound Multiplication* find the value, weight, length, &c. of any number of units of the same kind.

Ex. The price of a maund of sugar is *Rs.*10. 15*a.* 6*p.*; find the price of 35 maunds.

*Rs.*10 15*a.* 6*p.*	The price of 1 maund = *Rs.*10. 15*a.* 6*p.*
35	∴ the price of 35 mds. = *Rs.*10. 15*a.* 6*p.* × 35
*Rs.*383. 14*a.* 6*p.*	= *Rs.*383. 14*a.* 6*p.*

(2) The value, weight, length, &c. **of any number** of units being given, we can by *Compound Division* find the value, weight, length, &c. of one unit of the same kind.

Ex. If 30 mds. of rice cost *Rs.*134. 1*a.*; what is the price per maund ?

```
30) Rs.134. 1a.( 4Rs.
    120
    ---
     14
     16
    ---
    225(7a.
    210
    ---
     15
     12
    ---
    180(6p.
    180
```

The price of 30 mds. = *Rs.*134. 1*a.*
∴ the price of a md. = *Rs.*134. 1*a.* ÷ 30
= *Rs.*4. 7*a.* 6*p.*

(3) The value, weight, &c. of a certain number of units being given, to find the value, weight, &c. of a certain other number of units of the same kind.

Proceed as in the following Examples :—

Ex. 1. If 7 yards of cloth cost *Rs.*26. 4*a.*, what will be the cost of 15 yds. of the same?

7)*Rs.*26. 4*a.*	7 yards cost *Rs.*26. 4*a.*
*Rs.*3. 12*a.*	∴ 1 yard costs *Rs.*26. 4*a.* ÷ 7 = *Rs.*3. 12*a.*
15	∴ 15 yards cost *Rs.*3. 12*a.* × 15 = *Rs.*56. 4*a.*
*Rs.*56. 4*a.*	

Ex. 2. If 7 lbs. of tea cost 15*s.* 9*d.*, what will be the cost of 12 lbs.?

7)15*s.* 9*d.*	7 lbs. cost 15*s* 9*d.*
2*s.* 3*d.*	∴ 1 lb costs 15*s.* 9*d.* ÷ 7 = 2*s.* 3*d.*
12	∴ 12 lbs. cost 2*s.* 3*d.* × 12 = £1. 7*s.*
£1. 7*s.*	

(4) The value, weight, &c. of a certain number of units being given, to find the number of units of the same kind corresponding to some other value, weight, &c.

Proceed as in the following Examples :—

Ex. 1. If 12 maunds of rice cost *Rs.*35, find how many maunds of the same can be bought for *Rs.*20. 6*a.* 8*p.*

12)*Rs.*35. *Rs.*20. 6*a.* 8*p.* = 3920*p.* ; *Rs.*2. 14*a.* 8*p.* = 560*p.*

*Rs.*2. 14*a.* 8*p.* ∴ the no. of mds. required = 3920 ÷ 560
= the price of a maund. ∴ = 7. *Ans.*

Ex. 2. If 25 men finish a piece of work in 16 days, in how many days will 20 men finish it?

25	
16	25 men finish the work in 16 days,
20)400 days.	∴ 1 man will finish in (25 × 16) or 400 days.
20 days.	∴ 20 men will finish in 400 ÷ 20 or 20 days. *Ans.*

Ex. 3. How many men can perform in 24 days a piece of work which 15 men can perform in 40 days?

15	In 40 days the work is done by 15 men.
40	∴ in 1 day the work is done by (15 × 40) or 600 men.
24)600 men.	∴ in 24 days, the work is done by 600 ÷ 24 or 25 men.
25 men.	*Ans.*

Note. In questions such as the two above, it should be noticed that to a *diminution* in the number of men corresponds an *increase* in the number of days, and *vice versâ*.

Examples XLI.

1. What is the value of 72 reams of paper, at 13*s*. 8*d*. a ream ?
2. Find the cost of 120 ounces of silver, at 5*s*. $3\frac{3}{4}$*d*. an ounce.
3. What will be the price of 1 lb., when 1 cwt. costs £137. 18*s*. ?
4. If 41 cwt. cost £52. 10*s*. $7\frac{1}{2}$*d*., what is the price of a cwt. ?
5. If 6 chairs cost *Rs*.32. 12*a*., what will 3 dozen cost?
6. If a workman's wages for 12 days be *Rs*.14. 4*a*. 6*p*., what would it amount to in 18 days ?
7. If 4 yards of flannel cost *Rs*.3. 13*a*. 4*p*., what is the cost of 57 yards of the same ?
8. If 42 bighas of land be rented for *Rs*.640. 8*a*., what would be the rent of 61 bighas ?
9. If a man earn *Rs*.15. 12*a*. in 6 days, in how many days will he earn *Rs*.189 ?
10. If I travel by Railway 85 miles for *Rs*.7. 15*a*. 6*p*., how far may I travel for *Rs*.9. 6*a*. ?
11. If 13 sheep cost *Rs*.175. 8*a*., how many may be purchased for *Rs*.2160 ?
12. If 7 seers of tea cost *Rs*.7. 9*a*. 4*p*., what will be the cost of 1 md. 24 sr. 8 ch. ?
13. A clerk's salary is *Rs*.1916. 4*a*. per annum ; what ought he to receive for 60 days' service ?
14. How much land may be rented for *Rs*.705. 4*a*., if 5 acres are rented for *Rs*.46. 10*a*. 8*p*. ?
15. How many men can perform in 12 days a piece of work, which 15 men can perform in 20 days ?
16. If 3 mds. 12 sr. 8 ch. of sugar cost *Rs*.16. 9*a*., what will 2 mds. 14 sr. 10 ch. cost ?
17. Find the quantity of rice which can be purchased for *Rs*.86. 3*a*. $9\frac{1}{2}$*p*., when 70 mds. 10 sr. cost *Rs*.270. 12*a*. 1*p*.
18. If 3 cwt. 69 lbs. cost £14. 3*s*. 6*d*., how much may be bought for £23. 12*s*. 6*d*. ?
19. If 2 cwt. 3 qrs. 7 lbs. cost £5. 17*s*. $8\frac{1}{2}$*d*., what is the cost of 9 cwt. ?
20. In how many days would 171 men perform a piece of work, which 108 men can perform in 266 days ?

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178. Revolution of Wheels.

A **wheel** in making one **revolution** passes over a length of ground exactly equal to its **circumference**. Hence, if we multiply

9

the circumference by the number of revolutions made, we shall find the distance passed over; and conversely, if we divide the distance passed over by the circumference, we shall find the number of revolutions, or by the number of revolutions we shall find the circumference.

Ex. 1. A carriage-wheel is 4 yds. 2 ft. 7 in. in circumference, and makes 1456 revolutions on a journey. What is the length of the journey?

	mi.	fur.	yds.	ft.	in.
$1456 = 8 \times 13 \times 14$			4	2	7
					8
(220 yds. = 1 furlong).			38	2	8
					13
∴ the distance passed over is 4 mi. 0 fur. 37 yds. 2 ft. 4 in.		2	65	1	8
					14
	4	0	37	2	4

Ex. 2. A wheel makes 131 revolutions in passing over 669 yds. 1 ft. 8 in.; what is its circumference?

The circumference = 669 yds. 1 ft. 8 in. ÷ 131 = 5 yds. 4 in.

Ex. 3. How many revolutions will a carriage-wheel 3 yds. 2 ft. 6 in. in circumference, make in a journey of 7 miles 3 fur. 34 po. 4 yds. 1 ft.?

```
3 yds. 2 ft. 6 in.      7 mi. 3 fur. 34 po. 4 yds. 1 ft.
  3                       8
 11 ft.                  59 fur.                  138)474168(3436
 12                      40                           414
138 in.                2394 po.                        601
                         11                            552
                     2)26334                            496
                       13167 yds. + 4 yds.              414
                     = 13171 yds.                        828
                           3                             828
                       39514 ft.
                          12                 ∴ the number of revolutions
                      474168 in.               required = 3436. Ans.
```

Examples XLII.

1. If a wheel 5 yds. 2 ft. 4 in. in circumference makes 1080 revolutions on a journey, how far will the carriage go?

2. If a wheel 5 yds. 1 ft. 6 in. in circumference makes 64640 revolutions, what space will it pass over?

3. How many revolutions will the wheel of a carriage, 4 ft. 7 in. in circumference, make in 2 mi. 4 fur.?

4. A wheel makes 514 revolutions in passing over 1 mi. 467 yds. 1 ft.; what is its circumference?

5. A boy's hoop is 3 yds. 10 in. round; how many miles of ground will it pass over in 2501 turns?

6. The fore-wheel of a carriage is 4 ft. 6 in. round, and the hind-wheel a foot longer; how many more turns will the former make than the latter in a distance of 30 miles?

7. A wheel makes 1540 revolutions in passing over 2 mi. 458 yds. 1 ft.; what is its circumference?

8. How many revolutions will a wheel 4 yds. 2 ft. in circumference make on a journey of 12 mi. 696 yds. 2 ft.?

9. The circumference of the fore-wheel of a carriage being 8 ft. 3 in., and that of the hind-wheel 11 ft. 11 in., how many more revolutions would be made by the fore-wheel than by the hind-wheel in going a distance of 52 miles?

10. The driving wheel of a locomotive is 5 yds. 2 ft. 9 in. in circumference, and makes on an average 3 revolutions a second; find the rate of the train per hour.

11. The fore-wheel of a carriage which is 2 yds. 2 ft. 6 in. in circumference makes 4350 more revolutions than the hind-wheel in going over a distance of 19 miles 2 fur. 120 yds.; what is the circumference of the hind-wheel?

12. Find the circumference of the wheel of a locomotive which makes on an average 4 revolutions in a second, and which performs a journey of 76 miles in 1 hour 36 min.

13. A wheel revolves 1028 times in going 2 mi. 934 yds. 2 ft. What is its circumference?

14. In going over a distance of 205 miles the fore-wheel turns 98400 times and the hind-wheel 78720 times. How much longer is the circumference of the hind-wheel than that of the fore-wheel?

15. The circumference of the fore-wheel of a carriage is 8 ft. and that of the hind-wheel is 10 ft.; in what distance will the fore-wheel make 100 revolutions more than the hind-wheel?

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179. Averages.

The **Average** or **Mean** of any number of given quantities o the same kind, is that quantity which when substituted for each of the given quantities makes their sum the same. Hence, to find the *Average* of any number of quantities we divide the sum of them by their number.

Ex. The receipts at a Railway Station are as follow: Jan., *Rs.*2458. 14*a.* 8*p.*; Feb., *Rs.*2019. 6*a.*; March, *Rs.*2857. 4*a.* 8*p.*; April,

*Rs.*3051. 1*a.* 4*p.*; May, *Rs.*3463. 13*a.* 4*p.*; and June, *Rs.*4007. 10*a.*; find the average receipts per month.

Rs.	*a.*	*p.*
2458	14	8
2019	6	0
2857	4	8
3051	1	4
3463	13	4
4007	10	0
6)17858	2	0
*Rs.*2976	5	8

The sum of the receipts for the 6 months is found to be *Rs.*17858. 2*a.*; hence the average month's receipt is found by dividing this sum by 6, and is

*Rs.*2976. 5*a.* 8*p.*

180. Nearest money.

When there is a **remainder** after division, we observe, that if the quotient be multiplied by the divisor the product will be *less* than the dividend; also that if the quotient be increased by 1 and be then multiplied by the divisor the product will be *greater* than the dividend. Hence, in all cases, a **nearest** sum can be found, which will be exactly divisible by the divisor. Also a quotient correct to the **nearest lowest denomination.** (Art. 149.)

Ex. 1. Find the *nearest* sum of money to £197. 11*s.* 6*d.* that can be divided by 23 without remainder.

```
   £.   s.  d.   £.  s.  d.
23)197  11  6   (8   11  9½
   184
    13
    20
   271 ( 11s.
   23
    41
    23
    18
    12
   222 ( 9d.
   207
    15
     4
    60 ( 2q.
    46
    14q.
```

From the work it appears that if the given sum be diminished by 14*q.*, or $3\frac{1}{2}$*d.*, there will be no remainder, or if it be increased by 9*q.* or $2\frac{1}{4}$*d.*, so as to make the last partial dividend 69, there will be no remainder; hence the *nearest* sum required is £197. 11*s.* 6*d.* + $2\frac{1}{4}$*d.* or

£197. 11*s.* $8\frac{1}{4}$*d.* *Ans.*

Ex. 2. If £197. 11*s.* 6*d.* be given for 23 pieces of cloth, find to the *nearest* penny the price given for each piece.

From the last *Ex.*, it appears that £8. 11*s.* 9*d.* a piece would give 15*d.* too little, and £8. 11*s.* 10*d.* would give 8*d.* too much; hence, to the *nearest* penny the price would be £8. 11*s.* 10*d.* *Ans.*

Examples XLIII.

1. On Sunday I spent no money, on Monday *Rs.*43. 14*a.*, on Tuesday *Rs.*51. 12*a.* 8*p.*, on Wednesday *Rs* 46 14*a.* 6*p.*, on Thursday *Rs.*52. 8*a.*, on Friday *Rs.*32. 15*a.* 6*p.*, on Saturday *Rs.*26 4*a.*; find my average daily expenditure during the week.

2. The daily receipts of a grocer for the week are as follow :— Monday *Rs.*47. 10*a.* 2*p.*; Tuesday *Rs.*56. 8*a.* 4*p.*; Wednesday *Rs.*78. 7*a.*; Thursday (being a holiday) nothing; Friday *Rs.*39. 7*a.* 4*p.*; and Saturday *Rs.*159. 13*a.* 2*p.*; find his average daily receipts (1) excluding Thursday, and (2) including Thursday.

3. Find the least sum of money that must be subtracted from £663. 14*s.* 8*d.* to make the remainder divisible by 37.

4. Deduct *Rs.*26. 13*a.* 6*p.* from *Rs.*562. 8*a.*, and divide the resulting sum equally among 29 persons to the nearest pie; how much will each person receive, and how much will remain over?

5. The average price of a quarter of wheat for 19 years was 56*s.* 8*d.* a quarter; for the first five years the average price was 61*s.* $3\frac{1}{4}$*d.* a quarter, for the next 4 years 58*s.* $0\frac{1}{4}$*d.*, for the next 7 years 53*s.* $5\frac{3}{7}$*d.*, find the average of the last 3 years.

6. Find the nearest sum of money to *Rs.*3339. 10*a.* 10*p.* that can be divided by 29 without remainder.

7. The mean height of 6 mountains is 10357 feet: find what the height of the seventh mountain must be, in order that the mean height of the seven mountains may be 10643 ft.

8. 120 tons of coal are purchased for £87. 16*s.* 9*d.*; find to the nearest farthing the price at which they must be retailed per ton, so that no loss may be incurred.

9. Find the least sum of money that must be added to *Rs.*3658. 12*a.* 4*p.* to make the sum divisible by 127.

10. A tradesman's average annual income from 1830 to 1850 was *Rs.*3744. 13*a.* 4*p.* In 1830 his income was *Rs.*3699. 6*a.* 8*p.*, and in 1851 his income was *Rs.*3600. 8*a.* 8*p.*; what was his average annual income from 1831 to 1851 (inclusive)?

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181. Gain and Loss.

The price at which an article is bought is called its **cost** price; that at which it is sold, its **selling** price. If the selling price be greater than the cost price, it is **gain**; if less, it is **loss**. Hence the difference between the two prices is the *gain* or *loss*.

(1) Given the quantity sold, and also the cost and selling prices, to find the gain or loss.

Ex. 1. A person bought 524 yards of cloth at *Rs.*7. 14*a.* 6*p.* per yard and retailed it at *Rs.*8. 2*a.* 4*p.* per yard ; what was his profit?

Selling price per yard = *Rs.*8. 2*a.* 4*p.*
Cost............... = *Rs.*7. 14*a* 6*p.*
∴ gain per yard = 3*a.* 10*p.*
∴ gain on 524 yards = 3*a.* 10*p.* × 524 = *Rs.*125. 8*a.* 8*p.* *Ans.*

Ex. 2. A trader bought 1763 yards of cloth at 6*s.* 11*d.* per yard and retailed it at 5*s.* $3\frac{1}{2}$*d.* per yard ; what was his loss?

Cost price per yard = 6*s.* 11*d.*
Selling price .... = 5*s.* $3\frac{1}{2}$*d*
∴ loss per yard = 1*s.* $7\frac{1}{2}$*d.*
∴ loss on 1763 yards = 1*s.* $7\frac{1}{2}$*d.* × 1763 = £143. 4*s.* $10\frac{1}{2}$*d.* *Ans.*

(2) Given the gain or loss, and the cost and selling prices, to find the quantity sold.

Ex. 3 A mercer bought some gloves at 2*s.* $2\frac{1}{2}$*d.* a pair, and by selling them at 3*s.* 6*d* per pair, gained £9. 6*s.* ; how many pairs did he buy?

Selling price per pair = 3*s.* 6*d.*
Cost...... .... = 2*s.* $2\frac{1}{2}$*d.*
∴ gain per pair = 1*s.* $3\frac{1}{2}$*d.* = 62*q.*

Now, the whole gain = £9 6*s.* = 8928*q.*

∴ the number of pairs bought = 8928 ÷ 62 = 144. *Ans.*

Examples XLIV.

1. A person bought 500 yds. of cloth at *Rs.*7. 14*a.* per yard and retailed it at *Rs.*8. 2*a.* per yard ; what was his profit?

2. A person gave *Rs.*200 for 48 cwt. of goods ; what does he gain by selling them at *Rs.*5 a cwt. ?

3. A man buys 35 sheep for *Rs.*360 and 30 more for *Rs.*460 ; what will he gain or lose by selling them at *Rs.*15. 4*a.* each ?

4. A merchant bought 35 pieces of cloth measuring on an average 29 yards each at 3*s.* $10\frac{1}{2}$*d.* a yard, and sold them at 5*s.* 7*d.* a yard ; what profit did he make ?

5. I bought 360 yds. of cloth at *Rs.*2. 10*a* 8*p.* per yard, of which I sold 210 yds. at *Rs.*3. 9*a.* 4*p.* per yard ; but the article advancing in price, sold the remainder at *Rs.*4. 8*a.* per yard ; what did I gain on the whole?

6. I buy 84 books at *Re.*1. 15*a.* 8*p.* each, and sell them at a profit of *Rs.*70 ; what is the selling price of each ?

7. A shop-keeper purchases 35 reams of scribbling paper at *Rs.*7. 4*a.* per ream ; the carriage of the paper costs *Rs.*4. 12*a.* He

sells it at $8a.$ $8p.$ a quire with the exception of the out-side quires of each ream, which he sells at $5a.$ a quire. Find his gain.

8. A grocer gave *Rs.*500 for 16 cwt. 2 qrs. 18 lbs. of sugar, and he lost *Rs.*72. $6a.$ by retailing it ; at what rate did he sell it per lb. ?

9. I buy a number of books at *Re.*1. $6a.$ $4p.$ each and sell them at *Re.*1. $10a.$ each. If I thereby make a profit of *Rs.*22, how many books do I buy ?

10. A person gives *Rs.*556. $8a.$ for a certain number of gallons of wine. He sells it at *Rs.*2. $10a.$ a gallon, and thereby makes a profit of *Rs.*36. $12a.$ How many gallons does he buy ?

11. Find the cost of 20 dozen bottles of wine at *Rs.*2. $7a.$ $8p.$ per bottle ; and if 3 bottles be spoiled, what will the merchant gain by selling the remainder at *Rs.*2. $10a.$ $8p.$ per bottle ?

12. A cabinet dealer bought chairs at *Rs.*11. $15a.$ a piece, and lost *Rs.*9. $12a.$ by selling each at *Rs.*11. $2a.$ How many chairs did he buy ?

13. A person lays out £43. $9s.$ $4d.$ in spirits at $5s.$ $4d.$ a gallon ; 19 gallons leaked out in the carriage ; he however sold the remainder at $7s.$ $6d.$ a gallon ; what profit did he make ?

14. A merchant bought 7 pieces of cloth, each 27 yards, for £55. $12s.$; and sold 56 yards at $5s.$ $3\frac{1}{2}d.$ per yard and the rest at $6s.$ $8d.$ per yard. Find his whole gain.

15. A merchant laid out *Rs.*693 in spirits which he bought at *Rs.*6. $6a.$ $8p.$ a gallon ; he retailed it at *Rs.*8. $4a.$ a gallon, making a profit of *Rs.*115. $8a.$ How many gallons must he have lost by leakage ?

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182. Barter and Exchange.

When we **barter** we give or take one sort of goods in **exchange** for another of a different sort which is regarded as an equivalent. Hence, to find how much of the first sort be given in exchange for a fixed quantity of the second, we must *first* find the money value of the second sort and *then* find what quantity of the first sort is of equal value.

Ex. 1. How many pounds of tea at $3s.$ $2\frac{1}{2}d.$ a lb. must a grocer give in exchange for 35 yards of cloth at $12s.$ $4\frac{1}{2}d.$ a yard ?

$12s.$ $4\frac{1}{2}d.$	$594q.$	$3s.$ $2\frac{1}{2}d.$	154)20790(135
12	35	12	154
$148d.$	$20790q.$	$38d.$	539
4		4	462
$594q.$		$154q.$	770
			770

∴ the number of lbs. of tea = 135. *Ans.*

Ex. 2. What weight of sugar at 3*a.* a lb. must be given in exchange for a chest of tea weighing 84 lbs. at *Re.*1. 9*a.* a lb.?

*Re.*1. 9*a.*	25*a.*	
16	84	3)2100
25*a.*	2100*a.*	700

∴ the number of lbs. of sugar = 700. *Ans.*

Examples XLV.

1. How many dollars of 4*s.* $1\frac{1}{2}$*d.* each must be given in exchange for 4950 thalers of 2*s.* $11\frac{1}{2}$*d.* each?

2. How many francs of $9\frac{1}{2}$*d.* each will be given in exchange for 475 thalers at 2*s.* $11\frac{1}{2}$*d.* each?

3. How many lbs. of tea at *Re.*1. 9*a.* 8*p.* a lb. must be given in exchange for 46 yards of silk at *Rs.*4. 0*a.* 2*p.* a yard?

4. A man exchanges 45 sheep at *Rs.*22. 14*a.* each and 37 pigs at *Rs.*36. 12*a.* each for 13 oxen at *Rs.*173. 4*a.* each, the difference being paid or received in money; how much does he pay or receive?

5. The Calcutta rupee is worth 1*s.* $11\frac{3}{4}$*d.* each; how many must be given for £9895. 16*s.* 8*d.*?

6. How much coffee at 1*s.* $10\frac{1}{2}$*d.* a lb. should be given in exchange for 72 lbs. of tea at 3*s.* 4*d.* per lb.?

7. How many yards of cloth worth 3*s.* $7\frac{1}{2}$*d.* a yard must be given in exchange for 144 yards worth 18*s.* $1\frac{1}{2}$*d.* a yard?

8. How many Rubles at 3*s.* $4\frac{1}{2}$*d.* each are equal in value to 378 Napoleons, at 15*s.* $9\frac{3}{4}$*d.* each?

9. What quantity of tea at *Rs.*2. 6*a.* 6*p.* per lb., must be given in exchange for 5 cwt. 2 qrs. of sugar at *Rs.*3. 15*a.* per stone?

10. A person exchanged 18 dozen of wine for a gold snuff-box weighing 8 oz. 13 dwts. 10 grs. valued at £4. 10*s.* an oz. What did he value his wine at per dozen?

11. *A* gives *B* 98 gallons of brandy worth *Rs.*12. 12*a.* a gallon, and gets in return *Rs.*409. 8*a.* and 576 yards of cloth; what is the value of the cloth per yard?

12. A man sold 53 horses at *Rs.*168. 11*a.* 4*p.* each, and with the money he received for them and *Rs.*990 more he bought 355 cows and a certain number of calves; he gave for 198 of the cows *Rs.*22. 2*a.* a head, and for the rest of the cows *Rs.*18. 6*a.* 8*p.* a head, and for the calves *Rs.*14. 6*a.* a head. How many calves did he buy?

183. Allotment.

By **allotment** we divide a given quantity in a certain way into a proposed number of parts and thus ascertain the actual amount of each part.

Ex. 1. How many sovereigns, half-sovereigns, crowns, florins, shillings, six-pences and three-pences, and of each an equal number are there in £67. 16*s.* 3*d.* ?

s.	*d.*
20	0
10	0
5	0
2	0
1	0
	6
	3
38*s.*	9*d.*
12	
465*d.*	

£	*s.*	*d.*
67	16	3
20		
1356*s.*		
12		
16275*d.*		

465)16275(35
1395
2325
2325

Since every collection of one of each of these coins amounts to 38*s.* 9*d.* ; therefore there will be as many coins of each kind as £67. 16*s.* 3*d.*, contains 38*s.* 9*d.* Hence the number of coins of each kind = 35. *Ans.*

Ex. 2. An equal number of men, women and boys earned *Rs.*556. 8*a.* in 6 weeks; each man earned *Re.*1. 2*a.* 8*p.* a day, each woman 10*a.* and each boy 6*a.* 8*p* ; how many were there of each ?

Re.	*a.*	*p.*	
1	2	8	= a man's daily earning.
	10	0	= a woman's......
	6	8	= a boy's ....
*Rs.*2	3	4	= total daily earnings.
		7	
*Rs.*15	7	4	= weekly earnings.
		6	
*Rs.*92	12	0	= earnings of 6 weeks. ∴ no. of each sort = 6. *Ans.*

Rs.	*a.*
92	12
16	
1484*a.*	

Rs.	*a.*
556	8
16	
8904*a.*	

1484)8904(6
8904

Examples XLVI.

1. Divide £39 into four equal numbers of guineas, half-guineas, crowns and half-crowns respectively.

2. An equal number of gold-mohurs, rupees, eight-anna pieces, four-anna pieces, two-anna pieces and pice amount to *Rs.*447. 4*a.* 1*ps.* ; how many of each sort are there ?

3. An equal number of guineas, pounds, half-guineas, crowns, half-crowns and six-pences amount to £714 ; how many of each are there ?

4. An equal number of rupees, half-rupees, quarter-rupees, two-anna pieces, double-paisas and paisas amount to *Rs.*803. 5*a.* 2*ps.* ; find the number of each.

5. At the end of a week £54. 3*s.* is paid in wages to an equal number of men, women and boys ; a man is paid 4*s.* 6*d.*, a woman 3*s.* 3*d.* and a boy 1*s.* 9*d.* a day ; how many of each class are there ?

6. Tithes of the value of £448. 10*s.* are commuted for an equal number of bushels of wheat, barley and oats ; how many bushels of

each kind will be received when wheat is sold at *7s. 2d.* a bushel, barley at *4s. 9d.*, and oats at *3s. 5d.* ?

7. *Rs.*750 is paid in wages at the end of the week to a certain number of men, twice as many women, and three times as many children, each man earns *Rs.*2. *1a. 4p.* a day, each woman *Re.*1. *6a.* and each child *Rs.*1. *2a. 8p.* ; how many children are there ?

8. A bag contains a certain number of rupees, twice as many half-rupees, five times as many quarter-rupees, and eight times as many two-anna pieces, and the value of the whole sum in the bag is *Rs.*272. Find the number of each.

9. One farm produced 111 times as much rice as another ; both farms produced 1776 mds. 10 sr. ; how much did the smaller farm produce ?

10. How many packets of tea of 1 lb. 8 oz. and 1 lb. 12 oz. respectively, an equal number of each, can be made out of a chest of tea, in which the tea weighs 1 cwt. 1 qr. 3 lbs. ?

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184. Mixtures.

When several articles of the same kind but of different qualities or value are mixed together to form a **compound**, it is called a **mixture**. The parts forming the compound are called **ingredients** or **components** of the compound.

(1) Given the quantity and price of each of the component parts, to find the price of the mixture.

Ex. 1. A mixture is made of 9 gallons of spirit at *Rs.*6. *4a.* per gal., 16 gallons at *Rs.*9. *6a.* and 90 gallons at *Rs.*11. *2a.* ; what is the value of a gallon of it ?

The cost of 9 gals. = *Rs.*6. *4a.* × 9 = *Rs.* 56. *4a.*
................16 gals. = *Rs* 9. *6a.* × 16 = *Rs.* 150. *0a.*
.............90 gals. = *Rs.*11. *2a.* × 90 = *Rs.*1001. *4a.*
∴ the cost of 115 gals. = *Rs.*1207. *8a.*
∴ the cost of 1 gal. = *Rs.*1207. *8a.* ÷ 115 = *Rs.*10. *8a.* *Ans.*

Ex. 2. A man buys 16 lbs. of tea at *Rs.*2. *2a.* per lb., 12 lbs. at *Rs.*2. *5a. 4p.* per lb., and 24 lbs. at *Rs.*2. *6a. 10p.* per lb. At what price per lb. must he sell the mixture, so as to gain *Rs.*35. *12a.* on the whole ?

The cost of 16 lbs. = *Rs.*2. *2a.* × 16 = *Rs.* 34. *0a.*
.... 12 lbs. = *Rs.*2. *5a.* *4p.* × 12 = *Rs.* 28. *0a.*
.... 24 lbs. = *Rs.*2. *6a.* *10p.* × 24 = *Rs.* 58. *4a.*
∴ the cost of 52 lbs. = *Rs.*120. *4a.*
Gain = *Rs.* 35. *12a.*
∴ selling price of 52 lbs. = *Rs.*156. *0a.*
∴ selling price per lb. = *Rs.*156 ÷ 52 = *Rs.*3. *Ans.*

(2) To find the quantity to be added to a mixture under certain conditions.

Ex. 3. A pipe of wine containing 126 gallons is bought for £112; how much water must be added to it to allow of its being sold at 17*s.* 6*d.* a gallon?

£112 = 112 × 20 × 12*d.* = 26880*d.*; 17*s.* 6*d.* = 210*d.*

Now the quantity sold for £112 at 17*s.* 6*d.* a gal. = (26880 ÷ 210) or 128 gallons.

∴ the quantity of water mixed = (128 − 126) or 2 gallons. *Ans.*

Ex. 4. If a person gives *Rs.*556. 8*a.* for 184 gallons of wine; how much water must be added to it, if he wishes to sell it at *Rs.*2. 10*a.* a gallon and make a profit of *Rs.*36. 12*a.*?

The selling price of the mixture = *Rs.*556. 8*a.* + *Rs* 36. 12*a.*
= *Rs.*593. 4*a.* = 9492*a.*

Also the selling price per gal. = *Rs.*2. 10*a.* = 42*a.*

∴ the quantity sold = (9492 ÷ 42) or 226 gallons.

∴ the quantity of water added = (226 − 184) or 42 gallons. *Ans.*

Examples XLVII.

1. A grocer mixes 40 lbs of tea at *Re.*1. 3*a.* a lb., 48 lbs. at *Re.*1. 5*a.* 6*p.* a lb and 64 lbs. at *Re.*1. 9*a.* 10*p.* a lb.; find the value of 1 lb. of the mixture.

2. A grocer mixes 3 cwt. 24 lbs. of sugar at $6\frac{1}{2}$*d.* per lb. with 2 cwt. 64 lbs. at $4\frac{1}{4}$*d.*: at what price per lb. must he sell the mixture so as not to lose by the sale?

3. A tea merchant mixes 25 lbs. of tea at 14*a.* a lb., 40 lbs. at *Re.*1. 3*a.* 4*p.*, and 27 lbs. at *Re.*1. 9*a.* 4*p.*; at what rate per lb. must he sell the mixture, so as to gain *Rs.*23. 2*a.* on the transaction?

4. How many lbs. of tea-dust (which cost him nothing) must be put in the above mixture, to enable him to sell the tea at *Re.*1. 3*a.* 4*p.* per lb. and gain at the same time *Rs.*4. 4*a.* on the transaction?

5. A trader buys 756 cwt. of sugar at *Rs.*19. 7*a.* 8*p.* per cwt. with which he mixes 1921 cwt. of sugar which cost him *Rs.*21 per cwt.: at how much per lb. must he sell the mixture in order to make a profit of *Rs.*7396. 1*a.* 4*p.*?

6. A grocer mixes 19 lbs. of tea at 1*s.* $10\frac{3}{4}$*d.* per lb., 26 lbs. at 2*s.* $3\frac{1}{2}$*d.* per lb., and 27 lbs. at 2*s.* $6\frac{1}{4}$*d.* per lb.; at how much per lb. must he sell the mixture so as to gain £2. 3*s.* 4*d.* on his outlay?

7. A spirit merchant mixes 26 gallons of wine at 12*s.* 3*d.* a gallon with 39 gallons at 13*s.* 4*d.* a gallon; how many gallons of water must he add to the mixture so as to sell it at 10*s.* 9*d.* a gallon?

8. A man bought 150 eggs at 2 a penny, 150 more at 3 a penny, and mixed them and sold the whole at 5 for 2*d.*, how much does he lose?

9. A grocer buys 4 cwt. of sugar at $6d.$ per lb.; and 8 cwt. at $4\frac{1}{2}d.$ per lb. He sells 6 cwt. at $5\frac{1}{4}d.$ per lb.; at what rate per lb. must he sell the remainder so as neither to gain nor lose?

10. A merchant bought 84 gallons of whisky at *Rs.*8. 6*a.* a gallon, and sold it at *Rs.*8. 4*a.* a gallon, making a profit of *Rs.*105. How many gallons of water did he add to the whisky?

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185. Income and Expenditure.

Income including taxes and other rates is called **gross income**, but excluding these, it is **net income**. What a man lays by out of his income after meeting all necessary expenses, is called his **savings**.

Ex. 1. On the reduction of the income-tax from $9d.$ in the pound to $4d.$, a person saves £29. 15*s.* 10*d.*; find his gross income.

£29. 15*s.* 10*d.* = 7150*d.* = savings.

He saves $(9-4)$ or $5d.$ in every £ of his income.

∴ gross income required = £$(7150 \div 5)$ or £1430. *Ans.*

Ex. 2. A man has a yearly income of *Rs.*4867. 8*a.* and sets aside *Rs.*630 for charity, insurance and other purposes. What is the greatest sum he can spend per week, without getting into debt?

		Rs.	*a.*	*p.*	
		4867	8	0	= yearly income
		630	0	0	= charity, &c.
52	4	4237	8	0	= yearly expenditure.
	13	1059	6	0	
		*Rs.*81.	7	10...8*p.*	

Hence we see that he may spend *Rs.*81. 7*a.* 10*p.* every week, and have 8*p.* over at the end of the year. If he spends *Rs.*81. 8*a.* per week he will run into debt.

Examples XLVIII.

1. A man's annual income is *Rs.*10,000 and his daily expenses are *Rs.*18. 10*a.* 4*p.*; how much does he save in 9 years?

2. A man's income in the year 1895 was *Rs.*5250, out of which he saved *Rs.*1691. 4*a.*; what was his average daily expenditure?

3. A man spends *Rs.*105. 14*a.* in a week; how much does he spend in a year of 365 days?

4. If a person spends in 4 months, as much as he earns in three, how much can he lay by annually, supposing that he earns *Rs.*2505. every 6 months?

5. What annual income would enable a person to spend 8*s.* 9*d.* a day and save £7. 16*s.* $10\frac{1}{2}d.$, every calendar month?

6. If a person has an income of £535. 17*s.* 6*d.* a year, and he spends daily £1. 3*s.* $10\frac{1}{2}d.$, how much will he save at the end of the year?

7. Find the salary of a person who pays £7. 9*s*. 4*d*. income-tax, when the tax is 7*d*. in the pound.

8. A person after paying an income-tax of 4*p*. in the rupee, has *Rs*.8567. 11*a*. 4*p*. remaining; find his gross income.

9. If a person's yearly income be £65. 12*s*. 6*d*. and he lay by £20 a year, how much does he spend per day?

10. A person has an income of *Rs*.6706. 12*a*. 6*p*., and for the first 7 months he spends on an average *Rs*.588. 6*a*. 6*p*. a month; how much must he spend during each of the remaining 6 months, so as not to run into debt?

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186. Division of money.

When a given sum of money is divided among a number of persons in a proposed way, the amounts they severally receive are called their respective **shares**.

Ex. 1. Divide £16. 5*s*. 6*d*. among *A*, *B* and *C*, so that *A* may have £1. 2*s*. 6*d*. more than *B*, and *B* 16*s*. 9*d*. more than *C*.

	£.	s.	d.	
Here *B* has	0	16	9	more than *C*;
and *A* ...	1	2	6	more than *B*.
∴ *A* ...	1	19	3	more than *C*.

Now, if we take away these sums, to be subsequently given to *B* and *A* respectively, their shares will be equal to that of *C*.

Hence we have

£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
	16	9	16	5	6	4	9	10	4	9	10
1	19	3	2	16	0		16	9	1	19	3
£2	16	0	3)13	9	6	£5	6	7	£6	9	1
			£4	9	10						

∴ *A*'s share = £6. 9*s*. 1*d*.; *B*'s share = £5. 6*s*. 7*d*.;

and *C*'s share = £4. 9*s* 10*d*.

Ex. 2. Divide *Rs*.117. 11*a*. among *A*, *B* and *C*, so that *A* may receive twice as much as *B*, and *B* twice as much as *C*.

If *C*'s share is 1, *B*'s share is 2 and *A*'s share is 4.

Now, 1+2+4=7; 7)*Rs*.117. 11*a*.
Rs. 16. 13*a*.

∴ *C*'s share = *Rs*.16. 13*a*.
B's share = *Rs*.16. 13*a*. × 2 = *Rs*.33. 10*a*.
and *A*'s share = *Rs*.16. 13*a*. × 4 = *Rs*.67. 4*a*. } *Ans.*

Ex. 3. Divide *Rs.*2415 among A, B and C in such a way that for every *Rs.*20 that A gets, B gets *Rs.*15, and C gets *Rs.*11; how much does each receive?

$20+15+11=46$; $46)$*Rs.*2415 / *Rs.* 52. 8*a.*

∴ A's share = *Rs.*52. 8*a.* × 20 = *Rs.*1050
B's share = *Rs.*52. 8*a.* × 15 = *Rs.*787. 8*a.*
and C's share = *Rs.*52. 8*a.* × 11 = *Rs.*577. 8*a.* } *Ans.*

Examples XLIX.

1. Divide *Rs.*24. 9*a.* 4*p.* among A, B and C, so that B may have *Rs.*3. 5*a.* 4*p.* more than A, and C's share may be double of B's.

2. Divide *Rs.*73. 4*a.* 6*p.* between two men so that one may receive as much again as the other.

3. Divide *Rs.*1845. 9*a.* 6*p.* equally among 39 persons; and supposing 15 of them to have received their portions, and of the rest only 21 to appear; how much might be given to each of these?

4. Divide £20. 2*s.* 6*d.* into two sums of money, one of which contains as many half-crowns as the other contains shillings.

5. Divide *Rs.*24515 among A, B and C, so that A may have *Rs.*1786. 12*a.* more than B, and C *Rs.*3257. 5*a.* less than B.

6. Divide *Rs.*2509. 14*a.* among A, B and C, so that B may receive 3 times, and C 5 times, as much as A.

7. Divide £189. 5*s.* 7½*d.* among 3 men, so that one of them may have 15 guineas more than either of the other two.

8. A purse and the money it contains are worth *Rs.*19. 4*a.*, and the money is 10 times the value of the purse; how much does the purse contain?

9. Divide *Rs.*690 between A, B and C, so that where A receives *Rs.*10, B may receive *Rs.*30, and where B receives *Rs.*20, C may receive *Rs.*50.

10. The sum of *Rs.*473. 6*a.* 4*p.* has to be divided among 5 persons, so that the first has 20 shares, the second 17, the third 12, the fourth 8, and the fifth 5; how much will each receive?

11. Divide £119. 16*s.* 3*d.* among 36 persons, in such a way that 17 of them may each receive 18*s.* 9*d.* more than each of the rest.

12. Divide *Rs.*68427. 3*a.* 4*p.* among 3 persons, so that the first shall have *Rs.*5687. 2*a.* 8*p.* more than the second, and the second *Rs.*7289. 1*a.* 4*p.* more than the third.

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187. Men, Women and Boys.

Ex. 1. Divide *Rs.*156. 4*a.* among 7 men, 9 women and 11 boys,

so that each man may receive three times as much as a boy, and each woman twice as much as a boy.

The 7 men will receive as much as 7×3 or 21 boys and the 9 women as much as 9×2 or 18 boys; therefore 7 men, 9 women and 11 boys will receive as much as $21 + 18 + 11$ or 50 boys. Thus,

7 men	= 21	boys
9 women	= 18	...
11 boys	= 11	...
	50	

Rs. *a.*
50 { 5)156 4
10) 31 4
*Rs.*3 2

Hence a boy's share = *Rs.*3. 2*a.*,

a woman's = *Rs.*6. 4*a.*, and a man's = *Rs.*9. 6*a.*

Ex. 2. A man and a woman together have *Rs.*40. 6*a.* 8*p.*, a woman and a boy together have *Rs* 30. 8*a.*, a man and a boy together have *Rs.*35. 7*a.* 6*p.*; find how much a man, a woman and a boy together have.

Here, adding the three given items, we have
twice a man's money + twice a woman's money + twice a boy's money = *Rs.*40. 6*a.* 8*p.* + *Rs.*30. 8*a.* + *Rs.*35. 7*a.* 6*p.* = *Rs* 106. 6*a.* 2*p.*

∴ a man + a woman + a boy together have *Rs.*106. 6*a.* 2*p.* ÷ 2
= *Rs.*53. 3*a.* 1*p.* *Ans.*

Examples L.

1. Divide £2. 10*s.* $10\frac{1}{2}$*d.* between 3 men and 2 women, giving to each of the men 3 times as much as to each of the women.

2. A gentleman divided *Rs.*103. 2*a.* among 12 men, 16 women and 30 children; he gave to each man twice as much as to each woman, and to each woman three times as much as to each child. What did each woman receive?

3. Divide *Rs.*3993. 8*a.* among one man, one woman and 15 boys, in such a way that the man's share is 10 times, and the woman's share 3 times as much as that of each boy; what is the value of the share of each?

4. Divide *Rs.*5501. 9*a.* among 4 men, 6 women and 8 boys, giving to each man double that of a woman and to each woman triple that of a boy.

5. Divide £15. 6*s.* among 12 men, 17 women and 26 children, in such a way that a man shall receive 3 times as much as a child and a woman twice as much as a child; what does a woman receive?

6. Divide *Rs* 1151. 4*a.* among 20 women and 25 men, so that each woman may receive *Rs.*7. 8*a.* more than each man; how much will each woman receive?

7. If 20 men, 40 women and 50 children receive *Rs.*3500 among them for 7 weeks' work and 2 men receive as much as 3 women or 5 children, what sum does a woman receive per week?

8. The weekly wages at a mill amount to *Rs.*1862. In the mill a certain number of women are employed at *Re.*1. 6*a.* 8*p.* a day, five times as many men at *Rs.*2. 12*a.* a day, and 6 times as many boys at *Re.*1. 2*a.* 8*p.* a day; how many men are employed?

9. *A* and *B* together have *Rs.*48. 14*a.* 9*p.*, *B* and *C* together have *Rs.*45. 10*a.* 6*p.*, *A* and *C* together have *Rs.*54. 8*a.* 11*p.*; how much has *C*?

10. A goat and a lamb are together worth *Rs.*6. 10*a.*, a goat and a calf are together worth *Rs.*10. 4*a.* 8*p.*; and a calf and a lamb are together worth *Rs.*8. 5*a.* 6*p.*; find the price of a goat, of a lamb and of a calf.

Examples worked out.

Ex. 1. A man has a certain number of pice, twice as many two-anna pieces, three times as many four-anna pieces and four times as many rupees. If the total amount be *Rs.*501. 9*a.*, find the number of coins of each kind.

Here, 1 pice + 2 two-anna pieces + 3 four-anna pieces + 4 rupees $=(1+16+48+256)$ pice $=321$ pice; and *Rs.*501. 9*a.* $=32100$ pice.

$\therefore$ the number of pice $=(32100\div 321)$ or 100.

Hence, no. of two-anna coins $=2\times 100=200$; the no. of four-anna coins $=3\times 100=300$, and the no. of rupees $=4\times 100=400$. *Ans.*

Ex. 2. A man died on June 2 Monday, 1890, having lived 23025 days exclusive of the day of his death. Find the day and date of his birth.

A year $=365$ days; therefore 23025 days $\div 365=63$ years 30 days. Now in these 63 years, 16 are leap years (which $=366$ days); therefore 23025 days $=63$ years $+(30-16)$ days or 63 years 14 days.

Again, $1890-63=1827$, and reckoning 14 days backwards from June 1, we come to May 19.

Hence the man was born on May 19, 1827.

Now 23025 divided by 7 gives a remainder 2; therefore he was born on Saturday, reckoning 2 days backwards from Sunday.

Ex. 3. The total expenses of a family when rice is at *Rs.*4 per maund are *Rs.*55; when rice is at *Rs.*3. 12*a.* per maund, they are *Rs.*52. 8*a.* (other expenses remaining the same); find his total expenses when rice is at *Rs.*4. 4*a.* per maund.

Here, a decrease of (*Rs.*4 − *Rs.*3. 12*a.*) or 4*a.* per md. in the price of rice makes a decrease of (*Rs.*55 − *Rs.*52. 8*a.*) or *Rs.*2. 8*a.* $=40a.$ in the family expenses.

Hence, quantity of rice consumed by the family $=\frac{40}{4}$ or 10 mds. Therefore the expenditure on rice $=$ *Rs.*(4×10) or *Rs.*40 and the other expenses $=$ *Rs.*$(55-40)=$ *Rs.*15.

Now, the price of 10 mds. at *Rs.*4. 4*a.* per md. = *Rs.*4. 4*a.* × 10 = *Rs.*42. 8*a.*

Hence, the required expenses = *Rs.*42. 8*a.* + *Rs.*15 = <u>*Rs.*57. 8*a.*</u>

Ex. 4. A corn-merchant mixed 10 mds. of rice worth *Rs.*4 per md. with a certain quantity worth *Rs* 3. 8*a.* per md., and selling the mixture at *Rs.*3. 12*a.* per md. gained *Rs.*10 on the whole. How many mds. of the second kind did he mix?

By selling the first sort of rice at *Rs.*3. 12*a.* per md. he incurs a loss of (*Rs.*4 − *Rs.*3. 12*a.*) or 4*a.* per maund; therefore the loss in 10 mds. = 10 × 4*a.* = 40*a.* = *Rs.*2. 8*a.*

Now, gain per md. on the second sort = (*Rs.*3. 12*a.* − *Rs.*3. 8*a.*) = 4*a.* and as he shall have to make altogether *Rs.*10 + *Rs.*2. 8*a.* or *Rs.*12. 8*a.* = 200*a.*

∴ the quantity required = $\frac{200}{4}$ or 50 mds. *Ans.*

Ex. 5. A *gowala* mixed milk worth *Rs.*7 per md. with twice as much worth *Rs.*5. 8*a.* per md. and having sold the mixture at *Rs.*6. 4*a.* per md., cleared *Rs.*10. 8*a.* on the whole. How much did he mix of each sort?

The cost of 1 md. of first + 2 mds. of second = *Rs.*7 × 1 + *Rs.*5 8*a.* × 2 = *Rs.*18.

∴ the cost of 1 md. of the mixture = *Rs.*18 ÷ 3 = *Rs.*6.

The gain per md. = *Rs.*6. 4*a.* − *Rs.*6 = 4*a.* and the total gain is *Rs.*10. 8*a.* = 168*a.*

∴ the whole mixture contains $\frac{168}{4}$ or 42 mds.

$$\text{Now, } 1+2=3\ ;\ \therefore \left.\begin{array}{l}\text{quantity of first sort} = 42 \div 3 = 14 \text{ mds.}\\ \text{and} \quad \ldots \quad \text{second} \ldots = 14 \times 2 = 28 \text{ mds.}\end{array}\right\} \textit{Ans.}$$

Ex. 6. A supply of water suffices for 60 days if 10 maunds leak off every day, but only for 55 days if 15 maunds leak off daily. Find the total quantity of water in the supply.

In the first case 60 × 10 or 600 mds. leak off altogether, while in the second 55 × 15 or 825 mds. leak off.

∴ for (60 − 55) or 5 days' use (825 − 600) or 225 mds. of water are required.

∴ for daily use (225 ÷ 5) or 45 mds. of water are required.

Now, taking the first case, we find that the supply lasts for 60 days; and in that time (60 × 45) or 2700 mds. are required for use; and 60 × 10 or 600 mds. leak off.

Hence the total quantity reqd. = (2700 + 600)mds. = 3300 mds. *Ans.*

Ex. 7. On changing 3 four-anna pieces, I received 36 coins in single and double pice. How many did I get of each?

Here 3 four-anna pieces = 12*a.* = 48 pice.

Now had all been single pice, I would have received 48; but as I received $(48-36)$ or 12 single pice less, and the difference between a double and a single pice is one pice,

$$\left.\begin{array}{l}\therefore \text{ the number of double pice} = 12 \\ \text{and } \quad \ldots \quad \ldots \text{ single pice} = 36-12=24\end{array}\right\} \textit{Ans.}$$

Ex. 8. A man has three estates, and his incomes from the second and third are respectively twice and thrice as much as from the first. He has to pay an income-tax of 8 pies in the rupee for the first, 1*a.* in the rupee for the second, and 1*a.* 4*p.* in the rupee for the third. If the total income-tax be *Rs.*80, how much income does each estate yield?

Supposing his income from first to be *Re.*1, his income from second = *Rs.*2, and from third *Rs.*3.

Also, for the first he should have to pay (1×8) or 8*p.* in the *Re.*
... second ... .. (2×12) or 24*p.* in the *Re.*
... third ... ... (3×16) or 48*p.* in the *Re.*

$\therefore$ the total tax amounts to $(8+24+48)$ or 80 pies in the rupee.

Also *Rs.*$80=(80\times 16\times 12)$ pies, the total tax.

$$\left.\begin{array}{l}\text{Hence, income from first} = Rs.(80\times 16\times 12 \div 80) = Rs.\ 192 \\ \ldots \quad \ldots \text{ second} = \ldots \ 192\times 2 \qquad = Rs.\ 384 \\ \ldots \quad \ldots \text{ third} \ = \ldots \ 192\times 3 \qquad = Rs.\ 576\end{array}\right\} \textit{Ans.}$$

Ex. 9. *A* gave *B* as many sovereigns as is expressed by the sum of all the numbers that can be formed by different arrangements of the digits 2, 4 and 7 taken all together; and *B* gave *A* as many six-pences as is expressed by the sum of all the numbers that can be formed by different arrangements of the figures 4, 5, 8 and 9 taken all together. Who is the gainer and by how much?

The sum of all the numbers that can be formed by different arrangements of the digits 2, 4 and 7 taken all together $=2\times(2+4+7)\times(10^2+10+1)=2\times 13\times 111=2886$. [See *Ex.* 9, Page 70.]

Similarly, the sum of the numbers formed by the different arrangements of the digits 4, 5, 8 and 9 taken all together $=6\times(4+5+8+9)\times(10^3+10^2+10+1)=6\times 26\times 1111=173316$.

Hence *A* gave *B* 2886 sov. or £2886, and *B* gave *A* 173316 six-pences or £4332. 18*s.*

Therefore *A* is the gainer by (£4332. 18*s.* − £2886) or £1446. 18*s.* *Ans.*

Miscellaneous Examples II.

1. From 261 times *Rs.*352. 1*a.* 4*p.* take *Rs.*90892. 8*a.* and divide the remainder by 89.

2. How many Napoleons of 15*s.* $9\frac{1}{2}d.$ each can be obtained for 5685 thalers of 2*s.* $11\frac{1}{2}d.$ each?

3. How many Nobles are equivalent to £195. 13*s.* 4*d.* ?

4. 13 rupees, 9 half-crowns and 17 three-penny pieces amount to £2. 16*s.* ; find the value of a Rupee. Find the value of a lac of rupees in English money. (1 lac=1,00,000.)

5. A dealer bought 9 horses at *Rs.*118. 13*a.* 4*p.* each ; one died and the others he sold at a profit on each of *Rs.*21. 1*a.* 8*p.* Find his gain.

6. The value of a mark being 13*s.* 4*d.*, and that of a moidore 27*s.*, shew that there are twice as many farthings in 57 marks and 57 moidores, as there are drams in 1 cwt. 3 qrs. 19 lbs. 8 oz. 8 drs. of sugar.

7. To a certain stock-in-trade A and B together contributed *Rs.*22. 10*a.*, B and C together *Rs.*25. 8*a.* and A and C together *Rs.*27. 6*a.* ; how much did each contribute ?

8. A boy receiving 4*a.* per week has 2*a.* stopped every third week ; if there are 39 weeks in a school year, how much does he realize in 4 years ?

9. A has *Rs.*1002. 7*a.* 8*p.* and B 128786 pies ; if A receive from B 22222 pies and B from A *Rs.*115. 15*a.* 6*p.*, how much will A have more than B ?

10. Of 21 people 13 lose *Rs.*1163. 13*a.* 6*p.* each and 8 lose *Rs.*930. 1*a.* 9*p.* each. What is the average loss per man ?

11. A and B having an equal share in a heap of potatoes containing 86 maunds, A takes 24 mds. and B the rest, paying A *Rs.*27. 11*a.* 4*p.* What is the worth of a maund of potatoes ?

12. A grocer's bill amounts to *Rs.*1897. 8*a.* It happens to be made up of equal sums for tea at *Re.*1. 14*a.* 8*p.* per seer, sugar at 4*a.* per seer, rice at 3*a.* per seer, and coffee at 11*a.* per seer. How many seers are there of each sort ?

13. A person mixes together 10 lbs. of tea at *Re.*1. 4*a.* per lb., 12 lbs. at *Re.*1. 6*a.* and 14 lbs. at *Re.*1. 8*a.* per lb. He reserves 6lb. of the mixture for himself and sells the remainder at *Re.*1. 13*a.* per lb. How much does he gain ?

14. A manufacturer employs 50 men and 35 boys who work respectively 12 and 8 hours a day during 5 days of the week, and half the time the other day ; each man receives 4*a.* and each boy 1*a.* 4*p.* an hour. What is the whole amount of wages for a year ?

15. What quantity of water must I add to a pipe of wine which costs *Rs.*900, to reduce its price to *Rs.*5 a gallon ?

16. The yearly expense of a school is *Rs.*18993. 11*a.* : there is an endowment yielding *Rs.*4850. 15*a.* and subscriptions *Rs.*743. The rest is to be made up by the fees of the pupils of whom there are 217 ; what must each of them pay on an average ?

17. In what time will a tradesman, who gains *10a. 8p.* a day and spends *5a.* of it, be able to pay off a debt of *Rs.208. 9a. 8p.* ?

18. A man's weekly income is *Rs.18. 7a* and his quarterly expenditure is *Rs.182.* How much will he save at the year's end ? (a year = 52 weeks.)

19. I buy 80 lbs. of black tea at *Rs.2. 2a.* per lb. and 20 lbs. of green at *Rs.2. 12a.* per lb. and mix them ; at what rate must I sell the mixture so as to gain *1a. 4p.* in the rupee ?

20. Divide two fields, one of 6 ac. 3 po. 13 sq. yds., the other of 4 ac. 37 po. 27 sq. yds., between *A*, *B* and *C*, so that *A*'s no. of ro. = *B*'s no. of sq. po. = *C*'s no. of sq. yds.

21. September 17, 1893, was Sunday. What day of the week was September 17, 1891 ?

22. A wine merchant bought 2 pipes of wine at *£2. 13s. 4d.* per gallon. How much water must he mix with it that by selling a gallon of the mixture for *£2. 6s. 8d.*, he may gain on the whole *£14* ?

23. A factor bought 25 pieces of cloth for *Rs.185000* at *Rs.4 10a.* per yard. How many yards are there in each piece ?

24. A house and its furniture are together worth *£3367. 2s. 6d.* ; the house is worth 8 times the furniture. What is the house worth ?

25. A man's total expenses are *Rs.44*, when rice sells at *Rs.2. 8a.* per maund, and *Rs.46. 4a.* when rice sells at *Rs.2. 11a.* per md. What are his expenses when rice sells at *Rs.3. 3a.* per maund ?

26. Two persons buy mangoes at 16 per rupee ; one sells at 12 per rupee and the other 16 for *Re.1. 4a.* How much profit does one make more than the other ?

27. A man spending daily *Rs.2. 10a. 6p.* lays by *Rs.150. 2a. 11p.* in the year 1897 ; find his daily income.

28. I received 320 pieces in half-rupees and quarter-rupees in exchange for 100 rupees. How many of each did I get ?

29. *A* and *B* gave equa sums in buying 15 horses and 22 cows. *A* took 5 horses and 17 cows and *B* the rest. If a horse cost *Rs.56. 8a.* and a cow *Rs.35. 10a.*, how should they settle the account ?

30. A man was born on the 15th of May 1762, and died on the 17th of June 1825. How many days did he live, exclusive of the day of his death ?

31. A goldsmith manufactured 2 lbs. 3 dwts. 8 grs. of gold into rings, each containing 9 dwts. 16 grs. : he sold the rings at *Rs.25* each ; how much did he receive for them ?

32. A piano, table and carpet cost *Rs.632. 12a.* ; the piano and table cost *Rs.547. 6a.*, and the table and carpet cost *Rs.260. 2a. 8p.* Find the price of each.

33. A grocer buys 40 lbs. of tea at *Re.*1. 12*a.* per lb. and also some cheaper tea; he mixes the two kinds of tea and by selling all the tea for *Rs.*236. 4*a.* at *Re.*1. 11*a.* per lb. gains *Rs.*32. 14*a.* 8*p.* on his outlay; how many lbs. of the cheaper tea does he buy, and at what price per lb.?

34. Twice *A*'s money = 3 times *B*'s money, and the difference of their moneys is *Rs.*12. 10*a.* How much has each?

35 A bag contains a certain number of rupees, twice as many half-rupees, 4 times as many quarter-rupees and 8 times as many two-anna pieces, and total amount in the bag is *Rs.*100. How many of each are there?

36. *A*, *B* and *C* contributed equal sums in purchasing 22 horses, 28 cows and 56 sheep. *A* took 7 horses, 9 cows and 19 sheep, *B* took 8 horses, 8 cows and 17 sheep, and *C* the rest. If the price of a horse be *Rs.*68 8*a.*, of a cow *Rs.*44. 10*a.* and of a sheep *Rs.*7. 6*a.*, which of them shall have to pay and which to receive, and how much?

37. A landowner has three estates. The first estate yields an income of *Rs.*3000, the second *Rs.*4200 and the third *Rs.*6250. If the rate of tax be 1*a.* in the rupee for the first, 1*a.* 4*p.* in the rupee for the second and 1*a.* 3*p.* in the rupee for the third, how much tax has he to pay altogether?

38. Divide *Rs.*7890 among *A*, *B* and *C* in such a way that *A* may receive *Rs.*125 more than twice as much as *B*, and *C Rs.*250 more than thrice as much as *B*.

39. A certain weight of gold worth *Rs* 20. 14*a.* 6*p.* per tola is mixed with an equal weight worth *Rs.*18. 6*a.* 6*p.* per tola. Determine the weight of gold, so that by selling the mixed gold at *Rs.*19. 14*a.* 6*p.* per tola, a goldsmith may clear *Rs.*12. 8*a.* on the whole.

40. In making 50 benches, the cost of each for wood is *Re.*1. 2*a.*, for labour 13*a.*, for polish 2*a.* and for screws 1*a.* How much is gained on each bench by selling the whole lot for *Rs.*112. 8*a.* ?

41. The 15th of May 1890 was Thursday. What day of the week was the 27th April 1790 ?

42. The cost of maintaining a family is *Rs.*122. 8*a.* when milk sells at 2*a.* per seer, and *Rs.*125. 12*a.* when milk sells at 2*a.* 3*p.* per seer. Find the monthly consumption of milk in the family and the amount of other expenses, supposing the latter to be unchanged.

43. A besieged garrison has a supply of water for 50 days. Owing to a leak, however, in the bottom of the reservoir, 5 gallons waste every day, and then the supply suffices for ten days less. Find for how many days the supply would suffice if 20 gallons leak off every day.

44. A *gowala* mixes 12 mds. 16 sr. of milk at *Rs*.6. 9*a*. per md with 22 mds. 24 sr. at *Rs*.7. 8*a*. per md. He then adds 1 md. 20 sr. of water and sells the mixture at 6 seers per rupee. How much does he gain or lose ?

45. Divide *Rs*.10256. 12*a*. among three men, so that the first shall get *Rs*.1251. 4*a*. more than the second, and *Rs*.152 less than the third.

46. 8 men, 16 women and 24 boys earned *Rs*.136 in 8 days. A woman earns daily 2*a*. more than a boy, and a man daily earns as much as a woman and a boy together. Find how much a man, a woman and a boy daily earn.

47. If 50 pieces of coin consisting of single and double pice make up a rupee, find the number of each coin.

48. A man died on the 7th August, Thursday, 1890, having lived 21000 days (exclusive of the day of his death). Find the day and date of his birth.

49. A certain English landowner has three estates, for which he has to pay a total tax of £180. His income from the second and third estates are respectively twice and four times his income from the first. The rates of tax for the three are respectively 1*s*. 2*d*., 1*s*. 3*d*. and 1*s*. 4*d*. in the £. Determine his income from each estate.

50. *A* pays *B* as many rupees as is expressed by the sum of the numbers formed by all the different arrangements of the figures 2, 3 and 4 taken all together, and *B* pays *A* as many double pice as is expressed by the sum of the numbers formed by the figures 1, 2, 3 and 4 taken all together and arranged in all possible ways. Who shall be the gainer and by how much ?

51. Divide *Rs*.51. 10*a*. among 8 boys, 4 women and 3 men in such a manner that a woman shall receive 2*a*. more than twice as much as a boy, and a man 4*a*. more than as much as a boy and a woman together.

52. A man died on the 1st of August, Friday morning, 1890 He had lived 10000 days. Find the date and day of his birth.

53. Divide *Rs*.1780. 13*a*. into three such parts that the first part shall be *Rs*.125. 3*a*. more than the sum of the second and third, and the second part *Rs*.17. 12*a*. more than the third.

54. If the monthly expenditure of a family be *Rs*.57. 8*a*., when rice is at *Rs*.4. 6*a*. per maund and *Rs*.58, when rice is at *Rs*.4. 8*a*. per maund ; what should the expenditure be when rice would be at *Rs*.4. 12*a*. per maund ?

55. What sum of money is that which being multiplied by 16, *Rs* 24 added to the product, the sum divided by 13, and *Rs*.3. 13*a*. added to the quotient, the sum is *Rs*.7. 13*a*. ?

56. An equal number of men, women and boys together earned

*Rs.*62. 8*a.* in 5 days. A boy earns 2*a.*, a woman 3*a.* and a man 5*a.* daily. Find the number of boys.

57. A goldsmith mixes a certain number of tolas of gold worth *Rs.*20. 8*a.* per tola with twice that quantity worth *Rs.*19. 6*a.* per tola. On selling the mixed gold at *Rs.*20 per tola, he gained *Rs.*15. How much of each kind did he mix?

58. Sound travels at the rate of 1142 ft. per second; what is the distance of a thunder cloud when the sound of thunder follows the flash of lightning after an interval of 9 seconds?

59. *A* gives *B* 112 gallons of brandy at 32*s.* 6*d.* a gallon, and receives in return £40. 12*s.* 6*d.* and 780 yds. of cloth. What is the price of the cloth per yard?

60. There are 6 presses at work striking off sovereigns, half-sovereigns, florins, shillings, six-pences and four-penny-pieces respectively, and each at the rate of 2500 per hour; find the value of the money struck off in 13 days of 9 hours each.

61. What is the difference in seconds between the Mahomedan year of 354 days 8 hrs. 48 min. and the Hindu year of 365 days 6 hrs. 12 min. 30 sec.?

62. If 6 hats cost as much as 25 pairs of gloves, worth *Re.*1 10*a.* a pair, how many hats can be bought for *Rs.*616. 2*a.* 4*p.*?

63. If telegraph posts are placed 66 yards apart and a railway train passes one in every three seconds, how many miles an hour is the train running?

64. A person observed the flash of a cannon 7 seconds before he heard the report; how far was the cannon distant, supposing that sound moves at the rate of 1142 ft. per second?

65. In how many days of 8 hours each will a person be able to count 10 lacs of rupees at the rate of 80 per minute? How many will remain to be counted on the morning of the 26th day?

66. How much water must be mixed with 30 seers of milk worth 2*a.* per seer, in order to reduce its price to 1*a.* 6*p.* per seer?

67. By the payment of 2*s.* 1*d.* in London a banker will give credit at Calcutta for a rupee; how many rupees may be received in Calcutta for the payment of £5025. 6*s.* 3*d.* in London?

68. If 5 oz of silk can be spun into a thread 2 fur. 20 po. long; what weight of silk would supply a thread sufficient to reach to the Moon, if the distance be 240000 miles?

69. A ship's crew of 50 men have a supply of water for 30 days at 2 seers a head; if they lose 125 seers, and find that they will be 50 days at sea, what must be each man's daily allowance?

70. A landowner has four estates, for which he has to pay a tax of *Rs.*760. The second, third and fourth yield respectively twice,

thrice and four times as much income as the first. If the tax be levied at 10, 9, 8 and 6 pies in the rupee respectively, find the amount of his income from each estate.

71. A tradesman in India exchanges with a merchant in China as many maunds of sugar as is expressed by the sum of all the numbers that can be formed by the different arrangements of the digits 7, 8 and 9 taken all together, for as many pounds of tea as is expressed by the sum of all the numbers similarly formed by the digits 3, 0, 5 and 7 taken all together. How much tea does the Indian merchant get in return for 37 mds. of sugar?

72. A man's monthly expenditure consists of 5 mds. of rice, 1md. 20 sr. of flour, 15 sr. of ghee and 2 mds. 15 sr. of milk. When rice costs *Rs*.3. 10*a*. 6*p*. per md., flour *Rs*.4. 12*a*. per md., ghee *Rs*.37. 8*a*. per md. and milk *Rs*.5. 8*a*. per md., the total expenses amount to *Rs*.130. 10*a*. If the prices of other articles remain the same, what would his family expenses amount to, when rice would sell at *Rs*.4. 12*a*., flour at *Rs*.5. 4*a*., ghee at *Rs*.41. 8*a*., and milk at *Rs*.6. 4*a*. per maund?

73. A total weight of 12 mds. 10 sr. consists of a certain number of 10 seer-weights, three times as many of 5 seer-weights, 4 times as many of $2\frac{1}{2}$ seer-weights, 6 times as many of 1 seer-weights, 8 times as many of half seer-weights and 16 times as many of powa-weights. Find the number of each kind of weights.

74. A certain number of sovereigns, twice as many crowns, 5 times as many half-crowns, 8 times as many shillings and 12 times as many six-pences together amount to £28. 5*s*.; find the numbers of each coin.

75. A man mixed 3 mds. of milk at *Rs*.4. 8*a*. per md. with a certain quantity worth *Rs*.4. 4*a*. per md. and three times that quantity worth *Rs*.3. 12*a*. per md. He sold the mixture at *Rs*.4. 2*a*. per md. and thus cleared *Rs*.15 on the whole. How much of the second and third sort did he mix?

CHAPTER IV.

Numbers, Measures and Multiples.

1. NUMBERS.

188. Numbers which follow a regular order increasing by 1 are called **consecutive** numbers. The consecutive numbers commencing at 1 are called **natural** numbers.

Thus, 4, 5, 6, 7, 8, &c. are *consecutive*, and 1, 2, 3, 4, 5, 6, &c. are *natural* numbers.

189. Numbers are either **even** or **odd**.

Numbers are called **even** when they can be divided by 2 without a remainder, and **odd** when they cannot be so divided.

Thus, 4, 8, 10, 16, &c. are *even* and 3, 5, 7, 13, &c. are *odd* numbers.

190. A **measure** or **factor** of a number is *any* number which divides it without a remainder. It is said to *measure* the number by the **units** contained in the *quotient*.

Thus, 4 is a *measure* or *factor* of 24, because it is contained exactly 6 times in 24. All numbers have 1 for a measure.

191. An **aliquot part** of a number is *any* measure of it.

Thus, 4 is an *aliquot part* of 20, for 4 is a measure of 20.

192. A **multiple** of a number is *any* number which contains it an exact *number of times*.

Thus, 108 is a *multiple* of 12, because 12 is contained exactly 9 times in 108.

193. A measure is sometimes called a **submultiple**.

Thus, 4 is a *submultiple* of 16.

194. Numbers are either **prime** or **composite**.

A **prime number**, or a **prime**, is a number which can be divided exactly only by itself and by unity. A **composite number** is a number which can be separated into *factors* each greater than unity, or which, in other words, arises from the multiplication of *two* or *more* other numbers, termed *factors*.

Thus, 2, 5, 7, 11, &c. are *primes*, and 4, 8, 10, 12, &c. are *composite* numbers.

195. Two numbers are **prime** to each other, when their only common measure is 1.

196. One number is **divisible** by another when it can be divided by that other number exactly.

Thus, 20 is *divisible* by 5, for 20 contains 5 exactly 4 times.

197. The following RULES are important, and should be carefully committed to memory.

(1) If a number divide a product of two factors and be prime to one of them, it must divide the other.

Thus, if 4 divide 9×24, and 4 is prime to 9, then 4 must divide 24, for 4 is a measure of 24.

(2) If a number is divisible separately by two others which are prime to each other, it is divisible by their product.

Thus, if 240 be divisible by 3, and by 4, where 3 and 4 are prime to each other, it will be divisible by 3×4, for $240 = (3 \times 4) \times 20$.

(3) If one number is divisible by another, any multiple of the first is also divisible by the second.

Thus, 10 is divisible by 2 and 5; hence any number ending with 0, being a multiple of 10, is divisible by 2 and 5.

100 is divisible by 4 and 25, therefore all numbers ending with two ciphers are divisible by 4 and 25.

1000 is divisible by 8 and 125; hence all numbers ending with three ciphers are divisible by 8 and 125.

Again, $1001 = 7 \times 11 \times 13$, and therefore 1001 is divisible by 7, 11, and 13. Hence all numbers like 7007 or (7×1001), 18018, or (18×1001), 325325 or (325×1001) are all divisible by 7, 11 and 13.

(4) If each of two numbers is divisible by a third, their sum or difference is also divisible by the third.

Thus, $8654 = 8650 + 4$ and is divisible by 2, if 4 is;
$4235 = 4240 - 5$ ... ... 5, if 5 is;
$7336 = 7400 - 64$ ... ... 4, if 64 is;
$78664 = 78000 + 664$ ... ... 8, if 664 is;
$86184 = 86086 + 98$ and is divisible by 7, if 98 or $(184 - 86)$ is;
$429275 = 429429 - 154$ ... ... 11, if 154 or $(429 - 275)$ is;
$186459 = 186186 + 273$ ... ... 13, if 273 or $(459 - 186)$ is.

(5) If each of two numbers is divisible by a third, then the sum or difference of any multiple of the first and of any multiple of the second is also divisible by the third.

Thus, $627 = 600 + 20 + 7 = 6(99 + 1) + 2(9 + 1) + 7$
$= 6 \times 99 + 2 \times 9 + 6 + 2 + 7$;

$\therefore$ 627 is divisible by 3, if $6 + 2 + 7$ is.

$7362 = 7000 + 300 + 60 + 2 = 7(999 + 1) + 3(99 + 1) + 6(9 + 1) + 2$
$= 7 \times 999 + 3 \times 99 + 6 \times 9 + 7 + 3 + 6 + 2$;

$\therefore$ 7362 is divisible by 9, if $7 + 3 + 6 + 2$ is.

$82654 = 80000 + 2000 + 600 + 50 + 4$
$= 8(9999 + 1) + 2(1001 - 1) + 6(99 + 1) + 5(11 - 1) + 4$
$= 8 \times 9999 + 2 \times 1001 + 6 \times 99 + 5 \times 11 + 8 - 2 + 6 - 5 + 4$;

$\therefore$ 82654 is divisible by 11, if $(8 + 6 + 4) - (2 + 5)$ is.

(for 9999, 1001, 99 and 11 are all divisible by 11.)

198. Criteria of Divisibility.

A number is divisible by

2, if its *last* digit is divisible by 2; as 450, 326.
3, if the *sum* of the digits is divisible by 3; as 267, 531.
4, if its *last two* digits are divisible by 4; as 600, 520, 924.
5, if its *last* digit is 0 or 5; as 370, 865.
6, if it is divisible by both 2 *and* 3; as 318, 588.
8, if its *last three* digits are divisible by 8; as 3000, 5240, 2816.

9, if the *sum* of its digits is divisible by 9 ; as 648, 702.

10, if its *last* digit is 0 ; as 4570, 2300.

11, if the difference between the sum of its digits in the *odd* and in the *even* places is 0, or is divisible by 11 ; as 1067, 2695, 19613.

12, if it is divisible by both 3 *and* 4 ; as 708, 1164.

For **7** and **13**, see Art. 197 (4).

199. There is no direct method for determining primes, and so we give below a list of the prime numbers from 1 to 227.

1	11	29	47	71	97	113	149	173	197
2	13	31	53	73	101	127	151	179	199
3	17	37	59	79	103	131	157	181	211
5	19	41	61	83	107	137	163	191	223
7	23	43	67	89	109	139	167	193	227

200. *To ascertain what numbers are prime.*

(i) Every number whose last digit is 0, 2, 4, 6, or 8 is divisible by 2 (Art. 198), and therefore every such number except 2 itself is not a prime. Every number whose last digit is 0 or 5 is divisible by 5, and therefore every such number except 5 itself is not a prime. Hence the last digit of every prime number except 2 and 5, must be 1, 3, 7 or 9.

(ii) If then the last digit of the given number be 1, 3, 7, or 9 try as divisors one after another the primes 3, 7, 11, 13, &c. ; if there is a remainder in each case the given number is a prime. It is not necessary to try a divisor whose square is greater than the given number.

Ex. Are 689 and 947 primes ?

(1) 689 is not divisible by 3 (for $6+8+9=23$), nor by 7 (by trial), nor by 11 (for $6+9-8=7$), but is divisible by 13 ; therefore 689 is *not* a prime.

(2) 947 is not divisible by 3, 7, 11, 13, 17, 19, 23 or 29 ; and we need not try the next divisor 31, for the square of 31 is greater than 947. Hence 947 *is* a prime.

201. To **resolve** or **decompose** a composite number into its prime factors is to find those prime numbers which when multiplied together produce the given number.

Thus, $210=2\times3\times5\times7$; $504=2\times2\times2\times3\times3\times7=2^3\times3^2\times7$.

202. When the factors obtained are all primes, the number is said to be resolved or decomposed into its **prime or elementary factors**.

203. No number can be resolved into prime factors in more than **one** way.

204. *To resolve a number into its prime factors.*

RULE. Divide in succession by each of the primes 2, 3, 5, 7, 11, &c., which can be used as divisors, and in each case as often as

possible, until we obtain a quotient which is a prime; these divisors and the last quotient expressed in the form of a product make up the given number.

Ex. 1. Resolve 44856 into prime factors.

$$\begin{array}{r|l} 2^3=8)\underline{44856} & \\ 3^2=9)\underline{5607} & \\ 7)\underline{623} & \\ 89 & \end{array}$$

The last two digits form 56, which is divisible by 8; the sum of the digits $=4+4+8+5+6=27$. Hence the number is divisible by 8 and 9 or 2^3 and 3^2.

Also $623=7\times 89$, and that 89 is a prime.

$\therefore\ 44856=\underline{2^3\times 3^2\times 7\times 89}$.

Ex. 2. Decompose 8862777 into its prime factors.

$$\begin{array}{r} 3^2=9)\underline{8862777} \\ 3^2=9)\underline{984753} \\ 11)\underline{109417} \\ 7)\underline{9947} \\ 7)\underline{1421} \\ 7)\underline{203} \\ 29 \end{array}$$

The sum of the digits $=45$, which is divisible by 9 or 3^2; the sum of the digits of the quotient $=36$; also $(8+6+7+7)-(8+2+7)=11$. Hence the number is divisible by 9, 9 and 11.

Again, in 9947, we have $947-9=938$, which is divisible by 7; in like manner, again by 7, and 203 $=7\times 29$ and 29 is a prime.

$\therefore\ 8862777=9\times 9\times 11\times 7\times 7\times 7\times 29=\underline{3^4\times 11\times 7^3\times 29}$.

Examples LI.

1. Resolve *mentally* the following into elementary factors :—

(1) 6; 10; 14; 21; 35; 28; 45; 64; 81; 96; 72.

(2) 56; 30; 280; 144; 224; 285; 198; 176; 342.

2. Decompose the following numbers into their prime factors :—

(1) 320; 460; 462; 315; 612; 715; 846; 945; 735.

(2) 1188; 1309; 1827; 1331; 1456; 1485; 3675; 4620.

(3) 5250; 55020; 16632; 47089; 53599; 88725; 11025.

(4) 514250; 190463; 259811; 508079; 4149173; 4057690.

(5) 7507500; 73896433; 11176704; 119189070; 125023500.

3. Ascertain which of the following numbers are prime, and the prime factors of those which are composite :—

(1) 31; 53; 86; 96; 167; 132; 275; 480; 856; 873.

(2) 397; 289; 461; 727; 667; 851; 953; 971; 997.

(3) 1009; 1517; 1729; 4576; 2501; 4717; 3389.

4. Determine which of the following numbers are divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13 respectively :—

(1) 165; 216; 324; 425; 639; 936; 868; 512; 795.

(2) 3164 ; 4228 ; 11172 ; 2859 ; 11599 ; 14916 ; 53729.

(3) 1235 ; 6467 ; 38187 ; 123456 ; 777777 ; 601830.

(4) 2709344 ; 50707338 ; 6913580247 ; 726441196.

5. How many prime numbers are there between ?—

(1) 16 and 96. (2) 53 and 100. (3) 140 and 230.

(4) 330 and 350. (5) 556 and 600. (6) 790 and 1008.

6. By what numbers may 179, 313 and 799 be divided that the remainders may be 3, 5 and 7 respectively ?

II. GREATEST COMMON MEASURE.

205. A **common measure** or **common factor** of two or more numbers is *any* number, which will divide each of them without leaving a remainder.

Thus, each of the numbers 2, 3 and 6 is a *common measure* or *common factor* of 18 and 30, for each of the numbers 2, 3 and 6 divides 18 and 30 exactly.

206. The *greatest* number that divides each of two or more numbers exactly is called their **Greatest Common Measure (G. C. M.)** or **Highest Common Factor (H. C. F.)**

Thus, 6 is the *Greatest Common Measure* of 18 and 30, for it is the greatest number capable of dividing each of them exactly.

207. *If one number measure each of two others, it will measure their sum and difference ; also, any multiples of each, their sums and differences.*

Thus, 4 is a common measure of 20 and 12 ; and their sum $=20+12=32=4\times8$; their difference $=20-12=8=4\times2$; a multiple of $20=20\times5=100=4\times25$; of $12=12\times7=84=4\times21$;

also, $100+48=148=4\times37$; $100-48=52=4\times13$;

each of which evidently comprises the number 4 as a measure or factor ; and similarly of more numbers.

Examples LII.

Find, by *inspection*, the G. C. M. of :—

1. 4 and 6. **2.** 6 and 9. **3.** 8 and 12. **4.** 9 and 24.

5. 20, 32. **6.** 48, 27. **7.** 42, 28. **8.** 48, 54.

9. 91, 84. **10.** 30, 45. **11.** 45, 57. **12.** 42, 56.

13. 21, 28, 35. **14.** 30, 25, 45. **15.** 32, 40, 48. **16.** 30, 35, 40.

208. The G. C. M. of two or more numbers may often be found by resolving each number into its prime factors and then taking the product of **all** the prime factors common to them.

Ex. Find the G. C. M. of 63 and 168.

$$63=7\times 9=7\times 3\times 3\ ;\ 168=7\times 24=7\times 3\times 8=7\times 3\times 2\times 2\times 2.$$

Therefore the factors common to 63 and 168 are 7 and 3; hence the G. C. M. $=7\times 3=\underline{21}$. *Ans.*

209. In finding the G. C. M. of two or more numbers, it is sufficient to find the prime factors of **one** of the numbers, and then find by trial which of these factors divide each of the remaining numbers exactly; the product of all these common factors is the required G. C. M.

Ex. Find the G. C. M. of 492, 744 and 1044.

The prime factors of 492 are 2, 2, 3 and 41; of these factors 2, 2 and 3 divide 744 and 1044 exactly, but 41 does not divide them.

Hence, the required G. C. M. is $2\times 2\times 3$ or $\underline{12}$. *Ans.*

Examples LIII.

Find, by *method of factors*, the G. C. M. of :—

1. 45 and 72. **2.** 64 and 96. **3.** 48 and 72.
4. 56 and 140. **5.** 81 and 171. **6.** 74 and 259.
7. 205 and 287. **8.** 325 and 425. **9.** 230 and 414.
10. 490 and 546. **11.** 308 and 506. **12.** 247 and 323.
13. 1216 and 424. **14.** 620 and 2108. **15.** 45, 72 and 81.
16. 162, 729 and 4374. **17.** 1326, 3094 and 4420.
18. 372, 994 and 3132. **19.** 504, 5292 and 3040.
20. 102, 612, 476, 816 and 428.

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210. When numbers can easily be resolved into their prime factors we have shown in Art. 208, that their G. C. M. is formed by the product of the **least powers** of those factors which are common to all the given numbers, but when the numbers are large and their prime factors cannot be readily determined, we use a different method.

211. To find the G. C. M. of two numbers, whose prime factors cannot be readily ascertained, we use the following Rule.

RULE. Divide the greater of the numbers by the less, then the first divisor by the remainder, then the second divisor by the second remainder, and repeat this operation till there is no remainder; the **last divisor** will be the G. C. M. required.

Ex. Find the G. C. M. of 9756 and 8496.

```
8496)9756(1
     8496
     ----
     1260)8496(6
          7560
          ----
           936)1260(1
                936
                ---
                324)936(2
                    648
                    ---
                    288)324(1
                        288
                        ---
                         36)288(8
                            288
```

The first divisor is 8496 and the first remainder 1260.

The second divisor is 1260 and the second remainder 936.

The third divisor is 936 and the third remainder 324; and so on.

The final divisor is 36.

∴ the required G. C. M. is 36.

212. *To find the* G. C. M. *of three or more numbers.*

RULE. Find the G. C. M. of the first two numbers; then the G. C. M. of this G. C. M. and the third number; then the G. C. M. of this last G. C. M. and the fourth number; and continue this process to the last number; the **last G. C. M.** is the required G. C. M. of the given numbers.

Ex. Find the G. C. M. of 741, 1131, 1183 and 1989.

```
741)1131(1              39)1183(30            13)1989(153
    741                    117                   13
    ---                    ---                   --
    390)741(1               13)39(3               68
        390                    39                 65
        ---                                       --
        351)390(1                                  39
            351                                    39
            ---
             39)351(9
                351
```

∴ the required G. C. M. is 13. *Ans.*

Examples LIV.

Find the G. C. M. of :—

1. 126 and 444.	**2.** 646 and 950.	**3.** 54 and 258.
4. 366, 128.	**5.** 3556, 3444.	**6.** 5187, 5850.
7. 4833, 6237.	**8.** 9367, 14501.	**9.** 3252, 4248.
10. 2145, 3471.	**11.** 4081, 5141.	**12.** 1441, 1572.
13. 6441, 10283.	**14.** 13667, 14186.	**15.** 43365, 44688.
16. 12925, 63305.	**17.** 11050, 35581.	**18.** 109056, 179712.
19. 125075, 225025.	**20.** 105945, 945105.	**21.** 428571, 999999.
22. 143278, 1278142.	**23.** 385629, 7855323.	

24. 1257214, 11215246.
25. 703037, 5134083.
26. 3876519, 3101729671.
27. 41615795893, 877267019106.
28. 6186, 10310, 15465.
29. 12018, 20030, 30045.
30. 1617, 2871, 4213.
31. 13338, 14136, 15903.
32. 16442, 24663, 41105.
33. 2697, 3441, 1271.
34. 204, 1190, 1445, 2006.
35. 12558, 20769, 47403, 12581.
36. 5040, 23940, 28350, 31773.
37. 11573, 19397, 28036.
38. 70843288, 852706430 and 686138242.
39. 1070784, 1180608, 1455168 and 1520376.
40. 22680, 49140, 154980, 429660 and 925932.

---

213. Numbers which have no common measure greater than unity, are said to be **prime to each other**.

Thus, 15 and 29 are prime to each other.

Ex. Are 1726 and 1623 prime to each other.

```
1623)1726(1               78)103(1
     1623                    78
     ----                    --
      103)1623(15             25)78(3
          103                    75
          ---                    --
           593                    3)25(8
           515                      24
           ---                      --
            78                       1
```

∴ 1726 and 1623 are prime to each other.

214. Every common measure of two numbers is a measure of their G. C. M.

Thus, 2 and 3 being common measures of 18 and 30, is a measure of 6, the G. C. M. of 18 and 30.

215. The numbers of which the G. C. M. is required must refer to the same unit, and the G. C. M. refers to that unit.

Thus, the G. C. M. of *Rs.*429 and *Rs.*715 is *Rs.*143 ; the G. C. M. of 224 feet and 336 feet is 112 feet.

Examples LV.

1. Are the following numbers prime to each other :—

(1) 5789 and 7337. (2) 3375 and 5836. (3) 49561 and 97073.
(4) 58573 and 84329. (5) 9367 and 14501. (6) 19001 and 46253.
(7) 2698705 and 54987262. (8) 18432, 21952 and 42895.

2. Find the G. C. M. of :—

(1) 8029 and 23791. (2) 441441 and 844272. (3) 181896 and 147576. (4) 39835 and 162424. (5) 218707, 526769 and 695822.

Examples worked out.

Ex. 1. Find the greatest number that will divide 2293, 4245 and 5348 leaving the remainders 18, 20 and 23 respectively.

$2293-18=2275$; $4245-20=4225$; $5348-23=5325$.

The reqd. no. is the G. C. M. of 2275, 4225 and $5325=\underline{25}$. *Ans.*

Ex. 2. Two bills, one amounting to *Rs.*78. 12*a.* and the other to *Rs.*420 are to be paid in coins of one kind ; what is the largest coin that can be used ?

*Rs.*78. 12*a.* = 1260*a.* ; *Rs.*420 = 6720*a.*

∴ the largest coin required is the G. C. M. of 1260*a.* and 6720*a.*

= 420*a.* = $\underline{Rs.\ 26.\ 4a.}$ *Ans.*

Ex. 3. The sum of two numbers is 1144, and their G. C. M. is 143 ; how many pairs of such numbers can be formed ? Form them.

$1144 \div 143 = 8.$

Now $8=1+7=2+6=3+5=4+4$, and no more.

Of these parts the only pairs of numbers that are prime to each other are 1, 7 and 3, 5. Hence *only two* pairs of numbers can be formed.

$$\left.\begin{array}{l}\text{Thus, the first pair} = 1\times143 \text{ and } 7\times143, \text{ or } 143 \text{ and } 1001\text{ ;}\\ \text{and the 2nd pair} = 3\times143 \text{ and } 5\times143, \text{ or } 429 \text{ and } 715.\end{array}\right\} \textit{Ans.}$$

As regards the other pairs that can be formed, 143 will be a common measure, but not the G. C. M.

Ex. 4. The product of two numbers is 3240, and their G. C. M. is 18 ; how many pairs of such numbers can be formed ? Form them.

$3240 \div 18^2 = 10$; and $10 = 1\times10$ or 2×5.

Hence *only two* pairs of numbers can be formed.

$$\left.\begin{array}{l}\text{Thus, the 1st pair} = 18\times1 \text{ and } 18\times10, \text{ or } 18 \text{ and } 180\text{ ;}\\ \text{and the 2nd pair} = 18\times2 \text{ and } 18\times5 \text{ or } 36 \text{ and } 90.\end{array}\right\} \textit{Ans.}$$

Ex. 5. What number is that which, when divided by 6, the quotient again by 6, and that quotient again by 6, will give the G. C. M. of 35 and 135 ?

The G. C. M. of 35 and 135 is 5.

Now the question is, what number is that which, when divided by 6, the quotient again by 6, and that quotient again by 6, will give 5 ?

Since, 6, 6 and 6 are the three divisors and 5 the last quotient,

∴ the first dividend or the required number is $5\times(6\times6\times6)$

$=\underline{1080}.$ *Ans.*

Examples LVI.

1. What is the greatest sum of money contained exactly in *Rs.*34. 7*a.* 6*p.* and *Rs.*70. 12*a.* 6*p.*?

2. Find the greatest number that will divide 35 and 61, leaving remainders 3 and 5 respectively.

3. What number is that which, when divided by 12, the quotient again by 12, and that quotient again by 12, will give the G. C. M. of 148 and 772?

4. Find the greatest weight in grains, that will measure both pounds Avoir. and pounds Troy.

5. The sum of two numbers is 928, and their G. C. M. is 58; form as many pairs of numbers as convenient.

6. What is the greatest unit of time with which 15 hrs. 12 min. and 1 day 3 hrs. 33 min. can be both represented by integers?

7. Find the greatest number that will divide 1624, 2878 and 4220 leaving 7 as remainder after each division.

8. The product of two numbers is 5700, and their G. C. M. is 5 find as many pairs of numbers as convenient.

9. In working out a question in the G. C. M. of two numbers, I found the different remainders were 2388, 180, 48, 36 and 12, and the first two quotients 1 and 9; find the numbers, and the last three quotients.

10. In solving a question in the G. C. M. of two numbers, the quotients are 5, 1, 18, 1, 3, 1 and 2. The last divisor is 15. Find the numbers.

11. The sum of two numbers is 1394, and their G. C. M. is 34; how many pairs of numbers can be formed?

12. The product of two numbers is 4608, and their G. C. M. is 16; how many pairs of numbers can be formed?

13. What highest number will divide 287, 480 and 599 leaving the remainders 2, 5 and 10 respectively?

14. What is the greatest number by which, when 399, 695, 548, 1003 are divided, the respective remainders are 3, 2, 8 and 4?

15. Two bills, one of £4. 13*s.* 8*d.* and the other of £6. 9*s* 4*d.* are to be paid in the same coin. Find the largest coin that can be used.

16. *A* has *Rs.*679, *B Rs.*5901 and *C Rs.*6734; they agree to lay it out for sheep, at the highest price per head that will allow each exactly to invest his money; how much can they pay a head and how many can each purchase?

17. Find the two numbers nearest to 10000 that have 169 for their G. C. M.

18. A national school-master divided his scholars, consisting of 221 boys and 143 girls, into the largest possible equal classes so

that each class of boys should contain the same number as each class of girls. Find the number of classes.

19. A person wishes to distribute 805 mangoes, 1311 guavas, and 1978 plantains, equally among a number of beggars. Find the greatest number receiving the charity in this way.

20. A labourer was engaged for a certain number of days for *Rs.*10. 15*a.* 8*p.*, but being absent on some of these days he was paid only *Rs.*3. 3*a.* 8*p.*; shew that his daily wages cannot exceed 10*a.* 4*p.*

21. Find the greatest number of 4 digits and the least number of 5 digits that have 124 for their G. C. M.

22. Find the greatest and the least number of 6 digits that have 251 for their common measure. What is their G. C. M.?

III LEAST COMMON MULTIPLE.

216. A common multiple of two or more numbers is *any* number which is divisible by each of them separately.

Thus, 96 is a *common multiple* of 2, 3, 4, 6, 8 and 12, because it is divisible by each of them.

217. The **Least or Lowest Common Multiple** (L. C. M.) of two or more numbers is the *least* number that can be divided by each of them without a remainder.

Thus, 24 is the *Least Common Multiple* of 2, 3, 4, 6, 8 and 12, for it is the *least* number that the above numbers can divide without leaving a remainder.

218. The L. C. M. of two or more numbers may be obtained by resolving them into their prime factors, and taking the product of the **highest powers** of all the factors that are found in the given numbers.

Ex. Find the L. C. M. of 8, 12, 16, 20, 25 and 30.

$8=2\times2\times2=2^3$; $12=2\times2\times3=2^2\times3$;
$16=2\times2\times2\times2=2^4$; $20=2\times2\times5=2^2\times5$;
$25=5\times5=5^2$; $30=2\times3\times5=2\times3\times5$.

Here the factors that occur in the given numbers are 2, 3 and 5, of which the highest power of 2 is 2^4, and that of 5 is 5^2; therefore the L. C. M. is $2^4\times3\times5^2=16\times3\times25=\underline{1200}$. *Ans.*

Examples LVII.

1. Find *mentally* the L. C. M. of :—

(1) 6, 8. (2) 8, 16. (3) 10, 15. (4) 18, 30.
(5) 12, 27. (6) 10, 18. (7) 16, 24. (8) 12, 15.
(9) 3, 4, 5. (10) 2, 5, 7. (11) 3, 4, 16. (12) 5, 8, 20.
(13) 16, 12, 24. (14) 7, 10, 24. (15) 5, 12, 15. (16) 20, 40, 60.

2. Find, by *resolving into factors*, the L. C. M. of :—

(1) 12, 16, 18. (2) 16, 24, 30. (3) 24, 56, 84.
(4) 15, 35, 16, 56. (5) 25, 60, 84, 15. (6) 81, 27, 45, 18.
(7) 756, 6435. (8) 729, 1681. (9) 1008, 2064.
(10) 756, 350, 9075. (11) 735, 1575, 2205. (12) 225, 336, 360.
(13) 196, 350, 728, 924. (14) 11573, 19397, 28036.
(15) 72, 96, 144, 180, 450, 540. (16) 44, 126, 280, 198, 330.

---

219. To find the L. C. M. of two large numbers which cannot easily be resolved into prime factors, we use the following Rule.

RULE. Find the G. C. M. of the two numbers, and then multiply *either* of the numbers by the quotient arising from dividing the *other* by the G. C. M. The **product** will be the L. C. M. of the numbers.

Ex. Find the L. C. M. of 209 and 304.

Here, the G. C. M. is 19. Also $209 \div 19 = 11$.

$\therefore$ the L. C. M. $= 11 \times 304 = \underline{3344}$. *Ans.*

220. To find the L. C. M. of three or more numbers which cannot be readily resolved into factors, use the following Rule.

RULE. First find the L. C. M. of two of the numbers as in Art. 219; then the L. C. M. of this and another and so on, until all are taken. The **last L. C. M.** is the L. C. M. required.

Ex. Find the L. C. M. of 64, 250 and 432.

The G. C. M. of 64 and 250 is 2, and their L. C. M. is 8000.

The G. C. M. of 8000 and 432 is 16, and the L. C. M. is 216000.

Hence, the L. C. M. required $= \underline{216000}$. *Ans.*

Examples LVIII.

Find the L. C. M. of :—

1. 289, 323. **2.** 849, 1132. **3.** 508, 889.
4. 420, 798. **5.** 1287, 6281. **6.** 7247, 9365.
7. 12432, 36075. **8.** 15863, 21489. **9.** 24, 39, 376.
10. 84, 672, 472. **11.** 629, 851, 253. **12.** 64, 720, 960.
13. 1003, 2301, 4017. **14.** 14491, 16641, 3707.
15. 2523, 5887, 203, 8631. **16.** 1175, 4747, 5875, 9447.

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221. When the L. C. M. of several small numbers is required, the **easiest** method is that given by the following Rule.

RULE. Arrange the given numbers in a horizontal line from left to right, with a comma placed between every two. Divide by any one of the prime numbers 2, 3, 5, 7, 11 ...which will divide any two

at least of the given numbers exactly; set down the quotient so obtained and the undivided numbers in a line below, separated as before. Proceed in the same way with the numbers in the second, and each succeeding line, till we come to a line where no two numbers have a common divisor. The **product** of the numbers in the last line and of the several divisors is the L. C. M. of the given numbers.

Note. The work may often be shortened by *striking out* in the same line every number which exactly measures any other number in that line.

Ex. Find the L. C. M. of 2, 3, 8, 9, 15, 21 and 35.

$$\begin{array}{r|l} 2 & 2,\ 3,\ 8,\ 9,\ 15,\ 21,\ 35 \\ \hline 3 & 1,\ 3,\ 4,\ 9,\ 15,\ 21,\ 35 \\ \hline 5 & 1,\ 1,\ 4,\ 3,\ 5,\ 7,\ 35 \\ \hline 7 & 1,\ 1,\ 4,\ 3,\ 1,\ 7,\ 7 \\ \hline & 1,\ 1,\ 4,\ 3,\ 1,\ 1,\ 1 \end{array}$$

∴ the L. C. M. $= 2 \times 3 \times 5 \times 7 \times 4 \times 3$
$= \underline{2520}$ *Ans.*

$$\begin{array}{r|l} 3 & \cancel{2},\ \cancel{3},\ 8,\ 9,\ 15,\ 21,\ 35 \\ \hline & 8,\ 3,\ \cancel{5},\ \cancel{7},\ 35 \end{array}$$

∴ the L. C. M. $= 3 \times 8 \times 3 \times 35$
$= \underline{2520}$. *Ans.*

In the first line 2 is contained in 8, and 3 in 9, and ∴ struck off. In the second line 5 and 7 are both contained in 35, and ∴ struck off.

Examples LIX.

Find the L. C. M. of :—

1. 12, 15, 16. **2.** 8, 16, 20. **3.** 15, 25, 105.
4. 9, 15, 18, 20. **5.** 8, 12, 15, 20. **6.** 34, 68, 17, 2.
7. 16, 9, 12, 18. **8.** 36, 56, 75, 72. **9.** 81, 27, 45, 18.
10. 15, 35, 16, 56. **11.** 15, 20, 24, 21, 35. **12.** 24, 28, 36, 22, 16.
13. 3, 9, 7, 15, 28, 42. **14.** 8, 18, 28, 36, 54, 72, 90.
15. 9, 12, 15, 18, 21, 24, 27, 30. **16.** 32, 63, 25, 36, 42, 49, 84.
17. 12, 18, 28, 35, 60, 84, 100. **18.** 15, 16, 18, 20, 24, 25, 27, 30.
19. 48, 64, 27, 81, 33, 110, 165. **20.** 48, 64, 27, 33, 110, 165, 240.
21. 35, 52, 63, 77, 132, 117, 143. **22.** 27, 91, 42, 39, 63, 156, 234.
23. 27, 36, 54, 72, 84, 96, 215, 248, 324.
24. 18, 24, 35, 48, 56, 60, 72, 90, 120.
25. 7, 11, 21, 63, 91, 99, 117, 143.
26. 24, 35, 52, 60, 91, 108, 126, 156, 315.
27. 26, 30, 34, 39, 51, 65, 78, 85, 102, 195, 255.
28. 27, 87, 189, 126, 145, 210, 203, 261, 385.
29. 8, 9, 10, 11, 12, 14, 15, 18, 21, 24, 28, 35, 36, 40, 42, 44, 45, 50.
30. The first 12 numbers; the even numbers from 10 to 28 inclusive

---

222. Every common multiple of two numbers is a multiple of their L. C. M.

Thus, 48 a common multiple of 8 and 12 is a multiple of 24, the L. C. M. of 8 and 12.

223. *If* two numbers are prime to each other, their L. C. M. is their product.

Thus, the L. C. M. of 13 and 15 is $13 \times 15 = 195$.

224. Since the L. C. M. of two numbers is their product divided by their G. C. M. (Art. 219), therefore the L. C. M. $\times$ the G. C. M. of two numbers is equal to their product. Hence, if the G. C. M., the L. C. M., and one of the two numbers be given, we can find the other number by multiplying the G. C. M. and the L. C. M. and dividing the product by the given number.

Ex. The G. C. M. and the L. C. M. of two numbers are 11 and 11803 respectively, and one of them is 319; what is the other?

Here, the G. C. M. $\times$ the L. C. M. $= 11 \times 11803 = 129833$.

$\therefore$ the required number $= 129833 \div 319 = \underline{407}$. *Ans.*

225. (1) To find the *least* number that will contain each of two or more given numbers exactly.

RULE. The required *least* number is the L. C. M. of the given numbers.

Ex. 1. Find the least number that is divisible by 40, 63, 112.

The required number = the L. C. M. of 40, 63, $112 = \underline{5040}$. *Ans.*

Ex. 2. Five bells toll at intervals of 5, 8, 9, 10 and 12 seconds respectively; what interval will elapse between two of their successive tollings together?

The L. C. M. of 5, 8, 9, 10, 12 is 360.

$\therefore$ the required time = 360 sec. or $\underline{6}$ min. *Ans.*

(2) To find the *least* number which, when divided by each of several given numbers, leaves the same remainder.

RULE. Find the L. C. M. of the several given numbers and to it add the given remainder. The sum is the required *least* number.

Ex. Find the least number which, when divided by 4, 18, 21 and 20, leaves in each case a remainder 3.

The L. C. M. of 4, 18, 21 and 20 is 1260.

$\therefore$ the required number $= 1260 + 3 = \underline{1263}$. *Ans.*

Examples LX.

1. Find the least number which, when divided by 6, 8 and 9, gives in every case the remainder 5.

2. What is the smallest sum that can be paid either in guineas, or in half-crowns, or in florins or in half-sovereigns?

3. Five bells begin to toll simultaneously and they toll at intervals of 4, 6, 8, 9 and 10 seconds. After what time will they again toll simultaneously?

4. Find the least number which, when divided by 675, 1050 and 4368, will leave the same remainder 32.

5. Find the least weight that can be weighed by either pounds Avoir. or pounds Troy.

6. Six men fire at a target at intervals of 2, 5, 7, 10, 12 and 14 minutes respectively. After what time will they all *first* fire simultaneously, and how many times will each man have fired?

7. Seven bells are tolling, and they toll at intervals of 3, 5, 7, 8, 9, 10 and 12 seconds respectively. What interval will elapse between their once tolling together and tolling together again?

8. A can go round a circular course in 6 minutes, B in 8, C in 12, D in 15, and E in 18; if they all start together from the same place at the same time (7h. 13m. A. M.), when will they be together again?

9. Find the least sum of money that can be paid in pence, shillings, florins, half-crowns, crowns, sovereigns or half-sovereigns.

10. The G. C. M. and the L. C. M. of two numbers are 124 and 10540 respectively, and one of the numbers is 620; find the other.

11. A heap of pebbles can be made up exactly into groups of 25; but when made up into groups of 18, 27 and 32, there is always a remainder of 11; find the least number of pebbles such a heap can contain.

12. A basket contains a number of oranges ascertained to be between 500 and 900. If 2 fruits are taken away, the remainder may be distributed equally among 3, 4, 5, 6 or 7 boys. Find the number of oranges in the basket.

13. A book is divided into four parts, each part being divided into chapters. The number of pages in each part is the same. Each chapter in the first part contains 20 pages, each chapter in the second 40, each chapter in the third 60, and each chapter in the fourth 80. Find the number of pages and chapters in the book, the number of pages in the book is known to be between 900 and 1000.

14. Three horses are running round a race course of 5280 yards; the first horse runs 440 yards a minute, the second 352 yards and the third 264 yards; find the time between their once coming all together, and their coming all together again.

15. What is the least number which, when increased by 17, is divisible by 22, 25, 33, 44 and 45 separately?

16. The G. C. M. and the L. C. M. of two numbers are 19 and 49077 respectively, and one of them is 779; find the other.

17. What is the least number which, when diminished by 145 is exactly divisible by 24, 27, 28, 32, 36 and 56?

18. What is the least number which, when divided by all the digits except the first, leaves the remainder 1?

19. The G. C. M. of two numbers of 4 digits is 221, and their L. C. M. is 46189; determine the numbers.

20. Find all the numbers between 250 and 600 that have 1728 for their L. C. M.

21. Find the least sum of money that can be paid in coins worth either 8 pies, half-rupees, rupees, 5 *sikis*, 10 *sikis*, 14 *sikis*, *Rs.*5. 4*a.*, *Rs.*10. 8*a.*

22. There is an island 48 miles in circumference. Four persons *A*, *B*, *C* and *D* begin to walk continually round it starting from the same place at the same time. They walk 3, 4, 6 and 8 miles per hour respectively. How soon will they all be again together at the starting point?

23. Five men run round a circular park in 4, 5, 6, 7 and 8 hours respectively. If they all start at the same time from the same point, find the least number of hours in which they will again be at that point together.

24. Three round pillars are 10 ft. 5 in., 14 ft. 7 in. and 6 yds. 9 in. respectively in circumference; find the length of the shortest rope that can be wrapped round each an exact number of times.

25. The circumferences of the wheels of a carriage are 7 ft. 4 in. and 11 ft.; what is the least distance in which both the wheels will make an exact number of revolutions?

26 A cask is required to be exactly filled by any one of the following measures; 1 seer, 2 seers, 3 seers, 5 seers, 6 seers or 9 seers; find the smallest cask for this purpose.

27. I have travelled between 700 and 760 miles; had I travelled 20 miles less, I could have completed my journey in a train which goes at the rate of 40 miles an hour, or in a carriage which goes at the rate of 16 miles an hour, or on foot at the rate of 6 miles an hour in an exact number of hours. Find the distance I have travelled.

28. Find the least number of 8 digits that is divisible by 15, 18, 25, 35, 40 and 55 Also the greatest number of 5 digits that is divisible by 14, 20, 35, 45 and 75.

CHAPTER V.

The Doctrine of Fractions.

(USUALLY TERMED VULGAR FRACTIONS).

226. When a magnitude contains its unit a number of times exactly, the resulting number is called an **integer** or **whole number** (Art. 7). Hence all *whole numbers*, or *integers*, being supposed to be formed by the *repetition* of the unit, may therefore be regarded as the result of the *multiplication* of that element; but if the unit be considered capable of *division* into any number of *equal* portions, the quantities thence arising must be viewed in the light of *broken* magnitudes; and these are therefore termed **Fractions** or more generally, **Vulgar Fractions**, in order to distinguish them from fractions of a different *form*, whose nature will be discussed in the next chapter.

I. NOTATION AND NUMERATION OF FRACTIONS.

227. A **Fraction** denotes a part or parts of a unit; it is expressed in figures by two numbers placed one above the other with a bar or line between them.

228. If we suppose the *unit* to be divided into 2, 3, 4, 5, &c., equal portions, *one* of the portions in each case is represented by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., which may be regarded as the **primitive fractions** of their respective denominations and are called the **reciprocals** of the natural numbers 2, 3, 4, 5, &c.; also the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., are read, *one-half*, *one-third*, *one-fourth*, *one-fifth*, &c.

229. If *two or more* of these equal portions be taken together, the *aggregates* thence arising are expressed by repeating the unit as *often* as such portions are repeated, in the *form* of their sum, the number below the line remaining the same.

Thus, if the primitive fraction $\frac{1}{3}$ be taken *twice* there will arise a new fraction expressed by $\frac{2}{3}$; if $\frac{1}{4}$ be repeated *thrice*, there results a new fraction expressed by $\frac{3}{4}$; again, if $\frac{1}{5}$ be taken *four times*, the new fraction will be $\frac{4}{5}$: and similarly of all the other primitive fractions; also, the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, &c., are read *two-thirds*, *three-fourths*, *four-fifths*, &c., and all quantities of this *form* are called **Simple Fractions.**

230. Hence, the number *below* the line denotes the number of equal portions into which the unit is supposed to be divided, and is therefore called the **denominator**; and the number *above* the line expressing the number of such equal portions intended to be taken, is therefore termed the **numerator.** The numerator and denominator are called the **terms** of a fraction.

Thus, of the fraction $\frac{5}{7}$, whose *terms* are 5 and 7, the denominator 7 implies that the unit is supposed to be divided into *seven* equal portions; and the numerator 5 shews that *five* of such equal portions are here the object of our consideration.

231. The sum of a whole number and a fraction is called a **Mixed number**; as, $4+\frac{5}{7}$ or rather $4\frac{5}{7}$; for the addition sign is almost always omitted.

232. From what has been said above, it appears, that a fraction expressed in figures is read by first reading the numerator and then the denominator with the termination "ths"; thus $\frac{5}{7}$ is read five-sevenths. The exceptions are that fractions with denominator 2 or 3 are read as so many *halves* or *thirds*, and with denominator 4 as so many *quarters* as well as *fourths*. A mixed number is read by connecting the integer and the fraction by "*and*"; thus, $4\frac{5}{7}$ is read four *and* five-sevenths.

233. From Art. 230, it follows, that if the numerator be less than the denominator, the value of the fraction is less than the unit; if the numerator be equal to the denominator, the value of the frac-

tion is the unit; and if the numerator be greater than the denominator, the value of the fraction is greater than the unit.

234. Every whole number or integer may be expressed as a fraction whose denominator is 1.

Thus, $7=\frac{7}{1}$, for the unit is divided into 1 part, comprising the whole unit, and 7 of such parts, that is 7 units, are taken.

235. A fraction also expresses the quotient of the numerator by the denominator.

Thus, $\frac{5}{7}=5\div7$; since 1 unit is 7-sevenths, therefore 5 units is 35-sevenths, and therefore 5 divided by 7 is 35-sevenths divided by 7, and is therefore 5-sevenths; that is, $5\div7=\frac{5}{7}$. Hence $\frac{5}{7}$ is not only read 5 *sevenths* but also 5 *by* 7.

Similarly, $\frac{8}{4}=8\div4=2$; $\frac{7}{7}=7\div7=1$; and so on.

236. From the last Art. it follows, that if we multiply a fraction by its denominator we get its numerator.

Thus, since $\frac{5}{7}$ is the seventh part of 5, $\frac{5}{7}$ repeated 7 times gives 5, or $\frac{5}{7}\times7=5$; and 5 may therefore be expressed in a *Fractional Form* by $\frac{5}{1}$.

237. If we take a fractional magnitude, and considering it as a new unit, divide it into any number of equal parts and take one or more of these parts, we shall obtain a **fraction of a fraction**; as $\frac{4}{7}$ of $\frac{3}{8}$.

238. When fractions are represented in the manner above explained, they are called **Vulgar Fractions**, (*i.e.*) *common* or *ordinary* fractions.

239. We make the following distinctions in fractions :—

(1) A **proper fraction** is one in which the numerator is less than the denominator; thus $\frac{3}{4}$, $\frac{4}{7}$, $\frac{3}{8}$ are *proper* fractions.

(2) An **improper fraction** is one in which the numerator is either equal to or greater than the denominator; thus $\frac{7}{7}$, $\frac{9}{5}$, $\frac{13}{4}$ are *improper* fractions.

(3) A **simple fraction** is one in which numerator and denominator are both whole numbers. thus $\frac{3}{4}$, $\frac{10}{7}$ are *simple* fractions.

(4) A **compound fraction** is a fraction of a fraction; thus $\frac{3}{4}$ of $\frac{5}{9}$, $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{10}{9}$ are *compound* fractions.

(5) A **complex fraction** is one in which numerator or denominator or both are not whole numbers; thus $\frac{\frac{2}{5}}{\frac{3}{7}}$, $\frac{2\frac{1}{2}}{8}$, $\frac{3}{4\frac{1}{2}}$, $\frac{2\frac{1}{7}}{3\frac{1}{5}}$, $\frac{2\frac{1}{3}+1\frac{4}{5}}{2\frac{1}{3}-1\frac{4}{5}}$ are *complex* fractions.

240. The **reciprocal** of a fraction is the fraction formed by interchanging its terms; thus the *reciprocal* of $\frac{5}{7}$ is $\frac{7}{5}$; of 5 or $\frac{5}{1}$ is $\frac{1}{5}$.

241. We are hence enabled to find the results of the multiplication and division of a fraction by an integer, and these may be integers or fractions.

(1) To *multiply* a fraction by a whole number, only multiply the *numerator* by it.

Thus, $\frac{4}{13} \times 3 = \frac{4 \times 3}{13} = \frac{12}{13}$; because in $\frac{12}{13}$, *three times* as many parts of the unit are implied, as there are in $\frac{4}{13}$.

(2) To *divide* a fraction by a whole number, only multiply the *denominator* by it.

Thus, $\frac{2}{7} \div 5 = \frac{2}{7 \times 5} = \frac{2}{35}$; because the same number of parts are indicated in $\frac{2}{7}$ and $\frac{2}{35}$, and each part in the former is *five times* as great as each part in the latter, by Art. 230.

Examples LXI.

1. What fraction do we form in dividing a unit into 13 equal parts, and taking 11 of them; into 1000 equal parts, and taking 101?

2. Express in figures :—

One seventh; one quarter; seven halves; thirty-four thirds; forty-five seventy-ninths; seven-eighths; seven, and a half; nine, and seven-ninths; sixteen, and four twenty-oneths; two hundred, and three-elevenths; ninety-four, and five-seventeenths.

3. Express in words :—

$\frac{2}{3}$, $\frac{3}{4}$, $\frac{8}{9}$, $\frac{16}{17}$, $\frac{87}{100}$, $3\frac{4}{7}$, $8\frac{11}{16}$, $24\frac{27}{44}$ and $125\frac{98}{1000}$.

4. Multiply :—

(1) $\frac{4}{5}$ and $\frac{24}{37}$ each separately by 2, 3, 5, 7, 9, 11, 12, 13 and 18.

(2) $\frac{15}{23}$ and $\frac{87}{109}$ ... . 36, 68, 80, 95, 112 and 157.

5. Divide :—

(1) $\frac{5}{7}$ and $\frac{8}{13}$ each separately by 2, 3, 5, 7, 9, 11, 12, 13 and 18.

(2) $\frac{19}{37}$ and $\frac{26}{139}$ .. .. 36, 68, 80, 95, 112 and 157.

II. TRANSFORMATION OF FRACTIONS.

242. *If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the value of the fraction will not be altered.*

For, if the fraction $\frac{3}{7}$ be multiplied by 5, the product is $\frac{15}{7}$; and again if this be divided by 5, the quotient is $\frac{15}{35}$, by Art. 241; but since these two operations are the *reverse* of and therefore *neutralize* each other, it follows that—

$$\frac{3}{7} = \frac{15}{35} = \frac{3 \times 5}{7 \times 5}; \text{ and also, that } \frac{15}{35} = \frac{3}{7} = \frac{15 \div 5}{35 \div 5}.$$

243. It is clear from the above, that a whole number may be expressed in the form of a fraction with *any* denominator we please.

$$\text{Thus, } 5=\frac{5}{1}=\frac{5\times 2}{1\times 2}=\frac{10}{2}=\frac{20}{4}=\frac{35}{7}=\&c.$$

Also, a fraction may be transformed into another with a *given* denominator or numerator, provided it be a *multiple* or *sub-multiple* of the denominator or numerator of the proposed fraction.

Ex. 1. Convert $\frac{7}{8}$ into a fraction with 96 for its denominator, and reduce $\frac{24}{40}$ to a fraction with denominator 5.

$$(1)\ \frac{7}{8}=\frac{7\times 12}{8\times 12}=\frac{84}{96}; \qquad (2)\ \frac{24}{40}=\frac{24\div 8}{40\div 8}=\frac{3}{5}.$$

Ex. 2. Convert $\frac{5}{6}$ into a fraction with numerator 55, and $\frac{56}{64}$ into a fraction with numerator 7.

$$(1)\ \frac{5}{6}=\frac{5\times 11}{6\times 11}=\frac{55}{66} \qquad (2)\ \frac{56}{64}=\frac{56\div 8}{64\div 8}=\frac{7}{8}.$$

244. Since $\frac{5}{8}\times 4=\frac{20}{8}=\frac{5\times 4}{2\times 4}=\frac{5}{2}$;

therefore, *to multiply* a fraction by an integer, it appears to be immaterial whether the numerator be multiplied, or the denominator be divided, by it; and since

$$\frac{8}{9}\div 4=\frac{8}{36}=\frac{2\times 4}{9\times 4}=\frac{2}{9}:$$

therefore, *to divide* a fraction by a whole number, it amounts to the same thing whether we multiply the denominator, or divide the numerator by it.

245. Now, referring to Art. 241, we see that we have a choice of two methods both in the multiplication and division of a fraction by an integer, and we prefer the latter in accordance with the direction: "*Divide when you can, multiply when you are obliged.*"

Examples LXII.

1. Reduce each of the whole numbers 3, 5, 7, 8, 15, 18, 20, 25 to fraction with the denominator 13.

2. Convert 26, 117 and 125 into fractions with denominators 13, 25 and 35 respectively.

3. Convert $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{8}$ and $\frac{7}{15}$ into fractions having 120 for their denominator.

4. Express $\frac{14}{17}$, $\frac{18}{23}$, $\frac{21}{32}$ and $\frac{27}{35}$ as fractions having 756 for their common numerator.

5. Express $\frac{20}{45}$, $\frac{28}{63}$, $\frac{36}{81}$, $\frac{48}{108}$ and $\frac{60}{135}$ each as a fraction with denominator 9. Also express each as a fraction with numerator 5040.

6. Convert $\frac{48}{60}$, $\frac{81}{120}$, $\frac{98}{210}$, $\frac{396}{504}$ and $\frac{3341}{5140}$ into equivalent fractions with denominators 5, 40, 15, 14 and 20 respectively.

246. *To express a mixed number as an improper fraction.*

RULE. Multiply the integer by the denominator of the fraction ; to the product add the numerator and the result will be the new numerator, which placed over the given denominator will form the *improper* fraction required.

Ex. Represent $3\frac{4}{5}$ as an improper fraction.

$3\frac{4}{5}=\frac{3\times 5+4}{5}=\frac{15+4}{5}$	For $3\frac{4}{5}=\frac{3}{1}+\frac{4}{5}=\frac{3\times 5}{1\times 5}+\frac{4}{5}$ (Art. 242.)
$=\frac{19}{5}$. *Ans.*	$=\frac{15}{5}+\frac{4}{5}=\frac{19}{5}$. (Art 229.)

247. *To represent an improper fraction as a whole or mixed number.*

RULE. Divide the numerator by the denominator, and the quotient will be the integral part : and the fractional part will be formed by placing the remainder over the given denominator. If there be no remainder, the fraction is equivalent to the integer thus found.

Ex. Reduce $\frac{32}{8}$ and $\frac{327}{11}$ to whole or mixed numbers.

(1) $8)\underline{32}$ 4 $\therefore \frac{32}{8}=4$. *Ans.*	For $\frac{32}{8}=\frac{8\times 4}{8\times 1}=\frac{4}{1}$ (Art. 242)$=4$. (Art. 234.)
(2) $11)\underline{327}$ $29 \ldots 8$ $\therefore \frac{327}{11}=29\frac{8}{11}$. *Ans.*	For $\frac{327}{11}=\frac{319+8}{11}=\frac{319}{11}+\frac{8}{11}$ (Art. 229.) $=29+\frac{8}{11}=29\frac{8}{11}$. (Art. 231.)

248. The **complete quotient** of one number divided by another is the mixed number obtained by the above Rule.

Thus, the *complete quotient* of 79 divided by 15 is the mixed number $5\frac{4}{15}$, for $79\div 15=\frac{79}{15}=5\frac{4}{15}$.

Examples LXIII.

1. Express *orally* the following as improper fractions :—

(1) $1\frac{1}{3}$; $2\frac{1}{12}$; $3\frac{1}{15}$; $8\frac{2}{3}$; $9\frac{4}{7}$; $6\frac{4}{9}$; $5\frac{3}{8}$; $7\frac{5}{14}$; $4\frac{3}{14}$; $9\frac{7}{16}$.
(2) $13\frac{13}{14}$; $15\frac{9}{17}$; $16\frac{13}{15}$; $19\frac{7}{16}$; $14\frac{13}{16}$; $20\frac{11}{18}$; $17\frac{9}{17}$.

2. Convert into improper fractions :—

(1) $121\frac{5}{11}$; $54\frac{8}{11}$; $41\frac{7}{13}$; $123\frac{4}{17}$; $156\frac{5}{13}$; $95\frac{15}{16}$; $22\frac{7}{18}$.
(2) $275\frac{14}{15}$; $374\frac{64}{163}$; $344\frac{24}{29}$; $101\frac{13}{17}$; $49\frac{74}{87}$; $19\frac{18}{99}$; $44\frac{9}{59}$.
(3) $704\frac{12}{125}$; $5\frac{7}{680}$; $148\frac{237}{465}$; $25\frac{389}{2400}$; $685\frac{210}{999}$; $9879\frac{1457}{9879}$.

3. Express *orally* as mixed or whole numbers :—

(1) $\frac{7}{3}$; $\frac{5}{2}$; $\frac{16}{4}$; $\frac{13}{3}$; $\frac{47}{7}$; $\frac{19}{5}$; $\frac{38}{9}$; $\frac{76}{8}$; $\frac{750}{59}$; $\frac{21}{8}$; $\frac{24}{11}$; $\frac{19}{7}$.
(2) $\frac{149}{11}$; $\frac{199}{12}$; $\frac{91}{13}$; $\frac{231}{14}$; $\frac{132}{11}$; $\frac{143}{12}$; $\frac{187}{17}$; $\frac{112}{14}$; $\frac{81}{18}$.

4. Represent the following as mixed or whole numbers :—

(1) $\frac{440}{13}$; $\frac{2417}{19}$; $\frac{3797}{29}$; $\frac{9999}{31}$; $\frac{30471}{37}$; $\frac{523}{23}$; $\frac{747}{45}$; $\frac{775}{31}$.

(2) $\frac{3003}{217}$; $\frac{4521}{171}$; $\frac{6984}{481}$; $\frac{52504}{572}$; $\frac{51637}{152}$; $\frac{9999}{347}$; $\frac{19585}{144}$.

(3) $\frac{76845}{999}$; $\frac{830526}{9891}$; $\frac{907133}{7816}$; $\frac{4003187}{99999}$; $\frac{1516461}{30125}$; $\frac{1001010111}{100001}$.

5. Express the reciprocals of the following fractions as mixed numbers :—

$$\frac{7}{15};\ \frac{15}{49};\ \frac{17}{65};\ \frac{100}{6874};\ \frac{87}{3415};\ \frac{99}{4567};\ \frac{152}{51847};\ \frac{1251}{59956}.$$

6 Express $41\frac{7}{8}$, $25\frac{11}{12}$, $9\frac{15}{16}$, and $10\frac{9}{20}$ as fractions, with denominators 240 and 720.

7. Find the respective values of :—

(1) $\frac{17}{24}\times 8$; $\frac{13}{17}\times 17$; $6\frac{13}{35}\times 7$; $10\frac{8}{17}\times 17$; $6\frac{10}{11}\times 11$; $2\frac{11}{65}\times 13$.

(2) $\frac{45}{46}\div 9$; $\frac{14}{15}\div 7$; $\frac{117}{120}\div 13$; $\frac{143}{208}\div 11$; $6\frac{6}{7}\div 12$; $9\frac{12}{17}\div 15$.

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249. *To express a compound fraction as a simple one.*

A **Compound Fraction** is made up of two or more simple fractions connected by the word **of** ; as $\frac{1}{3}$ of $1\frac{4}{5}$ of $3\frac{6}{7}$.

RULE. Multiply all the numerators together for the numerator of the simple fraction, and all the denominators together for its denominator.

Ex. 1. Convert $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ into a simple fraction.

$$\tfrac{1}{3} \text{ of } \tfrac{4}{5} \text{ of } \tfrac{6}{7} = \frac{1\times 4\times 6}{3\times 5\times 7} = \tfrac{24}{105}.\ \textit{Ans.}$$

$$\left.\begin{array}{l} \text{For } \tfrac{1}{5} \text{ of } \tfrac{6}{7} = \tfrac{6}{7}\div 5 = \tfrac{6}{35} \\ \text{and } \tfrac{4}{5} \text{ of } \tfrac{6}{7} = \tfrac{6}{35}\times 4 = \tfrac{24}{35} \\ \text{whence, } \tfrac{1}{3} \text{ of } \tfrac{4}{5} \text{ of } \tfrac{6}{7} = \tfrac{1}{3} \text{ of } \tfrac{24}{35} = \tfrac{24}{35}\div 3 = \tfrac{24}{105}. \end{array}\right\} \text{(Art. 241.)}$$

$$\therefore \tfrac{1}{3} \text{ of } \tfrac{4}{5} \text{ of } \tfrac{6}{7} = \frac{1\times 4\times 6}{3\times 5\times 7} = \frac{24}{105}.$$

Note 1. Before applying the above Rule mixed numbers must be expressed as improper fractions.

Note 2. If there are factors common to both numerator and denominator, they may be *cancelled* or struck out, before obtaining the final result ; for this is in fact simply dividing the numerator and denominator of a fraction by the same number. (Art. 242.)

Ex. 2. Reduce $\frac{3}{5}$ of $2\frac{1}{12}$ of $5\frac{1}{15}$ to a simple fraction.

$$\tfrac{3}{5} \text{ of } 2\tfrac{1}{12} \text{ of } 5\tfrac{1}{15} = \tfrac{3}{5} \text{ of } \tfrac{25}{12} \text{ of } \tfrac{76}{15} = \frac{3\times(5\times 5)\times(4\times 19)}{5\times(3\times 4)\times(3\times 5)}$$

$$= \frac{3\times 5\times 5\times 4\times 19}{5\times 3\times 4\times 3\times 5} = \tfrac{19}{3} = 6\tfrac{1}{3}.\ \textit{Ans.}$$

(dividing numerator and denominator by the factors 3, 5, 5, 4 common to both).

Examples LXIV.

Reduce the following compound fractions to simple ones :—

1. $\frac{3}{4}$ of $\frac{4}{5}$; $\frac{4}{5}$ of $\frac{10}{12}$; $\frac{3}{16}$ of $\frac{8}{15}$; $\frac{5}{6}$ of $\frac{21}{8}$; $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{4}{7}$ of $\frac{10}{3}$.

2. $\frac{22}{5}$ of $\frac{8}{9}$; $\frac{3}{5}$ of $\frac{30}{14}$; $\frac{5}{6}$ of $\frac{10}{3}$ of 9 ; $\frac{4}{7}$ of $\frac{15}{4}$ of $\frac{7}{2}$; $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{25}{2}$.

3. $\frac{9}{7}$ of $\frac{5}{9}$ of $\frac{3}{8}$ of $\frac{3}{5}$; $\frac{3}{4}$ of $\frac{4}{7}$ of $\frac{7}{11}$ of $\frac{11}{5}$ of $\frac{5}{7}$; $\frac{4}{7}$ of $\frac{14}{12}$ of $\frac{6}{5}$ of $\frac{10}{12}$.

4. $\frac{1}{2}$ of $\frac{5}{6}$ of 4 of $\frac{2}{3}$ of $\frac{5}{7}$ of 6 ; $\frac{2}{5}$ of $\frac{5}{8}$ of $\frac{3}{16}$ of $\frac{9}{7}$ of $\frac{5}{8}$ of $\frac{7}{2}$.

5. $\frac{2}{3}$ of $\frac{3}{8}$ of $\frac{5}{9}$ of $\frac{2}{5}$ of $\frac{34}{11}$ of $\frac{55}{6}$. **6.** $\frac{21}{5}$ of $\frac{15}{4}$ of 10 of $\frac{5}{2}$ of $\frac{3}{5}$ of $\frac{22}{3}$ of $\frac{2}{13}$.

7. $\frac{7}{31}$ of $\frac{58}{9}$ of $\frac{35}{2}$ of $\frac{23}{9}$ of $\frac{205}{29}$. **8.** $\frac{13}{3}$ of $\frac{14}{15}$ of $\frac{30}{16}$ of $\frac{24}{7}$ of $\frac{35}{42}$ of $\frac{98}{171}$.

9. $\frac{6}{7}$ of $\frac{11}{12}$ of 9 of $\frac{49}{8}$ of $\frac{12}{13}$ of $\frac{5}{3}$ of $\frac{26}{35}$ of $\frac{75}{64}$ of $\frac{8}{21}$ of $\frac{2}{11}$.

10. $\frac{3}{5}$ of $\frac{3}{11}$ of $\frac{20}{3}$ of $\frac{3}{7}$ of $\frac{19}{9}$ of $\frac{42}{19}$ of $\frac{3}{2}$ of $\frac{5}{2}$ of $\frac{15}{5}$ of $\frac{1}{20}$ of $\frac{11}{8}$.

11. $15\frac{7}{11}$ of $8\frac{4}{5}$ of $13\frac{3}{7}$. **12.** $\frac{3}{14}$ of $4\frac{3}{8}$ of $\frac{6}{37}$ of $6\frac{8}{11}$ of $\frac{7}{82}$.

13. $5\frac{1}{9}$ of $\frac{20}{23}$ of $7\frac{1}{5}$ of $\frac{15}{29}$ of $7\frac{1}{4}$. **14.** $5\frac{1}{2}$ of $\frac{7}{22}$ of $1\frac{3}{14}$ of $3\frac{1}{5}$.

15. $4\frac{1}{3}$ of $\frac{3}{10}$ of $3\frac{1}{13}$ of $\frac{3}{4}$ of $\frac{5}{7}$ of $5\frac{1}{3}$ of $\frac{9}{25}$ of $\frac{4}{6}$ of $2\frac{3}{16}$.

16. $5\frac{1}{4}$ of $\frac{19}{25}$ of $\frac{4}{7}$ of $3\frac{4}{7}$ of $\frac{1}{5}$ of $\frac{5}{16}$ of $\frac{5}{8}$ of $\frac{7}{10}$ of $\frac{7}{11}$ of $1\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{7}$.

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250. A fraction is in its **lowest terms**, or in its **simplest form** when there are no factors common to both numerator and denominator. This will be the case when the numerator and denominator are **prime** to each other.

251. *To reduce a fraction to its lowest terms.*

RULE. Divide the numerator and denominator by their G. C. M.

Ex. Express the fraction $\frac{825}{960}$ in its lowest terms.

The G. C. M. of 825 and 960 is 15.

$$\therefore \frac{825}{960} = \frac{825 \div 15}{960 \div 15} = \frac{55}{64}. \quad \textit{Ans.}$$

252. In many instances, it is unnecessary to find the G. C. M. at *first*, the fractions being reducible to lower terms by successive divisions of the numerators and denominators by common factors discovered by *inspection*, or by employing the tests of divisibility given in Art. 198.

Ex. Reduce $\frac{4968}{5904}$ to its lowest terms.

$$\frac{4968}{5904} = \frac{2484}{2952} = \frac{1242}{1476} = \frac{621}{738} = \frac{207}{246} = \frac{69}{82}. \quad \textit{Ans.}$$

from *three* successive divisions of the numerator and denominator by 2, and then from *two* successive divisions by 3 ; and these are the terms which would have been obtained from dividing at *once* by 72 which is their G. C. M.

253. In examples like the following, it is convenient to break

up both the numerator and denominator into factors, and then *cancel* those which are common to both.

Note. It should be remarked that when a factor is *cancelled* it is to be replaced by 1 and not by 0.

Ex. Reduce $\frac{35 \times 63}{60 \times 77}$ to its lowest terms.

$$\frac{35 \times 63}{60 \times 77} = \frac{(5 \times 7) \times (3 \times 3 \times 7)}{(3 \times 5 \times 4) \times (7 \times 11)} = \frac{5 \times 7 \times 3 \times 3 \times 7}{3 \times 5 \times 7 \times 4 \times 11} = \frac{3 \times 7}{4 \times 11} = \frac{21}{44}. \quad \textit{Ans.}$$

Examples LXV.

1. Reduce to their lowest terms (by *inspection*) :—

(1) $\frac{3}{9}$; $\frac{4}{12}$; $\frac{5}{15}$; $\frac{7}{28}$; $\frac{4}{6}$; $\frac{10}{15}$; $\frac{14}{21}$; $\frac{9}{24}$; $\frac{9}{27}$; $\frac{24}{87}$; $\frac{80}{96}$; $\frac{27}{45}$; $\frac{48}{56}$.

(2) $\frac{28}{63}$; $\frac{64}{84}$; $\frac{18}{24}$; $\frac{24}{28}$; $\frac{26}{39}$; $\frac{42}{48}$; $\frac{21}{28}$; $\frac{36}{48}$; $\frac{46}{58}$; $\frac{56}{94}$; $\frac{138}{150}$; $\frac{96}{144}$.

(3) $\frac{77}{121}$; $\frac{25}{100}$; $\frac{75}{100}$; $\frac{63}{144}$; $\frac{72}{108}$; $\frac{35}{135}$; $\frac{39}{52}$; $\frac{46}{115}$; $\frac{76}{114}$; $\frac{75}{135}$.

2. Reduce the following fractions to their lowest terms :—

(1) $\frac{435}{957}$; $\frac{203}{315}$; $\frac{455}{1645}$; $\frac{256}{1024}$; $\frac{444}{703}$; $\frac{925}{1025}$; $\frac{768}{2592}$; $\frac{1476}{1764}$.

(2) $\frac{3094}{3042}$; $\frac{3444}{3556}$; $\frac{5239}{6076}$; $\frac{5565}{8533}$; $\frac{7568}{9504}$; $\frac{1775}{2350}$; $\frac{3565}{4930}$; $\frac{9050}{17919}$.

(3) $\frac{1261}{1649}$; $\frac{6435}{7293}$; $\frac{1236}{4764}$; $\frac{6006}{8008}$; $\frac{9504}{10692}$; $\frac{7497}{15729}$; $\frac{48510}{49005}$.

(4) $\frac{8991}{10989}$; $\frac{12540}{21945}$; $\frac{13478}{16701}$; $\frac{8398}{29393}$; $\frac{43365}{44688}$; $\frac{13667}{14186}$; $\frac{217800}{245025}$.

(5) $\frac{11050}{35581}$; $\frac{20301}{33633}$; $\frac{714285}{999999}$; $\frac{109375}{1000000}$; $\frac{135795}{222210}$; $\frac{99715}{113960}$.

(6) $\frac{95469}{359784}$; $\frac{180194}{1973594}$; $\frac{256417}{7006987}$; $\frac{1854432}{3171276}$; $\frac{1832051}{2592525}$; $\frac{496606401}{1006110363}$.

3. Reduce (by *cancelling*) to their lowest terms :—

(1) $\frac{18 \times 32}{27 \times 52}$; $\frac{16 \times 45}{24 \times 75}$; $\frac{21 \times 24}{56 \times 84}$; $\frac{45 \times 70 \times 15}{81 \times 90 \times 100}$; $\frac{48 \times 64 \times 49}{96 \times 88 \times 63}$.

(2) $\frac{51 \times 39 \times 42}{68 \times 52 \times 70}$; $\frac{19 \times 23 \times 26 \times 56}{57 \times 92 \times 78 \times 98}$; $\frac{85 \times 84 \times 38}{102 \times 154 \times 95}$; $\frac{76 \times 87 \times 65}{114 \times 145 \times 143}$.

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254. *To reduce two or more fractions having different denominators to equivalent fractions having a common denominator.*

RULE. Multiply each numerator by all the denominators *except* the one placed under it, for the new numerator : and multiply all the denominators together for the common denominator.

Ex. Express $\frac{1}{2}$, $\frac{2}{5}$ and $\frac{3}{7}$ as equivalent fractions with a common denominator.

Here, first $\left.\begin{array}{l}1\times5\times7=35\\2\times2\times7=28\\3\times2\times5=30\end{array}\right\}$ the new numerators : For $\frac{1}{2}=\frac{1\times5\times7}{2\times5\times7}=\frac{35}{70}$, $\frac{2}{5}=\frac{2\times2\times7}{5\times2\times7}=\frac{28}{70}$,

and $2\times5\times7=70$, the com. denr. : and $\frac{3}{7}=\frac{3\times2\times5}{7\times2\times5}=\frac{30}{70}$.

∴ the equivalent fractions are $\frac{35}{70}$, $\frac{28}{70}$ and $\frac{30}{70}$. *Ans.*

255. If two or more of the denominators have a common measure, the equivalent fractions may be expressed in simpler terms than obtainable by the above Rule, and having a least common denominator (L. C. D.) by the following Rule.

RULE. Find the L. C. M. of the denominators ; this will be the least common denominator. Then divide the L. C. M. so found by the denominator of each fraction, and multiply each quotient so found into the numerator of the fraction which belongs to it for the new numerator of that fraction.

Note. Before applying the above Rules, reduce mixed numbers to improper fractions, and compound fractions to simple ones ; moreover, if the L. C. D. be required, the given fractions should be reduced to their lowest terms.

Ex. Reduce $\frac{4}{5}$, $\frac{11}{12}$ and $\frac{3}{20}$ to equivalent fractions having the least common denominator.

The L. C. M. of 5, 12 and 20 is 60, which is here the L. C. D.

$60\div5=12$; $60\div12=5$; $60\div20=3$.

$$\therefore\ \frac{4}{5}=\frac{4\times12}{5\times12}=\frac{48}{60};\ \frac{11}{12}=\frac{11\times5}{12\times5}=\frac{55}{60};\ \frac{3}{20}=\frac{3\times3}{20\times3}=\frac{9}{60}.$$

Hence, the equivalent fractions are $\frac{48}{60}$, $\frac{55}{60}$ and $\frac{9}{60}$. *Ans.*

256. Similarly we can reduce fractions to equivalent ones having a least common numerator (L. C. N.).

Ex. Reduce $\frac{5}{6}$, $\frac{4}{9}$, $\frac{8}{9}$ and $\frac{16}{17}$ to fractions having a least common numerator.

The L. C. M. of 5, 4, 8 and 16=80, which is here the L. C. N.

$80\div5=16$; $80\div4=20$; $80\div8=10$; $80\div16=5$.

$$\therefore\ \frac{5}{6}=\frac{5\times16}{6\times16}=\frac{80}{96};\ \frac{4}{9}=\frac{4\times20}{9\times20}=\frac{80}{180};$$

$$\frac{8}{9}=\frac{8\times10}{9\times10}=\frac{80}{90};\ \frac{16}{17}=\frac{16\times5}{17\times5}=\frac{80}{85}.$$

∴ the fractions with a L. C. N. are $\frac{80}{96}$, $\frac{80}{180}$, $\frac{80}{90}$, $\frac{80}{85}$. *Ans.*

Examples LXVI.

1. Reduce to equivalent fractions with a common denominator :—

(1) $\frac{2}{3}, \frac{4}{5}$. (2) $\frac{1}{7}, \frac{2}{9}$. (3) $\frac{3}{4}, \frac{9}{11}$. (4) $\frac{1}{2}, \frac{4}{5}, \frac{6}{7}$. (5) $\frac{2}{3}, \frac{3}{7}, \frac{4}{11}$.
(6) $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{9}{13}$. (7) $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}$. (8) $1\frac{1}{3}, 2\frac{1}{5}, 3\frac{1}{7}$.
(9) $\frac{4}{5}, 2\frac{5}{9}, 3\frac{2}{11}$. (10) $7, \frac{5}{8}, 10\frac{10}{11}, 26\frac{1}{7}$. (11) $\frac{2}{3}$ of $\frac{9}{16}, \frac{1}{15}$ of $5\frac{1}{4}, \frac{1}{4}$ of $1\frac{1}{5}$.

2. Reduce the fractions in each of the following sets to equivalent fractions, having the least common denominator :—

(1) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$. (2) $\frac{2}{3}, \frac{3}{4}, \frac{7}{8}$. (3) $\frac{19}{21}, \frac{32}{35}$. (4) $\frac{3}{16}, \frac{8}{21}, \frac{5}{9}$.
(5) $\frac{7}{15}, \frac{11}{21}, \frac{23}{30}$. (6) $\frac{5}{6}, \frac{7}{9}, \frac{8}{15}, \frac{9}{10}$. (7) $\frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{11}{12}$. (8) $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{8}{9}$.
(9) $\frac{7}{12}, \frac{17}{30}, \frac{13}{18}, \frac{15}{20}, \frac{7}{15}$. (10) $\frac{14}{15}, \frac{15}{16}, \frac{17}{18}, \frac{19}{20}, \frac{23}{24}, \frac{26}{27}$.
(11) $\frac{1}{2}, \frac{4}{5}$ of $\frac{1}{2}$ of $\frac{5}{3}, \frac{1}{2}$ of $\frac{2}{5}$ of $2\frac{1}{2}, \frac{4}{5}, \frac{5}{6}$. (12) $1\frac{1}{2}, 3\frac{1}{3}, 4\frac{1}{4}, 6\frac{1}{6}$.
(13) $3\frac{2}{15}, \frac{37}{40}, 1\frac{12}{35}, \frac{16}{27}, 13\frac{11}{12}$. (14) $\frac{3}{7}$ of $\frac{21}{10}, \frac{5}{6}$ of $2\frac{5}{8}, \frac{4}{7}$ of $3\frac{3}{4}$ of $3\frac{1}{2}$.

3. Reduce the following fractions to equivalent ones with the least common numerator :—

(1) $\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{2}{7}$. (2) $\frac{4}{5}, \frac{8}{9}, \frac{11}{12}, \frac{13}{15}$. (3) $\frac{7}{12}, \frac{9}{20}, \frac{14}{15}, \frac{7}{15}$.
(4) $\frac{3}{7}, \frac{7}{15}, \frac{9}{22}, \frac{21}{26}$. (5) $\frac{2}{27}, \frac{13}{24}, \frac{9}{15}, \frac{24}{31}$. (6) $\frac{4}{9}, \frac{2}{3}, \frac{16}{19}, \frac{12}{13}, \frac{8}{13}$.

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257. *To compare the magnitudes of different fractions.*

(1) RULE. Reduce the fractions to equivalent ones with the least common denominator (L. C. D.), and then compare the numerators so obtained. That fraction which has the greatest numerator is the *greatest*, and that which has the least is the *least*.

Ex. 1. Find the *greatest* and *least* of the fractions $\frac{7}{9}$, $\frac{5}{8}$ and $\frac{11}{14}$.

The L. C. M. of the denominators $= 504$.

$504 \div 9 = 56$; $504 \div 8 = 63$; $504 \div 14 = 36$.

$$\therefore \frac{7}{9} = \frac{7 \times 56}{9 \times 56} = \frac{392}{504};\ \frac{5}{8} = \frac{5 \times 63}{8 \times 63} = \frac{315}{504};\ \frac{11}{14} = \frac{11 \times 36}{14 \times 36} = \frac{396}{504}.$$

Hence $\frac{11}{14}$ is the *greatest* and $\frac{5}{8}$ is the *least*. *Ans.*

Ex. 2. Arrange $\frac{4}{5}$, $\frac{11}{12}$, $\frac{13}{15}$ and $\frac{8}{9}$ in order of magnitude.

The L. C. M. of the denominators $= 180$.

$180 \div 5 = 36$; $180 \div 12 = 15$; $180 \div 15 = 12$; $180 \div 9 = 20$.

$$\therefore \frac{4}{5} = \frac{4 \times 36}{5 \times 36} = \frac{144}{180};\ \frac{11}{12} = \frac{11 \times 15}{12 \times 15} = \frac{165}{180};$$

$$\frac{13}{15} = \frac{13 \times 12}{15 \times 12} = \frac{156}{180};\ \frac{8}{9} = \frac{8 \times 20}{9 \times 20} = \frac{160}{180}.$$

Hence the fractions arranged in order of magnitude stand thus :—

$\frac{11}{12}, \frac{8}{9}, \frac{13}{15}$ and $\frac{4}{5}$. *Ans.*

(2) Fractions may also be compared by reducing them to a least common numerator (L. C. N.). In this case, the new fraction that has the least denominator is the *greatest*, and that which has the greatest denominator is the *least*.

Ex. Find the *greatest* and the *least* of $\frac{2}{5}$, $\frac{3}{10}$, $\frac{7}{15}$ and $\frac{9}{16}$.

The L. C. M. of the numerators = 126.

$126 \div 2 = 63$; $126 \div 3 = 42$; $126 \div 7 = 18$; $126 \div 9 = 14$.

$$\therefore \frac{2}{5} = \frac{2 \times 63}{5 \times 63} = \frac{126}{315}; \quad \frac{3}{10} = \frac{3 \times 42}{10 \times 42} = \frac{126}{420};$$

$$\frac{7}{15} = \frac{7 \times 18}{15 \times 18} = \frac{126}{270}; \quad \frac{9}{16} = \frac{9 \times 14}{16 \times 14} = \frac{126}{224}.$$

Hence $\frac{9}{16}$ is the *greatest* and $\frac{3}{10}$ is the *least*. *Ans.*

258. The defect of a fraction from 1 is called its **complement.**

Thus, $\frac{1}{4}$ and $\frac{2}{9}$ are respectively the *complements* of $\frac{3}{4}$ and $\frac{7}{9}$.

(3) Fractions may also be compared by taking their complements, provided that each of the complements has 1 for its numerator.

The *greatest* and *least* fractions will be those that have the least and the greatest complement.

Ex. Find the *greatest* and the *least* of the fractions $\frac{4}{5}$, $\frac{24}{25}$, and $\frac{5}{6}$.

The complements of these fractions are $\frac{1}{5}$, $\frac{1}{25}$ and $\frac{1}{6}$ respectively.

Now, of these complements $\frac{1}{25}$ is the least and $\frac{1}{5}$ the greatest;.

$\therefore$ $\frac{24}{25}$ is the *greatest* and $\frac{4}{5}$ is the *least*. *Ans.*

(4) Fractions may also be compared by the method illustrated by the following example.

Ex. Arrange in order of magnitude $\frac{3}{7}$, $\frac{5}{21}$ and $\frac{6}{19}$.

$$\frac{3}{7} = \frac{3 \div 3}{7 \div 3} = \frac{1}{2\frac{1}{3}}; \quad \frac{5}{21} = \frac{5 \div 5}{21 \div 5} = \frac{1}{4\frac{1}{5}}; \quad \frac{6}{19} = \frac{6 \div 6}{19 \div 6} = \frac{1}{3\frac{1}{6}}. \text{ (Art. 242.)}$$

The given fractions $= \frac{1}{2\frac{1}{3}}$, $\frac{1}{4\frac{1}{5}}$, $\frac{1}{3\frac{1}{6}}$ respectively. Of these $\frac{1}{2\frac{1}{3}}$ is the greatest and $\frac{1}{4\frac{1}{5}}$ is the least, for they have respectively the least and greatest denominators.

$\therefore$ the order of magnitude is $\frac{3}{7}$, $\frac{6}{19}$ and $\frac{5}{21}$. *Ans.*

Examples LXVII.

1. Which is the greater? (by the *first method*).

$\frac{2}{3}$ or $\frac{4}{5}$; $\frac{7}{8}$ or $\frac{9}{12}$; $\frac{17}{24}$ or $\frac{19}{26}$; $\frac{11}{24}$ or $\frac{15}{32}$; $\frac{5}{14}$ or $\frac{8}{21}$; $\frac{15}{19}$ or $\frac{15+8}{19+8}$.

2. Which is the less? (by the *second method.*)

$\frac{7}{18}$ or $\frac{17}{45}$; $\frac{4}{5}$ or $\frac{7}{12}$; $\frac{64}{81}$ or $\frac{85}{108}$; $\frac{31}{63}$ or $\frac{11}{14}$; $\frac{3}{7}$ of $\frac{5}{9}$ or $\frac{1}{2}$ of $\frac{3}{7}$; $\frac{13}{19}$ or $\frac{15-8}{19-8}$.

3. Which is the greatest, and which is the least of the following?

(1) $\frac{3}{5}$, $\frac{11}{12}$, $\frac{1}{18}$; $\frac{4}{9}$, $\frac{3}{8}$, $\frac{8}{15}$; $\frac{15}{42}$, $\frac{22}{49}$, $\frac{48}{63}$; $\frac{5}{18}$, $\frac{4}{9}$, $\frac{7}{27}$; $\frac{7}{9}$, $\frac{11}{18}$, $\frac{11}{12}$.
(2) $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{15}{14}$; $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{9}$, $\frac{9}{10}$; $\frac{4}{7}$, $\frac{26}{21}$, $\frac{65}{102}$, $\frac{15}{17}$; $\frac{2}{9}$, $\frac{5}{16}$, $\frac{11}{24}$, $\frac{5}{27}$.
(3) $\frac{1}{2}$ of $\frac{5}{6}$ of 4, $\frac{2}{3}$ of $\frac{5}{7}$ of 6, $\frac{3}{4}$ of $\frac{4}{5}$ of 3; $\frac{5}{32}$, $\frac{3}{28}$, $\frac{1}{24}$, $\frac{13}{42}$.
(4) $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{3}{10}$, $\frac{2}{7}$ of $\frac{5}{8}$ of $3\frac{1}{2}$, $4\frac{1}{5}$ of $3\frac{3}{4}$ of 10; $\frac{3}{7}$ of $\frac{5}{6}$, $\frac{1}{2}$ of $\frac{3}{7}$, $7\frac{3}{8}$.
(5) $\frac{17}{24}$, $\frac{11}{18}$, $\frac{13}{36}$, $\frac{25}{48}$, $\frac{7}{15}$; $\frac{4}{11}$, $\frac{9}{25}$, $\frac{18}{55}$, $\frac{9}{22}$; $\frac{4}{5}$, $\frac{5}{6}$, $\frac{4+5}{5+6}$.

4. Arrange in order of magnitude the following :—

(1) $\frac{2}{3}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{19}{25}$. (2) $\frac{8}{25}$, $\frac{7}{10}$, $\frac{8}{30}$, $\frac{7}{65}$. (3) $\frac{7}{13}$, $\frac{10}{21}$, $\frac{16}{35}$.
(4) $1\frac{1}{7}$ of $\frac{1}{4}$, $\frac{5}{21}$ of $3\frac{1}{2}$, $\frac{1}{5}$ of $2\frac{4}{5}$. (5) $\frac{5}{8}$ of $2\frac{2}{7}$, $1\frac{2}{5}$, $3\frac{1}{2}$ of $\frac{5}{14}$ of $\frac{3}{5}$.
(6) $\frac{13}{14}$, $\frac{15}{16}$, $\frac{13+15}{14+16}$. (7) $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{5+6+7+9}{6+7+8+10}$.

5. Arrange in order of magnitude :—(by the *third method.*)

(1) $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{2}$, $\frac{3}{4}$. (2) $\frac{9}{10}$, $\frac{11}{12}$, $\frac{8}{9}$, $\frac{6}{7}$. (3) $\frac{13}{14}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{45}{46}$.
(4) $\frac{89}{90}$, $\frac{99}{100}$, $\frac{11}{12}$, $\frac{15}{16}$. (5) $\frac{49}{50}$, $\frac{47}{48}$, $\frac{61}{62}$, $\frac{74}{75}$. (6) $\frac{15}{16}$, $\frac{17}{18}$, $\frac{19}{20}$, $\frac{24}{25}$.

6. Arrange in order of magnitude :—(by the *fourth method*).

(1) $\frac{1}{4}$, $\frac{2}{5}$, $\frac{7}{8}$, $\frac{2}{3}$. (2) $\frac{21}{22}$, $\frac{15}{16}$, $\frac{5}{48}$, $\frac{14}{33}$. (3) $\frac{5}{6}$, $\frac{2}{3}$, $\frac{1}{17}$, $\frac{1}{18}$.
(4) $\frac{9}{10}$, $\frac{15}{16}$, $\frac{19}{20}$, $\frac{28}{30}$. (5) $\frac{4}{9}$, $\frac{2}{5}$, $\frac{16}{19}$, $\frac{12}{13}$, $\frac{8}{13}$. (6) $\frac{3}{5}$, $\frac{5}{9}$, $\frac{6}{13}$, $\frac{13}{15}$.

7. Find a fraction intermediate in value to $\frac{5}{6}$ and $\frac{6}{7}$ whose denominator is 84; to $\frac{2}{5}$ and $\frac{29}{72}$ whose denominator is 720.

III. ADDITION OF FRACTIONS.

259. *To find the sum of two or more given fractions.*

(1) If the given fractions have the *same* denominator.

RULE. Add the numerators of the given fractions together for the numerator of the **sum**, and take their denominator for its denominator.

Ex. Find the sum of $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{8}{7}$.

Here, $\frac{4}{7}+\frac{5}{7}+\frac{8}{7}=\frac{4+5+8}{7}$ For 4 *sevenths* + 5 *sevenths* + 8 *sevenths* = (4 + 5 + 8) *sevenths*.

$=\frac{17}{7}=2\frac{3}{7}$. $= 17$ *sevenths* $=\frac{17}{7}$.

(2) If the given fractions have *different* denominators.

RULE. Express the fractions with a least common denominator; add together the new numerators for the numerator of their **sum**, and take he least common denominator for its denominator

The sum should always be expressed in its lowest terms; and, if an improper fraction, should be reduced to a mixed number.

Ex. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$.

The L. C. M. of the denominators $=24$.

$$\frac{2}{3}=\frac{2\times8}{3\times8}=\frac{16}{24};\quad \frac{3}{4}=\frac{3\times6}{4\times6}=\frac{18}{24};$$

$$\frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24};\quad \frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24}.$$

$$\therefore \text{ the sum}=\frac{16}{24}+\frac{18}{24}+\frac{20}{24}+\frac{21}{24}=\frac{16+18+20+21}{24}$$

$$=\frac{75}{24}=\frac{25\times3}{8\times3}=\frac{25}{8}=3\tfrac{1}{8}. \quad \textit{Ans.}$$

260. All fractions should be reduced to their lowest terms, improper fractions to whole or mixed numbers, and compound fractions to simple ones, before the application of the Rule.

261. If any one of the given numbers be whole or mixed numbers, add together the whole numbers as in simple addition and the fractional parts by the Rule given above.

Ex. Add together $5\frac{5}{12}$, $\frac{19}{6}$, $2\frac{21}{48}$ and $\frac{3}{4}$ of $3\frac{2}{3}$.

Here, $\frac{19}{6}=3\frac{1}{6}$; $2\frac{21}{48}=2\frac{7}{16}$; $\frac{3}{4}$ of $3\frac{2}{3}=\frac{3}{4}$ of $\frac{11}{3}=\frac{11}{4}=2\frac{3}{4}$.

$$\therefore \text{ sum of the fractions}=5\tfrac{5}{12}+3\tfrac{1}{6}+2\tfrac{7}{16}+2\tfrac{3}{4}$$

$$=(5+3+2+2)+(\tfrac{5}{12}+\tfrac{1}{6}+\tfrac{7}{16}+\tfrac{3}{4})$$

$$=12+\frac{20+8+21+36}{48}$$

$$=12+\tfrac{85}{48}=12+1\tfrac{37}{48}=13\tfrac{37}{48}. \quad \textit{Ans.}$$

Examples LXVIII.

1. Add together *orally* the following fractions :—

(1) $\frac{2}{7}+\frac{3}{7}$; $\frac{1}{8}+\frac{7}{8}$; $\frac{4}{5}+\frac{3}{5}$; $\frac{2}{9}+\frac{5}{9}+\frac{7}{9}$; $\frac{5}{6}+\frac{1}{6}+\frac{7}{6}$; $\frac{3}{10}+\frac{7}{10}$; $\frac{7}{15}+\frac{11}{15}$.

(2) $\frac{1}{2}+\frac{1}{3}$; $\frac{1}{3}+\frac{1}{4}$; $\frac{1}{4}+\frac{1}{5}$; $\frac{1}{2}+\frac{1}{6}$; $\frac{1}{7}+\frac{1}{14}$; $\frac{7}{10}+\frac{8}{15}$; $\frac{2}{5}+\frac{3}{10}+\frac{7}{15}$.

(3) $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$; $\frac{1}{3}+\frac{1}{4}+\frac{1}{12}$; $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}$; $\frac{3}{4}+\frac{5}{6}+\frac{7}{12}$; $\frac{1}{4}+\frac{5}{12}+\frac{8}{15}+\frac{7}{20}$.

(4) $3\frac{1}{2}+2\frac{1}{3}$; $4\frac{2}{3}+3\frac{1}{2}$; $1\frac{1}{4}+3\frac{3}{4}+2\frac{1}{2}$; $3\frac{1}{6}+4\frac{1}{2}+7\frac{7}{12}$.

2. Find the values of the following :—

(1) $\frac{11}{30}+\frac{2}{45}$; $\frac{2}{5}+\frac{4}{7}$; $\frac{3}{7}+\frac{5}{9}$; $\frac{3}{8}+\frac{7}{9}$; $\frac{5}{8}+\frac{12}{17}$; $\frac{3}{8}+\frac{5}{14}$; $\frac{3}{7}+\frac{9}{16}+\frac{15}{28}$.

(2) $1\frac{1}{2}+7\frac{1}{5}$; $2\frac{6}{7}+13\frac{3}{10}$; $5\frac{1}{6}+12\frac{4}{5}$; $37\frac{2}{11}+24\frac{12}{13}$; $7\frac{2}{7}+4\frac{7}{9}$.

(3) $\frac{31}{7}+\frac{47}{8}$; $\frac{41}{18}+\frac{21}{13}$; $\frac{57}{5}+\frac{38}{15}$; $\frac{31}{9}+\frac{49}{8}$; $\frac{27}{16}+\frac{71}{24}$; $\frac{17}{35}+\frac{22}{45}$.

(4) $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$; $\frac{1}{7}+\frac{2}{5}+\frac{3}{11}$; $\frac{4}{5}+\frac{6}{7}+\frac{9}{10}$; $\frac{5}{6}+\frac{4}{9}+\frac{14}{21}$; $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\frac{9}{10}$.

(5) $\frac{11}{16}+\frac{45}{8}+\frac{97}{2}$; $2\frac{1}{5}+3\frac{2}{9}+5\frac{3}{8}$; $8\frac{4}{7}+13\frac{5}{9}+27\frac{8}{11}$; $\frac{15}{4}+3\frac{1}{3}+11+\frac{14}{5}$.

(6) $\frac{2}{7}+\frac{5}{14}+\frac{11}{42}+\frac{20}{21}$; $\frac{113}{226}+\frac{226}{339}+\frac{565}{678}$; $\frac{133}{182}+\frac{325}{325}+\frac{261}{351}$.

(7) $\frac{3}{13}$ of $9\frac{8}{15}+\frac{4}{17}$ of $8\frac{1}{2}$; $14\frac{3}{4}+\frac{2}{3}$ of $\frac{5}{6}$ of 8 ; $\frac{2}{7}+4\frac{1}{3}+\frac{3}{5}$ of 2.

(8) $1\frac{2}{7}+\frac{5}{9}+\frac{8}{11}+3\frac{1}{5}$. (9) $3\frac{5}{8}+2\frac{3}{5}+\frac{7}{11}+7\frac{1}{6}$.

(10) $2\frac{7}{12}+3\frac{5}{18}+4\frac{1}{21}+5\frac{11}{60}$. (11) $3\frac{5}{12}+7\frac{1}{2}+8\frac{2}{3}+4\frac{1}{6}$.

(12) $1\frac{1}{12}+2\frac{2}{15}+3\frac{3}{20}+4\frac{4}{27}$. (13) $4\frac{2}{7}+8\frac{2}{3}+3\frac{3}{14}+8\frac{5}{6}$.

(14) $\frac{4}{11}$ of $\frac{2}{3}$ of $\frac{1}{2}+\frac{7}{8}$ of $\frac{1}{5}$ of $\frac{10}{33}+\frac{2}{5}$ of $1\frac{2}{3}+2\frac{5}{7}$.

(15) $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{9}{20}$ of $11\frac{1}{9}$; $\frac{3}{5}+2\frac{1}{2}$ of $\frac{7}{25}+3\frac{1}{3}$ of $\frac{9}{20}$.

(16) $387\frac{1}{2}+285\frac{1}{4}+394\frac{1}{5}+\frac{2}{5}$ of 3704 ; $1\frac{5}{19}+2\frac{6}{7}+4\frac{11}{38}+6\frac{1}{14}$.

(17) $275\frac{1}{3}+62\frac{11}{120}+1031\frac{1}{5}+\frac{7}{8}$ of $4150\frac{1}{7}$.

(18) $\frac{1}{3}+\frac{3}{4}+\frac{5}{8}+\frac{4}{9}+\frac{7}{12}+\frac{11}{18}+\frac{13}{24}$. (19) $\frac{1}{2}+\frac{2}{3}+\frac{3}{5}+\frac{7}{15}+\frac{23}{30}+\frac{11}{40}+\frac{23}{60}+\frac{41}{120}$.

(20) $\frac{3}{4}$ of $\frac{5}{7}$ of $52\frac{4}{15}+\frac{2}{5}$ of $\frac{8}{9}$ of $506\frac{1}{4}+\frac{2}{5}$ of $\frac{3}{8}$ of 1864.

(21) $\frac{1}{7}$ of $\frac{5}{7}+\frac{4}{5}$ of $\frac{1}{10}+\frac{2}{5}$ of $(\frac{1}{2}+\frac{11}{14})+\frac{3}{70}$ of $(\frac{2}{7}+\frac{4}{5})$.

IV. SUBTRACTION OF FRACTIONS.

262. *To subtract one fraction from another fraction.*

(1) When the given fractions have the *same* denominator.

RULE. Find the difference of the numerators of the given fractions for the numerator of the **remainder**, and take their denominator for its denominator.

Ex. Subtract $\frac{4}{17}$ from $\frac{10}{17}$.

Here, $\frac{10}{17}-\frac{4}{17}=\frac{10-4}{17}$
$=\frac{6}{17}$.

For, 10 *seventeenths* $-$ 4 *seventeenths*
$=(10-4)$ *seventeenths* $=6$ *seventeenths*
$=\frac{6}{17}$.

(2) When the given fractions have *different* denominators.

RULE. Reduce the fractions to a least common denominator ; subtract the less numerator from the greater ; under the **remainder** place the least common denominator, and the result, properly reduced, will be the required difference.

Note. Before applying the Rule, reduce fractions to their lowest terms, improper fractions to whole or mixed numbers, and compound fractions to simple ones.

Ex. Subtract $\frac{13}{18}$ from $\frac{19}{24}$, and $\frac{2}{3}$ of $\frac{5}{16}$ from $\frac{7}{8}$ of $\frac{3}{4}$.

(1) The L. C. M. of 18 and $24=72$.

$\therefore \frac{19}{24}-\frac{13}{18}=\frac{57}{72}-\frac{52}{72}=\frac{57-52}{72}=\frac{5}{72}$. *Ans.*

(2) Here, $\frac{7}{8}$ of $\frac{3}{4}=\frac{21}{32}$, and $\frac{2}{3}$ of $\frac{5}{16}=\frac{5}{24}$. Also L. C. D. $=96$.

$\therefore$ their difference $=\frac{21}{32}-\frac{5}{24}=\frac{63-20}{96}=\frac{43}{96}$. *Ans.*

263. *Additions and subtractions of fractions may be performed in any order.*

Thus, $7\frac{3}{4}-4\frac{4}{7}=(7+\frac{3}{4})-(4+\frac{4}{7})=7+\frac{3}{4}-4-\frac{4}{7}$, (Art. 107)
$=(7-4)+(\frac{3}{4}-\frac{4}{7})=3+\frac{3}{4}-\frac{4}{7}$.

Hence, if either of the given fractions be a whole or mixed number, it is most convenient to take separately the difference of the integral parts and that of the fractional parts, and then add the two results together.

Ex. 1. From $3\frac{2}{3}$ take $2\frac{1}{6}$.

Here, $3\frac{2}{3} - 2\frac{1}{6} = (3 + \frac{2}{3}) - (2 + \frac{1}{6}) = (3 - 2) + (\frac{2}{3} - \frac{1}{6})$

$= 1 + (\frac{4}{6} - \frac{1}{6}) = 1\frac{3}{6} = 1\frac{1}{2}$. *Ans.*

Ex. 2. Find the difference between $2\frac{4}{7}$ and $5\frac{1}{3}$.

Here, $5\frac{1}{3} - 2\frac{4}{7} = (4 + 1 + \frac{1}{3}) - (2 + \frac{4}{7}) = (4 - 2) + (1 + \frac{1}{3} - \frac{4}{7})$

$= 2 + (\frac{4}{3} - \frac{4}{7}) = 2 + (\frac{28}{21} - \frac{12}{21}) = 2 + \frac{16}{21} = 2\frac{16}{21}$. *Ans.*

264. The following peculiarities in *Subtraction of Fractions* should be carefully noticed.

(1) *When both the fractions have a common numerator.*

RULE. Multiply the difference of the denominators by the common numerator for the *new numerator*, and take the product of the denominators for the *new denominator*. The resulting fraction is the required difference.

Ex. 1. $\frac{8}{11} - \frac{8}{13} = \frac{(13 - 11) \times 8}{13 \times 11} = \frac{2 \times 8}{13 \times 11} = \frac{16}{143}$. *Ans.*

(2) *To subtract fractions when both have* 1 *for numerator.*

RULE. Find the difference between the denominators for a *new numerator* and multiply the denominators for a *new denominator*. The resulting fraction is the required difference.

Ex. 2. $\frac{1}{8} - \frac{1}{9} = \frac{9 - 8}{8 \times 9} = \frac{1}{72}$.

(3) *To subtract a proper fraction from unity.*

RULE. Subtract the numerator from the denominator for the *new numerator*, and underneath place the given denominator. The resulting fraction is the required difference.

Ex. 3. $1 - \frac{5}{11} = \frac{11 - 5}{11} = \frac{6}{11}$. *Ans.*

(4) *To subtract a mixed number from an integer.*

RULE. Subtract the fractional part from unity as in (3) and the integral part from the integer diminished by unity.

Ex. 4. $7 - 3\frac{4}{7} = (6 - 3) + (1 - \frac{4}{7}) = 3\frac{3}{7}$. *Ans.*

(5) *To subtract a mixed number from another, when the fractional part of the subtrahend is greater than that of the minuend.*

RULE. Subtract the subtrahend (composed of the integral and fractional part) from the integral part of the minuend as in (4) and to this difference add the fractional part of the minuend.

Ex. 5. $15\frac{3}{7} - 7\frac{2}{3} = (15 - 7\frac{2}{3}) + \frac{3}{7} = 7\frac{1}{3} + \frac{3}{7} = 7\frac{16}{21}$. *Ans.*

265. *An expression made up of additions and subtractions of fractions may be made equal to the difference of two sums.*

Thus, $5\frac{3}{4} - 1\frac{4}{7} + 2\frac{5}{6} + \frac{2}{3} - \frac{3}{5} = (5\frac{3}{4} + 2\frac{5}{6} + \frac{2}{3}) - (1\frac{4}{7} + \frac{3}{5})$. (Art. 107).

Examples LXIX.

1. Perform *orally* the following subtractions :—

(1) $\frac{5}{6} - \frac{1}{6}$; $\frac{7}{9} - \frac{2}{9}$; $\frac{4}{5} - \frac{2}{5}$; $\frac{1}{2} - \frac{1}{3}$; $\frac{1}{3} - \frac{1}{5}$; $\frac{1}{2} - \frac{1}{6}$; $\frac{2}{3} - \frac{1}{2}$; $\frac{1}{4} - \frac{1}{8}$.

(2) $1 - \frac{7}{8}$; $2 - \frac{4}{5}$; $2 - \frac{5}{16}$; $1 - \frac{2}{3}$; $1 - \frac{3}{4}$; $\frac{2}{3} - \frac{1}{2}$; $\frac{3}{4} - \frac{2}{3}$; $\frac{4}{5} - \frac{1}{4}$.

(3) $1 - \frac{7}{9}$; $3 - \frac{4}{7}$; $2\frac{1}{3} - 1\frac{1}{8}$; $4\frac{2}{3} - 3\frac{1}{2}$; $6\frac{1}{2} - 4\frac{1}{3}$; $7\frac{7}{8} - 4\frac{1}{4}$; $4\frac{9}{10} - 2\frac{9}{20}$.

2. Perform the following subtractions :—

(1) $\frac{3}{5} - \frac{1}{6}$; $\frac{7}{9} - \frac{3}{7}$; $\frac{8}{11} - \frac{2}{9}$; $\frac{5}{6} - \frac{8}{15}$; $\frac{12}{13} - \frac{5}{18}$; $\frac{14}{15} - \frac{11}{12}$.

(2) $1\frac{23}{28} - 1\frac{2}{21}$; $\frac{31}{56} - \frac{13}{98}$; $8\frac{4}{9} - \frac{8}{9}$; $9\frac{3}{13} - 2\frac{7}{13}$; $3\frac{17}{22} - 2\frac{13}{24}$; $8\frac{7}{8} - 5\frac{4}{9}$.

(3) $19\frac{2}{7} - 13\frac{2}{11}$; $18\frac{12}{25} - 17\frac{13}{27}$; $1000 - 384\frac{7}{30}$; $279\frac{2}{7} - 168\frac{8}{9}$.

(4) $2\frac{7}{9}$ of $16\frac{2}{7} - 1\frac{2}{3}$ of $3\frac{5}{8}$; $\frac{3}{7}$ of $\frac{2}{11}$ of $25 - \frac{3}{20}$ of $\frac{5}{9}$; $7\frac{1}{7}$ of $10\frac{1}{9} - 2\frac{1}{2}$ of $4\frac{1}{4}$.

3. Find the values of :—

(1) $\frac{5}{6} - \frac{3}{4} + \frac{2}{3} - \frac{1}{2}$.

(2) $\frac{15}{16} - \frac{14}{15} + \frac{13}{14} - \frac{11}{12}$.

(3) $\frac{1}{2} + \frac{2}{3} - \frac{1}{6} + \frac{3}{8} - \frac{1}{12}$.

(4) $\frac{1}{2} - \frac{3}{8} + \frac{7}{15} - \frac{11}{40} + \frac{5}{6} + \frac{3}{9} - \frac{5}{12} + \frac{3}{5}$.

(5) $13\frac{4}{5} - 9\frac{2}{9} - 1\frac{8}{45}$.

(6) $3\frac{8}{13} - \frac{6}{23} - 1\frac{3}{26} + \frac{5}{46}$.

(7) $7\frac{2}{3} + 6\frac{3}{4} - 3\frac{4}{15} - 2\frac{17}{25} + \frac{11}{20}$.

(8) $3\frac{1}{5} + 2\frac{4}{7} - (5\frac{1}{2} + 1\frac{3}{5}) + 2\frac{8}{35}$.

(9) $10\frac{1}{2} - (4\frac{6}{7} + 6\frac{3}{14}) + 7\frac{5}{6} + (8\frac{2}{3} - 6\frac{7}{12})$.

(10) $6\frac{7}{8}$ of $2\frac{2}{11} - (6\frac{7}{8} - 2\frac{2}{11})$.

(11) $2\frac{1}{2} - (4\frac{7}{8} + 10\frac{5}{12} + 3\frac{9}{20}) + 3\frac{1}{9} + 20\frac{2}{5}$.

(12) $\frac{3}{8}$ of $\frac{4}{7} - \frac{2}{11}$ of $3\frac{1}{7} + \frac{5}{9}$ of $3\frac{3}{8}$.

(13) $22\frac{11}{17} - (9\frac{23}{34} - 7\frac{16}{51} + \frac{1}{3}$ of $\frac{16}{17}) + \frac{2}{5}$ of $3\frac{3}{4}$.

(14) $8\frac{3}{4} - 3\frac{13}{14} + 2\frac{7}{9}$ of $\frac{3}{14}$ of $4\frac{1}{5} - (5\frac{11}{12} - 2\frac{7}{8})$.

(15) $47\frac{5}{27} - (3\frac{2}{3} + 31\frac{5}{54} + 2\frac{5}{8}) + 6\frac{5}{8} - (2\frac{8}{25} - 1\frac{9}{104})$.

V. MULTIPLICATION OF FRACTIONS.

266. *To multiply a fraction by a whole number.*

[We have already given an outline of this method in Arts. 241 and 244. Now, we propose to treat it at length.]

RULE Multiply the numerator by the whole number for the *new numerator*, and leave the denominator unchanged. The resulting fraction should always be expressed in its lowest terms, by cancelling those factors that are common to the multiplier and to the denominator of the fraction.

Thus, $8 \times \frac{5}{9} = \frac{8 \times 5}{9} = \frac{40}{9} = 4\frac{4}{9}$

For $8 \times \frac{1}{9} = 8 \times 5$ *ninths* $= 40$ *ninths* $= \frac{40}{9} = \frac{8 \times 5}{9}$.

Also, $9 \times \frac{4}{15} = \frac{3 \times 3 \times 4}{3 \times 5} = \frac{3 \times 4}{5} = \frac{12}{5} = 2\frac{2}{5}$.

267. *To multiply a mixed number by an integer.*

RULE. Either reduce the mixed number to an improper fraction and multiply as above, or multiply the integral part and the fractional part separately, and add the two products.

Thus, (1) $6\frac{4}{5} \times 3 = \frac{34}{5} \times 3 = \frac{102}{5} = 20\frac{2}{5}$.

(2) $6\frac{4}{5} \times 3 = 6 \times 3 + \frac{4}{5} \times 3 = 18 + \frac{12}{5} = 18 + 2\frac{2}{5} = 20\frac{2}{5}$.

268. To multiply a proper fraction differing very little from 1, or a mixed number differing very little from the next superior integer by a whole number, we have recourse to such artifices as is explained in Art. 264.

Thus, (1) $\frac{99}{100} \times 35 = (1 - \frac{1}{100}) \times 35 = 35 - \frac{35}{100} = 35 - \frac{7}{20} = 34\frac{13}{20}$.

(2) $15\frac{74}{75} \times 12 = (16 - \frac{1}{75}) \times 12 = 192 - \frac{12}{75} = 192 - \frac{4}{25} = 191\frac{21}{25}$.

(3) $99\frac{13}{14} \times 46 = (100 - \frac{1}{14}) \times 46 = 4600 - \frac{46}{14} = 4600 - 3\frac{2}{7} = 4596\frac{5}{7}$.

Examples LXX.

1. Multiply *orally* :—

(1) $\frac{1}{4}$ by 3 ; $\frac{3}{11}$ by 2 ; $\frac{5}{13}$ by 3 ; $\frac{1}{6}$ by 4 ; $\frac{3}{28}$ by 5 ; $\frac{5}{12}$ by 7 ; $\frac{3}{4}$ by 3 ; $\frac{5}{9}$ by 2.

(2) $\frac{17}{35}$ by 7 ; $\frac{17}{63}$ by 21 ; $\frac{23}{150}$ by 100 ; $\frac{63}{75}$ by 25 ; $\frac{28}{49}$ by 28 ; $\frac{8}{75}$ by 30.

2. Multiply :—

(1) $\frac{14}{33}$ separately by 55, 88, 90. (2) $\frac{5}{21}$ separately by 12, 36, 48, 49.

(3) $\frac{41}{112}$ ... ... 32, 128, 168. (4) $3\frac{17}{22}$ .... 11, 15, 21, 132.

(5) $3\frac{14}{33}$ .... 55, 77, 110. (6) $2\frac{8}{52}$ .... 13, 39, 42, 117.

(7) $159\frac{3}{8}$ by 12 ; $1625\frac{7}{11}$ by 23 ; $4\frac{88}{115}$ by 23 ; $1727\frac{8}{17}$ by 34 ; $3589\frac{11}{12}$ by 47.

3. Find the product of :—

(1) $99\frac{9}{10}$ separately by 6, 8, 15, 18, 25. (2) $999\frac{17}{18}$ by 99, 550.

(3) $499\frac{824}{825}$...... ...25, 50, 75, 100, 150, 200, 250.

(4) $74\frac{26}{27}$ by 43 ; $99\frac{423}{424}$ by 324 ; $999\frac{4845}{4846}$ by 999.

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269 The meaning of Multiplication as given in Art. 59 is not applicable when the multiplier is a fraction. Hence, to suit our purpose we make the following definition.

"To multiply by a fraction is to take that fraction of the multiplicand."

Thus, to multiply $\frac{5}{7}$ by $\frac{3}{8}$, we take $\frac{3}{8}$ of $\frac{5}{7}$ by the new definition.

But $\frac{3}{8}$ of $\frac{5}{7} = \frac{3 \times 5}{8 \times 7}$ by Art. 249 ; therefore $\frac{5}{7} \times \frac{3}{8} = \frac{3 \times 5}{8 \times 7}$.

Hence the Rule.

270. *To multiply a fraction by a fraction.*

RULE. Multiply together the respective numerators and denominators, reduced to fractional forms if necessary; and the fraction thence arising will be the product, which may be simplified by striking out any factor common to numerator and denominator.

Ex. 1. Multiply $\frac{2}{9}$ by $\frac{7}{8}$.

Here, $\frac{2}{9} \times \frac{7}{8} = \frac{2 \times 7}{9 \times 8} = \frac{2 \times 7}{9 \times 2 \times 4}$
$= \frac{7}{9 \times 4} = \frac{7}{36}$. *Ans.*

For, if $\frac{2}{9}$ be multiplied by 7, the product will be $\frac{14}{9}$ (Art. 241); but 7 being 8 times as great as $\frac{7}{8}$, the multiplier above used is 8 times too *large*, and the product $\frac{14}{9}$ will therefore be 8 times too *large* also: whence, the product required must be $\frac{14}{9} \div 8 = \frac{14}{72}$ (Art. 241) $= \frac{7}{36}$.

Ex. 2. Multiply $3\frac{24}{25}$ by $2\frac{37}{54}$; and $5\frac{4}{9}$ by $2\frac{3}{7}$ of $\frac{15}{17}$.

(1) Product $= \frac{99}{25} \times \frac{145}{54} = \frac{99 \times 145}{25 \times 54} = \frac{11 \times 9 \times 5 \times 29}{5 \times 5 \times 6 \times 9}$

$= \frac{11 \times 29}{5 \times 6} = \frac{319}{30} = 10\frac{19}{30}$. *Ans.*

(2) Product $= \frac{49}{9} \times \left(\frac{17}{7} \times \frac{15}{17}\right) = \frac{49 \times 17 \times 15}{9 \times 7 \times 17} = \frac{49 \times 15}{9 \times 7}$

$= \frac{7 \times 7 \times 3 \times 5}{3 \times 3 \times 7} = \frac{7 \times 5}{3} = \frac{35}{3} = 11\frac{2}{3}$. *Ans.*

271. *To find the continued product of three or more fractions.*

RULE. Multiply all the numerators together for the numerator of the continued product, and all the denominators for its denominator; cancelling all the factors common to numerator and denominator before obtaining the final result.

Ex. 1. Find the continued product of $\frac{3}{4}$, $\frac{5}{7}$ and $\frac{8}{15}$.

Here, Product $= \frac{3 \times 5 \times 8}{4 \times 7 \times 15} = \frac{3 \times 5 \times 2 \times 4}{4 \times 7 \times 3 \times 5} = \frac{2}{7}$. *Ans.*

Ex. 2. Multiply $\frac{5}{6}$, $3\frac{2}{11}$, $19\frac{1}{5}$ and $\frac{11}{56}$ together.

Product $= \frac{5}{6} \times \frac{35}{11} \times \frac{96}{5} \times \frac{11}{56} = \frac{5 \times (5 \times 7) \times (2 \times 2 \times 2 \times 2 \times 2 \times 3) \times 11}{(3 \times 2) \times 11 \times 5 \times (2 \times 2 \times 2 \times 7)}$

$= \frac{5 \times 2}{1} = \frac{10}{1} = 10$. *Ans.*

Examples LXXI.

1. Multiply *orally*:—

(1) $\frac{1}{2}$ separately by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{7}$, $\frac{7}{6}$.

(2) $\frac{1}{4}$ separately by $\frac{1}{3}$, $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{5}$.

(3) $\frac{3}{7}$ ... ... $\frac{4}{7}$, $\frac{5}{8}$, $\frac{2}{5}$, $\frac{1}{2}$.

(4) $\frac{4}{11}$ ... ... $\frac{7}{13}$, $\frac{4}{7}$, $\frac{6}{11}$, $\frac{10}{13}$.

2. Multiply :—

(1) $\frac{3}{5}$ by $\frac{3}{7}$; $\frac{3}{8}$ by $\frac{5}{9}$; $\frac{7}{9}$ by $\frac{5}{8}$; $\frac{4}{13}$ by $\frac{3}{4}$; $\frac{6}{35}$ by $\frac{7}{18}$; $\frac{18}{35}$ by $\frac{39}{80}$.
(2) $\frac{14}{27}$ by $\frac{12}{35}$; $\frac{6}{25}$ by $\frac{35}{54}$; $\frac{11}{12}$ by $\frac{15}{28}$; $\frac{36}{65}$ by $\frac{52}{99}$; $\frac{35}{72}$ by $\frac{27}{49}$.
(3) $2\frac{3}{5}$ by $7\frac{2}{9}$; $8\frac{2}{7}$ by $10\frac{5}{11}$; $6\frac{3}{8}$ by $14\frac{6}{17}$; $15\frac{1}{5}$ by $3\frac{8}{9}$; $6\frac{3}{4}$ by $2\frac{14}{27}$.
(4) $2\frac{3}{8}$ by $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{7}{9}$; $13\frac{5}{9}$ of $7\frac{1}{3}$ by $\frac{3}{4}$ of $\frac{4}{9}$ of $12\frac{1}{2}$; $\frac{8}{23}$ of $4\frac{3}{8}$ by $2\frac{13}{28}$.
(5) $\frac{8}{9}$ of $15\frac{6}{7}$ by $\frac{7}{12}$ of $3\frac{6}{37}$; $3\frac{5}{21}$ of $5\frac{5}{8}$ by $6\frac{2}{5}$ of $10\frac{11}{25}$; $\frac{3}{5}$ of $19\frac{5}{7}$ by $\frac{10}{23}$.

3. Find the values of :—

(1) $\frac{2}{3}\times\frac{3}{8}\times\frac{7}{12}$; $\frac{3}{8}\times\frac{11}{7}\times\frac{15}{11}$; $\frac{3}{4}\times\frac{5}{7}\times\frac{8}{9}$; $\frac{8}{11}\times16\frac{1}{2}\times\frac{5}{81}$; $4\frac{1}{2}\times\frac{1}{3}\times\frac{1}{3}$.
(2) $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}$; $\frac{3}{4}\times\frac{6}{7}\times\frac{8}{9}\times\frac{10}{11}$; $\frac{8}{13}\times2\frac{2}{3}\times1\frac{1}{25}\times1\frac{11}{64}$; $\frac{7}{12}$ of $1\frac{1}{7}\times3\frac{3}{20}\times\frac{13}{84}$.
(3) $\frac{5}{6}\times2\frac{3}{4}\times3\frac{5}{11}\times5\frac{2}{19}\times6\frac{1}{104}$; $12\frac{17}{19}\times8\frac{13}{18}\times\frac{51}{148}\times6\frac{1}{15}$ of $\frac{14}{17}\times2\frac{1}{7}$.
(4) $1\frac{3}{8}$ of $\frac{5}{6}\times\frac{5}{18}$ of $2\frac{1}{22}$ of $8\times12\frac{2}{5}$ of $6\frac{3}{5}$ of $1\frac{1}{17}\times\frac{5}{31}$.
(5) $4\frac{1}{2}$ of $3\frac{1}{8}$ of $7\frac{2}{30}\times\frac{2}{3}$ of $1\frac{1}{7}\times2\frac{2}{5}$ of $4\frac{3}{8}$ of $2\frac{2}{9}\times\frac{1}{5000}$.

4. Find the continued product of :—

(1) $\frac{49}{133}$, $\frac{76}{75}$ and $\frac{28}{98}$.
(2) $\frac{428}{515}$, $\frac{5253}{1819}$ and $\frac{615}{492}$.
(3) $\frac{17}{24}$, $\frac{384}{391}$, $\frac{851}{864}$, and $\frac{1584}{1591}$.
(4) $\frac{5687}{319}$, $\frac{667}{22011}$, $\frac{221}{629}$ and $\frac{72816}{8528}$.
(5) $\frac{324}{361}$, $\frac{1444}{1296}$, $\frac{441}{529}$ and $\frac{2116}{1764}$.
(6) $\frac{36}{65}$, $\frac{35}{132}$, $\frac{39}{108}$ and $\frac{75}{144}$.

5. Simplify :—

(1) $(3\frac{5}{9}+2\frac{7}{8})\times10\frac{2}{7}$; $3\frac{5}{9}+2\frac{7}{8}\times10\frac{2}{7}$; $(\frac{1}{2}+\frac{1}{3}+\frac{1}{4})\times(\frac{1}{5}$ of $\frac{2}{7})$.
(2) $(\frac{1}{2}\times\frac{3}{8}+\frac{3}{7}\times\frac{2}{5})-(\frac{2}{7}\times\frac{5}{9}-\frac{1}{8}\times\frac{4}{11})$; $\frac{1}{2}$ of $(6\frac{1}{2}+2\frac{2}{3}-3)$; $(\frac{1}{3}+\frac{1}{5})\times(\frac{1}{2}-\frac{1}{7})$.
(3) $(19\frac{4}{5}-3\frac{3}{4})\times(3\frac{4}{5}-2\frac{3}{7})$; $19\frac{4}{5}-3\frac{3}{4}\times3\frac{4}{5}-2\frac{3}{7}$; $19\frac{4}{5}-3\frac{3}{4}\times(3\frac{4}{5}-2\frac{3}{7})$.
(4) $\{(\frac{1}{2}+\frac{1}{3})$ of $(1\frac{1}{3}+2\frac{3}{4})\}\times\{(2\frac{1}{14}-1\frac{1}{2})$ of $(3\frac{1}{10}-\frac{3}{7})\}$.
(5) $\{1\frac{2}{7}$ of $26\frac{1}{2}$ of $(1-\frac{2}{3})\}\times\{2\frac{3}{8}$ of $(4\frac{1}{5}-3\frac{2}{3})$ of $\frac{45}{106}\}$.
(6) $(1\frac{2}{3}$ of $2\frac{7}{9}-3\frac{17}{18})\times(5\frac{1}{3}$ of $4\frac{1}{2}-3\frac{1}{4}$ of $3\frac{1}{5})\times\frac{27}{37}$ of $\frac{3}{17}\times\frac{1}{2}$.

VI. DIVISION OF FRACTIONS.

272. *To divide a fraction by a whole number.*

[We have already given an outline of this method in Arts. 241 and 244. Now, we propose to treat it at length].

RULE. Multiply the denominator by the whole number, and leave the numerator unaltered. The resulting fraction should always be reduced to its lowest terms by removing all factors common to numerator and denominator.

$$\text{Thus, } \frac{35}{36}\div 28=\frac{35}{36\times28}=\frac{7\times5}{36\times7\times4}=\frac{5}{36\times4}=\frac{5}{144}.$$

273. The meaning of Division as given in Art. 87 is not applicable when the divisor is a fraction. Hence, *Division* may be extended to express the finding of the fraction, the product of which and the divisor is the dividend and the *quotient* shows what *part* or *parts* the dividend is of the divisor.

Thus, to divide $\frac{3}{8}$ by $\frac{5}{7}$, we have, by definition,

$$\text{quotient} \times \tfrac{5}{7} = \tfrac{3}{8};$$

multiply each term of this equality by $\frac{7}{5}$,

$$\text{therefore quotient} \times \tfrac{5}{7} \times \tfrac{7}{5} = \tfrac{3}{8} \times \tfrac{7}{5},$$

$$\text{or quotient} = \tfrac{3}{8} \times \tfrac{7}{5},$$

that is, $\frac{3}{8} \div \frac{5}{7} = \frac{3}{8} \times \frac{7}{5}$. Hence the rule.

274. *To divide a fraction by a fraction.*

RULE. Multiply the dividend by the divisor *inverted*, and the result will be the quotient, which may be reduced to its lowest terms by cancelling any factors common to numerator and denominator: or, which is the same thing, *invert* the divisor, and then proceed by the Rule for the Multiplication of Fractions.

Ex. Divide $\frac{3}{7}$ by $\frac{4}{5}$.

Here, $\frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4}$
$= \frac{15}{28}$. *Ans.*

For, if $\frac{3}{7}$ be divided by 4, the quotient is $\frac{3}{28}$(Art. 241); but this quotient is 5 times too *small*, because the divisor has been taken 5 times too *great*; whence the quotient will be $\frac{3}{28} \times 5 = \frac{15}{28}$. (Art. 241.)

275. If the dividend be a whole number, or if dividend or divisor or both be mixed numbers, reduce them to improper fractions, and compound fractions to simple ones before the application of the Rule.

Ex. Divide $1\frac{1}{14}$ by $5\frac{5}{7}$; and $7\frac{7}{8}$ by $3\frac{3}{14}$ of $2\frac{1}{10}$.

(1) $1\frac{1}{14} \div 5\frac{5}{7} = \frac{15}{14} \div \frac{40}{7} = \frac{15}{14} \times \frac{7}{40} = \frac{3 \times 5 \times 7}{7 \times 2 \times 5 \times 8} = \frac{3}{2 \times 8} = \frac{3}{16}$. *Ans.*

(2) $7\frac{7}{8} \div 3\frac{3}{14}$ of $2\frac{1}{10} = \frac{63}{8} \div \frac{45}{14}$ of $\frac{21}{10} = \frac{63}{8} \div \frac{5 \times 9 \times 7 \times 3}{7 \times 2 \times 5 \times 2}$

$$= \frac{63}{8} \div \frac{9 \times 3}{2 \times 2} = \frac{63}{8} \times \frac{2 \times 2}{9 \times 3} = \frac{7 \times 9 \times 2 \times 2}{2 \times 2 \times 2 \times 9 \times 3}$$

$$= \frac{7}{2 \times 3} = \frac{7}{6} = 1\frac{1}{6}. \quad \textit{Ans.}$$

276. Numbers connected by *of* are considered a **single number.** The student should carefully notice the difference in meaning between $2\frac{1}{3} \div 1\frac{1}{2} \times \frac{3}{4}$ and $2\frac{1}{3} \div 1\frac{1}{2}$ of $\frac{3}{4}$. In the former, the sign $\div$ applies only to the next number $1\frac{1}{2}$; but in the latter, $1\frac{1}{2}$ of $\frac{3}{4}$ is a single number.

Thus, the former $= \frac{7}{3} \times \frac{2}{3} \times \frac{3}{4} = \frac{7}{6} = 1\frac{1}{6}$; the latter $= \frac{7}{3} \times \frac{2}{3} \times \frac{4}{3} = \frac{56}{27} = 2\frac{2}{27}$.

Examples LXXII.

1. Divide *orally* :—

(1) $\frac{1}{2}$ separately by 2, 3, 4, 5, 6. (2) $\frac{3}{5}$ separately by 4, 5, 7, 10, 12.
(3) $\frac{34}{37}$ ... ...12, 14, 15, 18, 20. (4) $\frac{15}{28}$ ... ... 3, 5, 30, 45.

2. Divide :—

(1) $\frac{40}{87}$ separately by 8, 16, 24, 36. (2) $\frac{145}{342}$ separately by 25, 75, 87.
(3) $2\frac{1}{6}$ ... ... 13, 65, 117 (4) $16\frac{2}{7}$ ... ... 19, 25, 32.

3. Divide *orally* :—

(1) $\frac{1}{2}$ separately by $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$. (2) $\frac{3}{5}$ separately by $\frac{6}{7}, \frac{2}{3}, \frac{4}{7}, \frac{1}{5}$.
(3) $\frac{7}{9}$ ... ... $\frac{5}{4}, \frac{2}{3}, \frac{4}{5}, \frac{1}{8}$. (4) $\frac{1}{2}$ ... ... $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}$.

4. Divide :—

(1) $\frac{7}{8}$ separately by $\frac{4}{7}, \frac{3}{8}, \frac{5}{16}, \frac{14}{15}$. (2) $\frac{13}{14}$ separately by $\frac{5}{8}, \frac{6}{7}, \frac{13}{15}, \frac{3}{14}$.
(3) $3\frac{3}{8}$ ... ... $5\frac{1}{2}, 2\frac{1}{3}, 4\frac{1}{5}, 6\frac{3}{5}$. (4) $4\frac{9}{16}$ ... ...$1\frac{7}{38}, 5\frac{2}{5}, 4\frac{1}{2}, 1\frac{8}{15}$.

Examples LXXIII.

1. Divide :—

(1) $\frac{2}{7}$ by $\frac{3}{8}$; $\frac{3}{5}$ by $\frac{4}{9}$; $\frac{4}{11}$ by $\frac{6}{13}$; $\frac{19}{5}$ by $\frac{83}{10}$; $\frac{26}{51}$ by $\frac{31}{34}$; $\frac{11}{12}$ by $\frac{47}{56}$; 143 by $\frac{11}{12}$.
(2) $2\frac{1}{5}$ by $3\frac{2}{9}$; $10\frac{4}{9}$ by $13\frac{2}{7}$; $17\frac{2}{3}$ by $7\frac{13}{14}$; $24\frac{3}{8}$ by $5\frac{5}{12}$; $2\frac{7}{8}$ by $\frac{3}{5}$ of $4\frac{1}{3}$.
(3) $3\frac{4}{9}$ by $\frac{1}{2}$ of $\frac{4}{7}$ of $\frac{5}{6}$; $3\frac{1}{125}$ of $3\frac{4}{7}$ by $\frac{47}{315}$ of 9 ; $15\frac{7}{11}$ of $8\frac{4}{5}$ by $\frac{4}{5}$ of $\frac{6}{11}$ of $15\frac{5}{6}$.
(4) $\frac{3}{5}$ of $1\frac{1}{7}\times 2\frac{2}{5}$ of $4\frac{3}{8}$ of $2\frac{2}{9}$ by $1\frac{3}{8}$ of $\frac{5}{6}\times\frac{5}{18}$ of $2\frac{1}{22}$ of 8.

2. Find the values of :—

(1) $(\frac{5}{7}\times\frac{3}{9}\times 13\frac{1}{2})\div(\frac{1}{9}\times\frac{3}{7}+54)$. (2) $(\frac{2}{7}+\frac{3}{4}$ of $\frac{133}{153})\div(\frac{4}{11}$ of $2\frac{5}{14}-\frac{14}{63})$.
(3) $6\frac{7}{8}+(1\frac{5}{6}+3\frac{8}{15})\div 6\frac{3}{7}$. (4) $6\frac{7}{8}+1\frac{5}{6}+3\frac{8}{15}\div 6\frac{3}{7}$.
(5) $10\frac{2}{3}\times 3\frac{3}{4}\div 4\frac{4}{5}-(2\frac{2}{3}+3\frac{3}{4})\div 4\frac{2}{5}+7\frac{1}{8}\div(3\frac{3}{4}+4\frac{4}{5})$.
(6) $2\frac{1}{4}\times 1\frac{1}{2}\div 1\frac{1}{8}$ of $2\frac{2}{3}$. (7) $2\frac{1}{4}$ of $1\frac{1}{2}\div 1\frac{1}{8}\times 2\frac{2}{3}$. (8) $2\frac{1}{4}$ of $1\frac{1}{2}\div 1\frac{1}{8}$ of $2\frac{2}{3}$.
(9) $\frac{2247}{1617}\div\frac{903}{1107}\times\frac{774}{875}\div\frac{1920}{5665}$. (10) $1\div[4-1\div\{2-1\div(1-\frac{5}{13})\}]$.
(11) $(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10})\div(\frac{1}{3}+\frac{1}{5}+\frac{1}{9}+\frac{1}{15})$.
(12) $(2+\frac{1}{3}+\frac{1}{8}+\frac{1}{12}+\frac{1}{15}+\frac{1}{18})\div(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5})$.

VII. COMPLEX AND CONTINUED FRACTIONS.

277. A fraction having a fraction or mixed number in the numerator or denominator or in both, is a **Complex Fraction.**

Thus, $\frac{\frac{2}{3}}{\frac{4}{5}}$, $\frac{8\frac{1}{4}}{9}$, $\frac{7}{9\frac{5}{8}}$, $\frac{4\frac{3}{7}}{5\frac{1}{9}}$, $\frac{1\frac{3}{5}+\frac{3}{4}}{4\frac{1}{2}\text{ of }3\frac{1}{3}}$ are *Complex Fractions.*

278. A complex fraction is read by inserting the word *by* for *divided by*, between the readings of numerator and denominator.

Thus, $\frac{3\frac{1}{4}}{4\frac{2}{5}}$ is read $3\frac{1}{4}$ *by* $4\frac{2}{5}$.

279. In the *sum* of a whole number and a fraction, when the fraction is either complex or simple (Art. 231), the sign is sometimes omitted, as in $5\frac{4\frac{3}{4}}{7}$ which means $5+\frac{4\frac{3}{4}}{7}$; and in a *product* when one of the factors is enclosed in a bracket the sign is often omitted, as in $\frac{2}{3}(\frac{7}{5}-\frac{3}{4})$, which means $\frac{2}{3}\times(\frac{7}{5}-\frac{3}{4})$.

280. Complex fractions are subject to the same rules as simple fractions, and can always be reduced to simple ones by treating them as the *quotient* of the numerator by the denominator. (Art. 235).

Thus, $\frac{2\frac{1}{5}}{3\frac{2}{9}}=2\frac{1}{5}\div3\frac{2}{9}=\frac{11}{5}\div\frac{29}{9}=\frac{11}{5}\times\frac{9}{29}=\frac{99}{145}$.

281. *To reduce a complex fraction to a simple fraction.*

RULE. Express the numerator and denominator of the complex fraction in the form of proper or improper fractions, and multiply the numerator by the denominator *inverted*; or more simply, multiply the numerator and denominator of the complex fraction by the L. C. M. of the denominators of the simple fractions.

Ex. 1. Reduce $\frac{5\frac{7}{9}}{9\frac{5}{11}}$ and $\frac{13\frac{1}{3}}{20}$ to simple fractions.

(1) $\frac{5\frac{7}{9}}{9\frac{5}{11}}=5\frac{7}{9}\div9\frac{5}{11}=\frac{52}{9}\div\frac{104}{11}=\frac{52}{9}\times\frac{11}{104}=\frac{52\times11}{9\times2\times52}=\frac{11}{18}$. *Ans.*

Or thus, $\frac{5\frac{7}{9}}{9\frac{5}{11}}=\frac{5\frac{7}{9}\times99}{9\frac{5}{11}\times99}=\frac{495+77}{891+45}=\frac{572}{936}=\frac{11\times52}{18\times52}=\frac{11}{18}$. *Ans.*

(2) $\frac{13\frac{1}{3}}{20}=13\frac{1}{3}\div20=\frac{40}{3}\div20=\frac{40}{3}\times\frac{1}{20}=\frac{20\times2}{3\times20}=\frac{2}{3}$. *Ans.*

Or thus, $\frac{13\frac{1}{3}}{20}=\frac{13\frac{1}{3}\times3}{20\times3}=\frac{40}{20\times3}=\frac{20\times2}{20\times3}=\frac{2}{3}$. *Ans.*

Ex. 2. Simplify $\frac{12\frac{2}{3}\text{ of }1\frac{8}{19}}{1\frac{5}{9}\text{ of }3\frac{3}{7}}$ and $\frac{8\frac{5}{6}-4\frac{2}{3}}{3\frac{3}{4}+7\frac{5}{12}}$.

(1) $\frac{12\frac{2}{3}\text{ of }1\frac{8}{19}}{1\frac{5}{9}\text{ of }3\frac{3}{7}}=12\frac{2}{3}\text{ of }1\frac{8}{19}\div1\frac{5}{9}\text{ of }3\frac{3}{7}=\frac{38}{3}\text{ of }\frac{27}{19}\div\frac{14}{9}\text{ of }\frac{24}{7}$

$=\frac{19\times2\times3\times}{3\times19}\div\frac{7\times2\times8\times3}{3\times3\times7}=2\times9\div\frac{2\times8}{3}$

$=\frac{2\times9\times3}{2\times8}=\frac{27}{8}=3\frac{3}{8}$ *Ans.*

(2) $\frac{8\frac{5}{6}-4\frac{2}{3}}{3\frac{3}{4}+7\frac{5}{12}}=\frac{(96+10)-(48+8)}{(36+9)-(84+5)}$, { Multiplying Numr. and Denr. by 12, the L. C. M. of the Denrs.

$=\frac{106-56}{45+89}=\frac{50}{134}=\frac{2\times25}{2\times67}=\frac{25}{67}$. *Ans.*

Examples LXXIV.

1. Reduce to their simplest forms :—

(1) $\dfrac{4\frac{3}{4}}{5\frac{4}{5}}$; $\dfrac{8\frac{3}{11}}{14\frac{6}{7}}$; $\dfrac{3\frac{3}{5}}{6}$; $\dfrac{18}{5\frac{1}{7}}$; $\dfrac{9\frac{7}{9}}{12\frac{5}{6}}$; $\dfrac{25\frac{7}{8}}{34\frac{11}{16}}$; $\dfrac{\frac{8}{27}}{\frac{13}{54}}$; $\dfrac{\frac{7}{45}}{4\frac{1}{30}}$; $\dfrac{2\frac{1}{2}}{3\frac{1}{3}+2\frac{1}{2}}$.

(2) $\dfrac{1\frac{3}{4} \text{ of } 1\frac{1}{7}}{1\frac{2}{3} \text{ of } \frac{6}{11}}$; $\dfrac{2\frac{1}{5} \text{ of } 8\frac{1}{3}}{2\frac{7}{9} \text{ of } 11\frac{2}{3}}$; $\dfrac{3\frac{1}{2} \text{ of } \frac{9}{14}}{3\frac{1}{2}+\frac{9}{14}}$; $\dfrac{3\frac{1}{7}-2\frac{2}{9}}{8\frac{2}{5} \text{ of } 17\frac{1}{2}}$; $\dfrac{5\frac{1}{7}+3\frac{1}{9}}{7\frac{11}{12}-1\frac{1}{4} \text{ of } 1\frac{8}{21}}$.

(3) $\dfrac{7\frac{5}{6}}{9\frac{7}{8}}$ of $\dfrac{14\frac{4}{11}}{13\frac{3}{7}}$; $\dfrac{6\frac{4}{9}}{13\frac{7}{14}}$ of $\dfrac{8\frac{5}{7}}{23\frac{1}{5}}$; $5\frac{1}{7}$ of $\dfrac{6\frac{3}{5}}{10\frac{5}{7}}$ of $\dfrac{15}{12\frac{4}{7}}$ of $\dfrac{8\frac{1}{3}}{42}$; $\dfrac{5}{6}$ of $\dfrac{13\frac{2}{5}}{4\frac{7}{9}}$.

(4) $\dfrac{2\frac{4}{11}}{2\frac{3}{5}} \div \dfrac{2\frac{7}{11}}{8\frac{7}{10}}$; $\dfrac{7\frac{1}{7}-3\frac{1}{8}}{6+4\frac{3}{4}} \div \dfrac{5\frac{1}{3}+1\frac{1}{7}}{6\frac{1}{7}-2\frac{1}{3}}$; $\dfrac{5\frac{1}{3}+4\frac{3}{4}}{3\frac{5}{6}+2\frac{7}{8}} \times \dfrac{5\frac{1}{3}-4\frac{3}{4}}{3\frac{5}{6}-2\frac{7}{8}} \div \dfrac{28\frac{4}{9}-22\frac{9}{16}}{14\frac{25}{36}-8\frac{17}{64}}$.

(5) $\dfrac{5\frac{2}{7}\div 7\frac{2}{5}}{2\frac{3}{8}-1\frac{4}{7}}$ of $\dfrac{2\frac{1}{4}\times 8\frac{1}{4}}{4\frac{1}{9}\div(\frac{1}{8}-\frac{1}{9})}$; $\dfrac{5\frac{4}{5}-2\frac{1}{3}}{3\frac{3}{4}+\frac{9}{10}}$ of $\dfrac{4\frac{1}{2}+5\frac{19}{25}}{4\frac{1}{20}}$; $\dfrac{3\frac{7}{11}-2\frac{5}{8}}{3\frac{7}{11}\times 2\frac{5}{8}} \times \dfrac{3\frac{7}{11}\div 2\frac{5}{8}}{3\frac{7}{11} \text{ of } 2\frac{5}{8}}$.

2. Reduce $\dfrac{2\frac{1}{3}}{7}$, $8\frac{1}{4}$, $\dfrac{9+\frac{1}{11}}{9\times\frac{1}{11}}$ and $16\frac{17}{18}$ to equivalent fractions with the least common denominator. Also reduce $\frac{3}{4}$ to a complex fraction having the denominator 5 and $\frac{7}{8}$ to a complex fraction having the numerator 10.

3. Compare the quantities $2\frac{1}{2}$, $\frac{2}{7}$ of $9\frac{2}{5}$ and $\dfrac{7\frac{1}{2}}{2\frac{5}{6}}$.

4. Find the values of :—

(1) $\frac{2}{3}$ of $3\frac{3}{10}+\dfrac{1\frac{3}{4}}{2\frac{5}{6}}$ of $17+\frac{2}{5}$ of $5\frac{3}{4}$ of $\frac{34}{51}$. (2) $\frac{2}{3}$ of $\frac{5}{7}+9+\dfrac{2\frac{4}{5}}{7}+\dfrac{1\frac{2}{3}}{2\frac{1}{2}}$.

(3) $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}+\frac{1}{5}$ of $\frac{2}{3}$ of $1\frac{7}{8}+\frac{37}{19}$ of $\frac{1}{9}$ of $\dfrac{28\frac{1}{2}}{2}$. (4) $1\frac{1}{3}+\frac{8}{3}$ of $\frac{41}{34}+\dfrac{4}{5\frac{1}{10}}$.

5. Find the difference between :—

(1) $\frac{3}{5}$ of $\dfrac{4\frac{1}{6}}{5\frac{1}{7}}$ and $\frac{2}{3}$ of $7\frac{1}{2}$. (2) $\dfrac{3\frac{2}{5}}{4\frac{7}{9}}$ and $\dfrac{6\frac{2}{3}}{12\frac{2}{7}}$. (3) $2\frac{3}{5}$ of $\dfrac{5\frac{1}{4}}{4\frac{1}{5}}$ and $\dfrac{7\frac{1}{3}}{11}$ of $15\frac{3}{8}$.

6. Find the values of :—

(1) $2\frac{3}{5}$ of $\dfrac{6\frac{1}{4}}{1\frac{3}{4}}\times\dfrac{3\frac{2}{5}}{5\frac{4}{7}}$. (2) $6\frac{2}{5}$ of $9\frac{5}{8}\times 12\frac{8}{11}$ of $\dfrac{1\frac{1}{2}}{112}$. (3) $\dfrac{7\frac{3}{7}}{40\frac{5}{9}}\div\dfrac{17\frac{1}{3}}{73}$

(4) $\dfrac{2}{3\frac{3}{16}}$ of $\dfrac{6\frac{1}{11}}{8}\times\frac{2}{3}$ of $8\frac{1}{4}$ of $\frac{35}{66}$. (5) $\dfrac{2\frac{3}{4}}{5\frac{7}{9}}$ of $\frac{1}{3}\times\frac{3}{5}$ of $\dfrac{4\frac{3}{8}}{7\frac{1}{2}}\times\dfrac{7\frac{3}{7}}{5\frac{1}{5}}$.

(6) $2\frac{10}{11}$ of $5\frac{1}{16}$ of $1331\div 3\frac{11}{12}$ of $\dfrac{44}{13\frac{3}{7}}$ of $202\frac{1}{2}$.

(7) $\dfrac{11\frac{3}{5}}{29}$ of $\dfrac{4\frac{1}{5}}{13}$ of $\dfrac{3\frac{1}{4}}{10\frac{3}{8}}\times 5\frac{1}{5}$ of $6\times 20\frac{3}{4}$. (8) $8\frac{1}{3}$ of $7\frac{1}{2}+2\frac{1}{9}$ of $\dfrac{4\frac{1}{2}}{14\frac{1}{4}}-8\frac{1}{6}$.

(9) $\dfrac{\frac{2}{11}+\frac{1}{21}+\frac{8}{77}-\frac{1}{5}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}$ of $\dfrac{\frac{4}{1\frac{1}{7}} \text{ of } \frac{3}{1\frac{1}{9}} - \frac{4}{3\frac{1}{3}} \text{ of } \frac{2\frac{1}{2}}{1\frac{1}{3}}}{\frac{3}{200} \text{ of } \frac{11}{7} \text{ of } 3\frac{1}{2}+2\frac{1}{55} \text{ of } 1\frac{5}{6} \text{ of } \frac{1}{19}}$.

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282. *To find the complete quotient in dividing a number by an integer.*

RULE. Divide in the usual way, and to the integral quotient add the fraction whose numerator is the remainder and denominator the divisor.

Ex. Divide 4148 by 117, and $3136\frac{7}{12}$ by 95, giving the complete quotient in each case.

(1)
$$\begin{array}{r} 117)4148(35 \\ 351 \\ \hline 638 \\ 585 \\ \hline 53 \end{array}$$
∴ the complete quotient $= 35\frac{53}{117}$.

(2)
$$\begin{array}{r} 95)3136\frac{7}{12}(33 \\ 285\phantom{\frac{7}{12}(33} \\ \hline 286\phantom{\frac{7}{12}(33} \\ 285\phantom{\frac{7}{12}(33} \\ \hline 1\phantom{\frac{7}{12}(33} \end{array}$$
In dividing by 95, the integral remainder is 1 and the full remainder is $1\frac{7}{12}$.

But $1\frac{7}{12}\div 95=\frac{19}{12}\times\frac{1}{95}=\frac{1}{60}$.

∴ the complete quotient $= 33\frac{1}{60}$.

283. Fractions of the nature given below are **Continued Fractions**, and can be simplified by beginning at the bottom and working upwards.

Ex.
$$\frac{1}{4-\cfrac{1}{2-\cfrac{1}{1-\frac{5}{13}}}}=\frac{1}{4-\cfrac{1}{2-\cfrac{13}{13-5}}}=\frac{1}{4-\cfrac{1}{2-\cfrac{13}{8}}}=\frac{1}{4-\cfrac{8}{16-13}}$$
$$=\frac{1}{4-\frac{8}{3}}=\frac{3}{12-8}=\frac{3}{4}. \quad \textit{Ans.}$$

Examples LXXV.

1. Find the complete quotient in dividing :—

(1) 3127 by 43. (2) 6556 by 401. (3) 2221 by 87.
(4) 8768 by 45. (5) $845\frac{14}{15}$ by 12. (6) $6739\frac{13}{18}$ by 37 and by 73.
(7) $4164\frac{7}{8}$ by 11, and by 132. (8) $56941\frac{5}{31}$ by 27.

2. Simplify :—

(1) $\dfrac{3}{1+\cfrac{2}{5+\frac{7}{8}}}$. (2) $\frac{1}{13}$ of $\dfrac{1}{1+\cfrac{\frac{1}{5}}{3+\frac{1}{4}}}$. (3) $\dfrac{2}{5+\cfrac{3}{4+\frac{1}{3}}}$. (4) $2+\dfrac{1}{3+\cfrac{4}{5+\frac{1}{6}}}$

(5) $3+\dfrac{1}{2+\dfrac{3}{5+\frac{1}{2}}}$.

(6) $2+\dfrac{3}{1+\dfrac{3}{6+\dfrac{4}{5+\frac{1}{3}}}}$.

(7) $6\frac{7}{8}+\dfrac{\dfrac{3\frac{3}{4}}{2}}{3\frac{1}{2}+\dfrac{3}{4\frac{4}{5}}}$.

(8) $2\frac{3}{4}+\dfrac{4}{5+\dfrac{6}{7+\frac{8}{9}}}$.

(9) $\dfrac{3}{4+\dfrac{3}{5+\dfrac{3}{6+\frac{3}{7}}}}+\dfrac{1}{2+\dfrac{1}{3+\dfrac{1}{4+\frac{1}{5}}}}$.

(10) $2\frac{1}{2}\times\dfrac{1}{3\frac{1}{3}+4\frac{1}{4}}+\dfrac{\frac{5}{7}-\frac{1}{3}\times(\frac{1}{2}+\frac{1}{7})}{1+\dfrac{1}{2-\frac{1}{4}}}$.

(11) $1+\dfrac{2}{3+\dfrac{1}{4+\dfrac{1}{5\frac{2}{3}}}}+\dfrac{18}{23}$.

(12) $\left(3-\dfrac{1}{2-\dfrac{1}{6+\frac{1}{5}}}\right)\div\left(1+\dfrac{1}{4+\dfrac{1}{3-\dfrac{1}{3+\frac{1}{2}}}}\right)$.

(13) $3\frac{4}{5}+\dfrac{5\frac{5}{6}}{7\frac{7}{8}+\dfrac{8\frac{9}{10}}{10\frac{1}{12}+\dfrac{13}{9\frac{3}{4}}}}$.

(14) $\dfrac{1}{3+\dfrac{1}{5+\frac{1}{7}}}+\dfrac{3}{8-\dfrac{7}{2-\frac{3}{4}}}+\dfrac{5}{6-\dfrac{5}{2-\frac{5}{6}}}$.

(15) $\dfrac{\dfrac{1}{1-\dfrac{1}{2-\frac{1}{2}}}+\frac{1}{3}+(1+\frac{1}{2})}{3\left(1+\dfrac{2}{3\frac{1}{2}}\right)-4}$.

(16) $1\frac{1}{4}\times12\frac{1}{5}$ of $\frac{3}{8}-\dfrac{6\frac{1}{4}+7\frac{15}{16}}{12-6\frac{2}{3}}-\dfrac{4+\dfrac{1}{4+\dfrac{1}{4-\frac{1}{4}}}}{4}$.

(17) $\dfrac{2}{4+\dfrac{2}{3+\frac{1}{4}}}+\dfrac{3}{3-\dfrac{2}{3-\frac{1}{4}}}+\dfrac{4}{4\div\dfrac{2}{4\div\frac{1}{4}}}+\dfrac{5}{3+\dfrac{\frac{1}{7}}{2+\frac{1}{3}}}$.

(18) $\dfrac{4\frac{2}{3}}{4\frac{2}{3}+\dfrac{4\frac{1}{3}}{4\frac{2}{3}+\dfrac{5}{4\frac{2}{3}+2\frac{1}{3}}}}$.

(19) $\dfrac{2}{2+\dfrac{2}{2+\dfrac{1}{2+\dfrac{2}{5+\frac{1}{4}}}}}+\dfrac{2}{5+\dfrac{1}{2+\dfrac{1}{3+\frac{1}{2}}}}+\dfrac{1\dfrac{1}{3}\times\dfrac{2\frac{1}{3}}{3\frac{1}{3}}\left(3-\dfrac{1}{4}\right)\div1\dfrac{1}{2}}{2+\dfrac{1}{3+\dfrac{1}{1+\dfrac{4}{5+\frac{1}{5}}}}}$.

(20) $1-\dfrac{1}{2+\dfrac{1}{1-\dfrac{1}{2+\dfrac{1}{1-\dfrac{1}{\frac{1}{5}\div\frac{2}{3}}}}}}\times1-\dfrac{1}{2+\dfrac{1}{2\times\dfrac{1}{1-\dfrac{1}{1\frac{2}{3}}}}}\times2\div\dfrac{1}{1-\dfrac{1}{1+\dfrac{1}{1-\dfrac{1}{1+\dfrac{1}{\frac{4}{5}}}}}}$.

3. Simplify :—

(1) $15\frac{1}{4}+1\frac{2}{3}\div(7\frac{3}{4}-6\frac{2}{3})$.
(2) $(15\frac{1}{4}+1\frac{2}{3})\div\frac{2}{15}(7\frac{3}{4}-6\frac{2}{3})$.

(3) $16-\{2\frac{1}{2}-(4\frac{1}{2}+1\frac{1}{4})\}$.
(4) $16+\{2\frac{1}{2}-(4\frac{1}{2}-1\frac{1}{4})\}$.

(5) $5\frac{1}{2}-\{5\frac{3}{4}-\frac{2}{3}(3\frac{2}{3}+2\frac{1}{5})\}$.
(6) $3\frac{2}{5}\times\frac{10}{11}\div3\frac{1}{7}$ of $3\frac{3}{7}$ of $\frac{5}{14}$ of $1\frac{16}{33}$.

(7) $(\frac{1}{2}+\frac{1}{3})$ of $(1\frac{1}{3}+2\frac{3}{4})$ of $(2\frac{1}{14}-1\frac{1}{2})$ of $(3\frac{1}{10}-\frac{3}{7})$.

(8) $(3\frac{7}{11}\div4\frac{2}{7})$ of $(10\frac{5}{7}\div7\frac{1}{2})$ of $\frac{77}{540}$.
(9) $3\frac{7}{11}\div(4\frac{2}{7}\div10\frac{5}{7})\div(7\frac{1}{2}\div\frac{77}{540})$.

(10) $1\frac{1}{5}$ of $5\frac{1}{4}+(4\frac{1}{2}-\frac{3}{10})\div1\frac{1}{15}-6\frac{3}{4}\div\frac{1}{2}$ of $2\frac{2}{3}$

4. Shew that the simple fraction equivalent to the value of $\frac{1}{3}$ of $\dfrac{\frac{1}{2}}{\frac{3}{4}}+\frac{1}{2}$ of $\dfrac{\frac{3}{4}}{\frac{9}{16}+1}+\frac{1}{6}$ of $\dfrac{\frac{1}{5}}{4}$, is of the same magnitude as that expressed by $\frac{3}{10}+\frac{1}{2}$ of $\frac{7}{15}$ of $\frac{7}{20}$.

5. Prove that $\frac{1}{7}$ of $(1-\frac{2}{7})+\frac{4}{5}$ of $\frac{1}{10}+\frac{3}{5}$ of $(\frac{1}{2}+\frac{1}{14})+\frac{3}{70}$ of $(\frac{2}{7}+\frac{1}{5})=1$.

VIII. SIMPLIFICATION OF FRACTIONS.

284. What has been proved in the adaptation of the fundamental operations to fractions, will furnish the means of simplifying arithmetical expressions formed by their combinations; and, in general, only very slight *mental* exertion will be required, if the attention of the *eye* be directed to the *composition* of the *terms* of the fractions concerned, and their *resolution* into the *factors* of which they are made up.

Ex. 1. Simplify $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}-\dfrac{13}{24}$ of $\dfrac{576}{264}$.

$$\text{The expression}=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{2}{5}+\frac{2}{7}+\frac{2}{9}}-\frac{13}{24}\text{ of }\frac{24\times24}{264}=\frac{\dfrac{6+4+3}{12}}{\dfrac{126+90+70}{315}}-\frac{13\times24}{264}$$

$$=\frac{\frac{13}{12}}{\frac{286}{315}}-\frac{13\times24}{11\times24}=\frac{13}{12}\times\frac{315}{286}-\frac{13}{11}$$

$$=\frac{13}{4\times3}\times\frac{105\times3}{13\times22}-\frac{13}{11}=\frac{105}{88}-\frac{13}{11}=\frac{105-104}{88}=\frac{1}{88}.\quad \textit{Ans.}$$

Ex. 2. Simplify $\left\{2\frac{3}{4}+\frac{5}{2}\text{ of }\dfrac{7}{3\frac{4}{5}}-\dfrac{1\frac{2}{3}}{2\frac{1}{2}}\right\}\div1\frac{77}{228}$.

$$\text{The expression}=\left\{\frac{11}{4}+\frac{5}{2}\text{ of }\frac{7\times5}{19}-\frac{\frac{5}{3}}{\frac{5}{2}}\right\}\div\frac{305}{228}$$

$$=\left\{\frac{11}{4}+\frac{5\times7\times5}{2\times19}-\frac{5}{3}\times\frac{2}{5}\right\}\times\frac{228}{305}$$

$$= \left\{\frac{11}{4} + \frac{175}{38} - \frac{2}{3}\right\} \times \frac{228}{305} = \frac{627 + 1050 - 152}{228} \times \frac{228}{305}$$

$$= \frac{1677 - 152}{228} \times \frac{228}{305} = \frac{1525}{305} = 5. \textit{ Ans.}$$

Examples LXXVI.

Simplify the following :—

1. $\frac{3}{8}$ of $\frac{4}{7} - \frac{2}{11}$ of $3\frac{1}{7} + \frac{5}{9}$ of $3\frac{3}{8}$.

2. $(\frac{2}{7} + \frac{3}{4}$ of $\frac{136}{153}) \div 2\frac{4}{11}$ of $(1\frac{5}{14} - \frac{14}{63})$.

3. $\dfrac{4\frac{1}{3} \times 4\frac{1}{3} \times 4\frac{1}{3} - 1}{4\frac{1}{3} \times 4\frac{1}{3} - 1}$.

4. $\dfrac{4\frac{2}{3} \times 4\frac{2}{3} - 3\frac{1}{4} \times 3\frac{1}{4}}{4\frac{2}{3} - 3\frac{1}{4}}$.

5. $\dfrac{1 + 6\frac{7}{8} \times (1 + 6\frac{7}{8})}{1 + 5\frac{4}{5} \times (1 + 5\frac{4}{5})}$.

6. $\dfrac{7\frac{1}{2}}{6\frac{1}{2}} + \dfrac{11\frac{1}{2} - 2\frac{2}{5}}{11\frac{1}{2} + 2\frac{2}{5}} \times 10\frac{9}{13} - 6\frac{42}{273}$.

7. $\dfrac{14\frac{1}{4} - 6\frac{1}{2}}{3\frac{1}{3} + 6\frac{13}{15}} \div \dfrac{4\frac{1}{12} + 6\frac{1}{8}}{9\frac{1}{5} - 3\frac{1}{4}} + (30\frac{7}{18} - 22\frac{8}{9})$.

8. $\dfrac{1 + 2 \times \frac{4}{5} + \frac{4}{5} \times \frac{4}{5}}{1 - \frac{4}{5} \times \frac{4}{5}}$.

9. $\dfrac{1}{3\frac{1}{5}} - \dfrac{2\frac{1}{4}}{9} + \dfrac{3\frac{3}{8}}{2} + \dfrac{\frac{4}{7}}{4\frac{4}{7}}$.

10. $(\frac{1}{26}$ of $3\frac{1}{4}) + (\frac{3}{8} \div \frac{27}{88}) - \left(\dfrac{1}{1\frac{1}{7}} - \dfrac{7\frac{2}{3}}{9}\right)$.

11. $\dfrac{\dfrac{3\frac{1}{3}}{7\frac{1}{5}} - \dfrac{1}{2\frac{2}{7}}}{4\frac{1}{5} \text{ of } \dfrac{2\frac{2}{3}}{6\frac{3}{8}}}$ of $\dfrac{11\frac{1}{3}}{17} \times 52\frac{4}{11}$.

12. $\dfrac{3 + \frac{5}{3} \text{ of } \dfrac{21}{7\frac{7}{8}} - \dfrac{1}{4} - \dfrac{1\frac{2}{3}}{2\frac{1}{2}}}{10 - \dfrac{151}{228} \text{ of } 5}$.

13. $\dfrac{1\frac{3}{5}}{3 + \dfrac{1}{3\frac{1}{3}}} + \dfrac{1\frac{2}{3} \text{ of } 4\frac{2}{7}}{1\frac{3}{9} \text{ of } 3\frac{3}{7}} + \dfrac{5\frac{2}{5} \text{ of } 7\frac{2}{9}}{8\frac{5}{24} - 3\frac{5}{12}}$.

14. $\dfrac{2\frac{1}{2}}{3\frac{1}{4}} + \dfrac{1\frac{1}{2} - \frac{5}{8}}{1\frac{1}{2} + \frac{5}{8}} - 1\frac{2}{35}$.

15. $\dfrac{2\frac{1}{2} - \frac{5}{6}}{2\frac{1}{2} + \frac{5}{6}} + \dfrac{7}{12}$ of $\dfrac{9 \times 10}{14 \times 3} - \dfrac{22\frac{1}{2}}{30}$.

16. $\dfrac{1\frac{2}{5}}{3\frac{1}{2}} - \dfrac{5\frac{5}{8}}{6\frac{1}{4}}$ of $\left(\dfrac{1}{5} - \dfrac{\frac{1}{2} - \frac{1}{3}}{4\frac{3}{4} - 3\frac{3}{8}}\right)$.

17. $\dfrac{1}{26}\left(5\dfrac{1}{2} - 2\dfrac{1}{4}\right) + \left(5\dfrac{1}{3} \text{ of } \dfrac{9}{128} \div \dfrac{9}{8} \text{ of } \dfrac{3}{8}\right) - \left(\dfrac{1}{1\frac{1}{7}} - \dfrac{1\frac{1}{3}}{3}\right) \div \dfrac{8}{7}\left(2 - \dfrac{4}{9}\right)$.

18. $\dfrac{\frac{3}{7} - \frac{2}{9}}{\frac{3}{7} + \frac{2}{9}}$ of $2\dfrac{11}{26} \div \dfrac{4}{13 - 3\frac{8}{9}} + 3\dfrac{11}{16} - \dfrac{3}{3 - 1\frac{10}{13}}$.

19. $\dfrac{1 + 2\frac{1}{2} + 3\frac{1}{3}}{\dfrac{1}{1\frac{1}{2}} + \dfrac{2}{2\frac{1}{3}} + \dfrac{3}{3\frac{1}{4}}} \times \dfrac{55\frac{2}{3} \div 11}{1\dfrac{2}{11} \text{ of } 13\dfrac{2}{3}}$.

20. $\dfrac{1 - \frac{4}{9}}{3\frac{1}{4} + 1\frac{1}{2} + \dfrac{1\frac{2}{3}}{6\frac{2}{3}}} \times \left(\dfrac{1 + \frac{1}{2}}{1 - \frac{1}{2}}\right)^2$.

21. $\dfrac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{\dfrac{1}{2\frac{1}{2}} + \dfrac{1}{3\frac{1}{2}} + \dfrac{1}{4\frac{1}{2}}}$ of $\dfrac{1}{13}$ of $\dfrac{1}{1 + \dfrac{\frac{1}{3}}{3 + \frac{1}{4}}}$.

22. $\left\{\dfrac{11\frac{3}{4} - 10\frac{1}{2}}{11\frac{3}{4} + 10\frac{1}{2}} \div \dfrac{10\frac{3}{8} + 11\frac{1}{8}}{10\frac{3}{8} - 9\frac{1}{8}}\right\} \times \dfrac{\frac{2}{7} + \frac{3}{11}}{\frac{2}{7} - \frac{3}{11}} \times \dfrac{8\frac{1}{11}}{2\frac{3}{11}}$.

23. $$\frac{2\frac{1}{4}}{2\frac{2}{3}}+\frac{2\frac{1}{2}+5\frac{1}{5}}{3\frac{1}{3}+9\frac{1}{2}}+\frac{5}{9}\text{ of }\frac{9}{10}+\frac{3}{8}\text{ of }\frac{3}{20}.$$

24. $$\left(6\frac{3}{7}\text{ of }\frac{5\frac{1}{6}-4\frac{7}{12}}{12\frac{2}{3}-7\frac{5}{12}}\right)\div\frac{1+\frac{1}{2\frac{1}{3}}}{2}.$$

25. $$\frac{4\frac{1}{6}}{1\frac{7}{16}\text{ of }3\frac{1}{5}}-\left(\frac{2}{7}\text{ of }\frac{2\frac{2}{3}+1\frac{1}{6}}{2-\frac{4}{25}}\right).$$

26. $$\frac{1}{3}\times\frac{17}{3}\text{ of }\frac{27}{85}+\left(2\frac{1}{3}+\frac{1}{3+\frac{1}{4}}\right)\times\frac{39}{103}.$$

27. $$1\frac{1}{11}-\frac{1-\frac{7}{22}}{2-\frac{1}{3}}+\frac{1\frac{2}{5}}{3\frac{1}{2}}-\frac{5\frac{5}{8}}{6\frac{1}{4}}\text{ of }\frac{2}{3}\left\{\frac{1}{5}-\frac{\frac{1}{2}-\frac{1}{3}}{4\frac{3}{4}-3\frac{5}{6}}\right\}.$$

28. $$\left\{\frac{7}{5-\frac{8}{3}}\div\frac{3-\frac{2}{3-1\frac{1}{2}}}{4-1\frac{1}{2}}\right\}-\frac{5}{7}\text{ of }\frac{3}{4}\left\{\frac{1}{1\frac{3}{7}}+\frac{6}{5}\text{ of }\frac{3\frac{1}{3}-2\frac{1}{2}}{\frac{47}{21}-2}\right\}.$$

29. $$\frac{1+5\frac{4}{9}(1+5\frac{4}{9})}{1+2\frac{1}{3}(1+2\frac{1}{3})}\times\frac{4\frac{1}{12}+2\frac{2}{3}}{13\frac{5}{12}-3\frac{1}{3}}\text{ of }\frac{11}{111}.$$

30. $$\frac{1\frac{3}{5}+7\frac{1}{3}\times1\frac{1}{11}-9\frac{1}{7}}{\frac{4\frac{3}{4}}{3\frac{4}{5}}\text{ of }\left(5\frac{5}{7}-3\frac{7}{10}\right)}.$$

31. $$\frac{5\frac{5}{8}\div\frac{2}{3}}{1\frac{1}{5}\text{ of }\frac{5}{9}\div10\frac{1}{3}}\times\frac{2}{5}\text{ of }\frac{1\frac{1}{2}\text{ of }4\frac{1}{9}}{13\frac{7}{8}\text{ of }5\frac{1}{3}}.$$

32. $$\frac{1}{4\frac{2}{3}}\text{ of }\frac{4\frac{3}{4}}{2\frac{1}{4}}+\frac{2}{4\frac{1}{2}}\text{ of }\frac{1\frac{1}{5}}{1\frac{2}{3}}-\frac{2\frac{3}{7}}{8}.$$

33. $$\frac{\frac{3}{5}\text{ of }(\frac{6}{7}-\frac{3}{4})}{\frac{7}{9}\text{ of }(\frac{3}{4}-\frac{5}{7})}\text{ of }\left\{\frac{\frac{8}{15}-\frac{6}{13}}{\frac{5}{13}-\frac{3}{11}}-\frac{9\frac{1}{5}+3\frac{4}{9}}{5\frac{11}{14}\text{ of }7\frac{1}{9}}+\frac{7\frac{5}{8}-4\frac{2}{5}}{\frac{21}{23}-\frac{19}{21}}\right\}.$$

34. $$7\frac{8}{13}\text{ of }\frac{1}{10+\frac{1}{3+\frac{1}{30}}}.$$

35. $$3\frac{1}{2}+\frac{2\frac{1}{4}}{3\frac{1}{4}+\frac{2}{5\frac{1}{6}+\frac{1}{4\frac{1}{2}}}}.$$

36. $$11+\frac{1}{1-\frac{1}{1+\frac{1}{8+\frac{1}{11}}}}.$$

37. $$\left(2+\frac{1}{3-\frac{1}{5+\frac{1}{6}}}\right)\div\{1\frac{3}{8}\div(1\frac{3}{4}\times14\frac{1}{2})\}.$$

38. $$\frac{3\frac{3}{7}}{1\frac{1}{5}\text{ of }2\frac{2}{3}}+\frac{8\frac{1}{4}\text{ of }2\frac{2}{7}}{11}-\frac{9\frac{3}{4}\text{ of }1\frac{1}{8}}{4(\frac{1}{2}+\frac{1}{24})}+\frac{2-\frac{4}{7}}{1-\frac{4}{9}}.$$

39. $$\frac{4\frac{1}{3}\text{ of }3-3\frac{1}{4}\text{ of }3\frac{1}{2}}{4\frac{1}{4}-3\frac{1}{3}}\div\left\{6\frac{1}{2}\text{ of }\frac{\frac{1}{3\frac{1}{3}}+\frac{1}{2\frac{1}{4}}}{3\frac{1}{3}-2\frac{1}{4}}\right\}\times6\frac{35}{117}.$$

40. $$\frac{5\frac{4}{5}-2\frac{1}{8}}{3\frac{3}{4}+\frac{9}{20}}\text{ of }\frac{4\frac{1}{2}+5\frac{19}{28}}{4\frac{1}{20}}\text{ of }\frac{2\frac{3}{5}+1\frac{2}{3}}{7\frac{19}{24}-2\frac{1}{4}}\text{ of }\frac{6\frac{1}{4}}{7\frac{1}{9}}.$$

41. $$\left\{\left(3\frac{8}{63}-\frac{2}{3}\text{ of }\frac{8\frac{1}{2}}{7}+\frac{2\frac{2}{5}}{5\frac{1}{4}}\right)\div\frac{2\frac{7}{9}+9\frac{2}{7}}{6\frac{1}{8}-4\frac{3}{4}}\right\}\text{ of }34\frac{18}{23}.$$

42. $$\frac{6\frac{3}{4}-1\frac{5}{14}}{2\frac{1}{6}+1\frac{2}{7}}\text{ of }\frac{(3\frac{4}{5}+5\frac{1}{6}-\frac{1}{45})(4\frac{1}{5}-3\frac{1}{4})}{1\frac{5}{11}+2\frac{1}{8}-(2\frac{9}{10}-\frac{1}{8}-\frac{1}{22})}.$$

43. $$8-8\times\frac{2\frac{1}{5}-1\frac{2}{7}}{2-\frac{1}{6-\frac{1}{6}}}.$$

44. $\left\{\dfrac{3\frac{1}{2}+2\frac{1}{3}}{\frac{3}{4}\text{ of }9\frac{1}{6}} \div \dfrac{2\frac{4}{5}-1\frac{1}{2}+9\frac{1}{11}}{4\frac{1}{5}-2\frac{1}{4}+13\frac{7}{11}}\right\}$ of $\dfrac{5\frac{1}{2}}{2\frac{1}{3}}$.

45. $\dfrac{1\frac{1}{5}-\frac{2}{3}\text{ of }1\frac{5}{7}+1\frac{1}{9}}{1\frac{4}{5}-\frac{3}{2}\text{ of }1\frac{1}{7}+1\frac{6}{9}}$ of $1\frac{1}{2}$ of $\dfrac{6\frac{3}{4}-1\frac{5}{14}}{2\frac{1}{6}+1\frac{3}{4}}$.

46. $\left\{\dfrac{2}{3-\dfrac{1}{1-\frac{1}{2}}}-\dfrac{1}{3}\text{ of }\left(5-\dfrac{2}{\frac{3}{2}-\frac{1}{6}}\right)\right\} \div \dfrac{\frac{1}{2}+\frac{3}{4}}{1\frac{1}{2}}$.

47. $\dfrac{17}{7+\dfrac{3}{4-2\frac{3}{4}}} \times \dfrac{2021}{2193} \div \left(1\dfrac{37}{48}-\dfrac{15}{16}\right)$.

48. $\left(\dfrac{1+\frac{1}{5}}{1-\frac{1}{5}}\right)^2 \div \left(\dfrac{1+\frac{5}{13}}{1-\frac{5}{13}}\right)^2$.

49. $\dfrac{8\frac{7}{8}-7\frac{6}{7}+5\frac{5}{6}-4\frac{4}{5}}{9\frac{9}{10}-8\frac{13}{15}+7\frac{7}{8}-6\frac{6}{7}} \div \left\{\frac{3}{7}\text{ of }2\frac{1}{17} \div \dfrac{1\frac{2}{3}}{2\frac{3}{7}}\right\}$.

50. $\left(\dfrac{2}{3-\frac{4}{5}}+\dfrac{3}{4-\frac{5}{6}}\right) \div \left(\dfrac{3}{2-\frac{1}{4}}-\dfrac{1}{3-\frac{4}{5}}\right) \times \left(\dfrac{1}{\frac{2}{3}-\frac{2}{9}}-\dfrac{1}{1\frac{1}{2}-\frac{19}{22}}\right)$

$\div \left(\dfrac{1}{1\frac{1}{7}+\frac{1}{8}}-\dfrac{2}{6\frac{3}{5}-2\frac{5}{11}}\right)$; $\dfrac{\frac{1}{4}+\frac{2}{9}}{4-\frac{5}{9}\text{ of }5\frac{1}{2}} \div \dfrac{\frac{3}{5}+\frac{1}{4}}{\frac{7}{10}\text{ of }4\frac{1}{2}-2\frac{1}{2}}$.

51. $\left\{\left(\dfrac{1}{\frac{2}{3}-\frac{2}{9}}-\dfrac{1}{1\frac{1}{2}-\frac{19}{22}}\right) \div \left(\dfrac{1}{1\frac{1}{7}-\frac{1}{8}}-\dfrac{2}{6\frac{3}{5}-2\frac{5}{11}}\right)\right\}$ of $\left\{\left(\dfrac{2}{3-\frac{4}{5}}+\dfrac{3}{4-\frac{5}{6}}\right)\right.$

$\left.\div \left(\dfrac{3}{2-\frac{1}{4}}-\dfrac{1}{3-\frac{4}{5}}\right)\right\}$; $\dfrac{3\frac{5}{7} \div 2\frac{4}{11}}{\frac{5}{7}\text{ of }(11\frac{3}{4}-2\frac{1}{8})} \div \dfrac{3\frac{2}{9}\text{ of }5\frac{2}{5}}{5\frac{1}{16}+\frac{1}{2}\text{ of }4\frac{3}{8}}$.

52. $1\frac{5}{6}$ of $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{2\frac{1}{2}-3\frac{1}{3}+4\frac{1}{4}} \times \left(\dfrac{2\frac{1}{2}}{3\frac{1}{7}}+\dfrac{\frac{3}{16}}{1\frac{3}{8}}\right) \div \left(\dfrac{3}{4\frac{1}{2}}+\dfrac{4\frac{1}{2}}{3}\right)$.

53. $\dfrac{11\frac{1}{9}-2\frac{1}{4}}{6\frac{1}{4}-3\frac{1}{16}} \div \dfrac{3\frac{1}{3}+1\frac{1}{2}}{2\frac{1}{2}+1\frac{3}{4}} \times \dfrac{3\frac{1}{3}-1\frac{1}{2}}{2\frac{1}{2}-1\frac{3}{4}}$.

54. $\dfrac{7\frac{1}{2}+1\frac{17}{22}}{8\frac{2}{3}+3\frac{8}{33}}-\dfrac{3\frac{4}{5}+\frac{29}{45}}{3\frac{3}{4}+14\frac{1}{36}}$.

55. $\left(\dfrac{\frac{1}{3}}{5\frac{3}{4}}+\dfrac{4}{4\frac{3}{5}}+\dfrac{3\frac{1}{4}-8\frac{1}{5}}{\frac{1}{2\frac{1}{2}}+\frac{1}{1\frac{1}{3}}}\text{ of }\frac{1}{5}\right) \times \left(\dfrac{\frac{1}{4}+\frac{1}{3}}{1\frac{3}{4}}-\frac{1}{2}+\dfrac{1}{2\frac{4}{5}}-\dfrac{\frac{1}{3}}{2\frac{11}{12}}+7\right)$ of $\dfrac{\frac{1}{3}+\frac{1}{9}}{\frac{1}{9}+\frac{5}{4}}$;

$3-\left(\dfrac{3\frac{1}{3}-\frac{1}{6}}{3\frac{3}{8}+\frac{1}{6}}-2\frac{5}{7}\text{ of }\dfrac{4}{19}\right)$.

56. $\left\{\dfrac{\frac{2}{5}\text{ of }\frac{7}{8}\text{ of }6\frac{2}{7}+7\frac{7}{11}+19\frac{13}{20}+8\frac{19}{22}}{3\frac{5}{8}+\frac{2}{7}+4\frac{1}{10}-\frac{11}{15}\text{ of }1\frac{6}{7}}-\dfrac{221}{680}\right\} \div \left\{\dfrac{1\frac{39}{52}+\frac{6}{65}}{\frac{9}{38}}+39\frac{11}{12}-24\frac{2}{5}\right\}$

IX. G. C. M. AND L. C. M. OF FRACTIONS.

285. The definitions that we have already given of the G. C. M. and L. C. M. of two or more whole numbers will also be applicable when the given numbers are fractions, provided that we understand by *exactly*, that the complete quotients must be *integers*.

286. *To find the* G. C. M. *of two or more fractions.*

RULE. Express the fractions in their lowest terms, if they be not already so. Then take the G. C. M. of the numerators for numerator and the L. C. M. of the denominators for denominator. The fraction so formed is the G. C. M. of the given fractions.

Ex. Find the G. C. M. of $\frac{8}{9}$, $\frac{12}{14}$, $\frac{48}{63}$.

Here, the fractions reduced to their lowest terms are $\frac{8}{9}$, $\frac{6}{7}$, $\frac{16}{21}$.

The G. C. M. of the numerators 8, 6, 16 is 2; and the L. C. M. of the denominators 9, 7, 21 is 63

Thus, the required G. C. M. $=\frac{2}{63}$. *Ans.*

287. *To find the* L. C. M. *of two or more fractions.*

RULE. Express the fractions in their lowest terms. Then take the L. C. M. of the numerators as numerator and the G. C. M. of the denominators as denominator. The fraction so formed is the L. C. M. of the given fractions.

Ex. Find the L. C. M. of $\frac{8}{25}$, $\frac{4}{14}$, $\frac{9}{20}$.

Here, the fractions reduced to their lowest terms are $\frac{8}{25}$, $\frac{2}{7}$, $\frac{9}{20}$.

The L. C. M. of the numerators 8, 2, 9 is 72; and the G. C. M. of the denominators 25, 7, 20 is 1.

Thus, the required L. C. M. $=\frac{72}{1}=\underline{72}$. *Ans.*

Note. Before applying the Rules given above, reduce mixed numbers to improper fractions and compound fractions to simple ones

Examples LXXVII.

1. Find the G. C. M. and the L. C. M. of :—

(1) $\frac{5}{6}, \frac{8}{9}$. (2) $\frac{11}{12}, \frac{14}{15}$. (3) $\frac{1}{9}, \frac{1}{12}, \frac{1}{16}$. (4) $\frac{7}{9}, 6\frac{5}{12}, \frac{1}{3}$.

(5) $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{8}{9}$. (6) $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$. (7) $1\frac{5}{28}, 2\frac{2}{21}, 3\frac{1}{7}$. (8) $\frac{15}{16}, 2\frac{1}{4}, 4, 5\frac{1}{3}$

(9) $1\frac{14}{25}, 5\frac{17}{20}, 14\frac{3}{10}, 6\frac{1}{15}$. (10) $1\frac{23}{55}, 2\frac{2}{77}, 3\frac{31}{33}, 8\frac{19}{22}$.

(11) $7\frac{1}{2}, 1\frac{13}{14}, 1\frac{1}{5}, 4\frac{1}{5}, \frac{3}{4}$. (12) $3\frac{2}{7}, 1\frac{2}{3}, 1\frac{3}{8}, 2\frac{2}{9}, 3\frac{10}{12}$.

2. What is the greatest length that is contained a whole number of times exactly in $26\frac{1}{4}$ ft., $28\frac{7}{8}$ ft. and $29\frac{1}{6}$ ft.?

3. A man gives away to each of five people $\frac{1}{12}$, $\frac{2}{15}$, $\frac{1}{10}$, $\frac{5}{18}$, $\frac{3}{20}$ of a basket of apples; how many has he left, supposing he has only just enough apples to do the above operation without dividing an apple?

4. What is the least number which, when divided by each of the fractions $\frac{3}{5}$, $\frac{7}{10}$, $\frac{14}{15}$, and $\frac{6}{25}$, gives a whole number as quotient in each case?

5. Three lines of paling run side by side for a distance of 150 yds. The upright posts are respectively $2\frac{1}{2}$, $3\frac{1}{3}$, $4\frac{1}{6}$ ft. apart.

How often will a person walking outside be able, on looking across, to see 3 posts in a line ?

6. Eight bells commence to toll simultaneously. They toll at intervals of $1\frac{1}{2}$, $2\frac{1}{2}$, 3. $4\frac{1}{2}$, 5, 6, 8 and 9 seconds respectively ; after what interval will they again toll together ?

7. Three wheels are respectively $10\frac{1}{2}$ ft., $6\frac{3}{8}$ ft. and $4\frac{9}{10}$ ft. round. Find the least distance travelled when they will make complete revolutions.

8. A man gives away to each of four people $\frac{1}{12}$, $\frac{7}{30}$, $\frac{5}{18}$ and $\frac{7}{48}$ of a basket of apples, and has only just enough apples to be able to do this without dividing an apple ; how many apples had he ?

X. MISCELLANEOUS EXAMPLES IN FRACTIONS.

288. The following Solutions, we hope, will be of service to students in acquiring a thorough knowledge of the principles of Vulgar Fractions.

Examples worked out.

Ex. 1. What fraction added to the sum of $\frac{5}{7}$, $\frac{2}{3}$, $\frac{4}{21}$ and $2\frac{1}{2}$ will make the sum equal to 5 ?

Here, $\frac{5}{7}+\frac{2}{3}+\frac{4}{21}+2\frac{1}{2}=2+2\frac{1}{14}=4\frac{1}{14}$.

$\therefore$ the required fraction $=5-4\frac{1}{14}=\frac{13}{14}$. *Ans.*

Ex. 2. What fraction is that from which if there be taken $\frac{2}{7}$ of $\frac{3}{8}$ and to the remainder be added $\frac{8}{15}$ of $\frac{5}{16}$, the sum will be 10 ?

Here, $\frac{2}{7}$ of $\frac{3}{8}=\frac{3}{28}$ and $\frac{8}{15}$ of $\frac{5}{16}=\frac{1}{6}$.

$\therefore$ the required number $=10-\frac{1}{6}+\frac{3}{28}=9\frac{5}{6}+\frac{3}{28}=9\frac{79}{84}$. *Ans.*

Ex. 3. Find what fraction multiplied by the sum of $2\frac{3}{5}$, $1\frac{1}{10}$ and $\frac{4}{15}$ will make the product equal to 17.

Here, $2\frac{3}{5}+1\frac{1}{10}+\frac{4}{15}=3+\frac{29}{30}=3\frac{29}{30}$.

$\therefore$ the required fraction $=17\div3\frac{29}{30}=17\times\frac{30}{119}=\frac{30}{7}=4\frac{2}{7}$. *Ans.*

Ex. 4. Find what least fraction added to the sum of $\frac{3}{4}$, $1\frac{7}{8}$ and $2\frac{5}{6}$ will make the result an integer.

Here, $\frac{3}{4}+1\frac{7}{8}+2\frac{5}{6}=3+2\frac{11}{24}=5\frac{11}{24}$.

$\therefore$ the required fraction $=1-\frac{11}{24}=\frac{13}{24}$. *Ans.*

Ex. 5. What number divided by $2\frac{1}{16}$ will produce $\frac{4}{15}$?

The required number $=\frac{4}{15}\times2\frac{1}{16}=\frac{4}{15}\times\frac{33}{16}=\frac{11}{20}$. *Ans.*

Ex. 6. A man has $\frac{3}{8}$ of an estate, he gives his son $\frac{1}{4}$ of his share ; what portion of the estate has he then left ?

$\frac{1}{4}$ of his share being given away, there remains $(1-\frac{1}{4})$ or $\frac{3}{4}$.

But his share $=\frac{3}{8}$ of the estate ; $\therefore$ he retains $\frac{3}{4}$ of $\frac{3}{8}=\frac{9}{32}$. *Ans.*

Examples LXXVIII.

1. What number added to $\frac{14}{27}$ makes $1\frac{2}{9}$? and what taken from $1\frac{22}{27}$ leaves $\frac{11}{15}$?

2. What number added to $\frac{10}{11}$, $\frac{11}{14}$, $\frac{2}{33}$, $\frac{41}{42}$, will make the sum total equal to 3?

3. Multiply the sum of $3\frac{2}{3}$, $4\frac{3}{4}$ and $4\frac{1}{5}$ by the difference of $7\frac{4}{7}$ and $5\frac{5}{6}$; and divide the product by the sum of $94\frac{1}{8}$ and $93\frac{1}{6}$.

4. Prove that the sum of $5\frac{1}{3}$ and $3\frac{1}{5}$ is equal to four times their difference.

5. Compare the product and quotient of $\frac{7}{9}$ by $\frac{10}{11}$.

6. Find what quantity multiplied by $\frac{2}{3}$ of $\frac{4}{5}$ of $3\frac{1}{2}$, gives a result equal to $\frac{4}{5}$ of $\frac{7}{25}$ of $6\frac{1}{4}$.

7. What number is that, whereof the part expressed by $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$ is 45? What number must be added to $\frac{1}{3}$ of $2\frac{1}{7}$ to give $3\frac{2}{5}$?

8. Find the least fraction which, added to the sum of $\frac{7}{8}$, $\frac{9}{10}$ and $\frac{29}{12}$, will make the result an integer.

9. To 479 add $1\frac{904}{905}$ and repeat the addition 6 times.

10. From $11\frac{9}{10}$ take the sum of $2\frac{1}{2}$, $3\frac{2}{3}$ and $4\frac{3}{4}$, and multiply the difference by $2\frac{1}{3}$ of $\frac{27}{65}$ of $6\frac{1}{2}$.

11. Multiply $49\frac{15}{16}$ by $50\frac{1}{16}$ and add $\frac{1}{256}$ to the result.

12. How many times does $\frac{3}{5}+\frac{2}{3}-\frac{1}{15}$ contain $\frac{2}{7}+\frac{1}{3}-\frac{1}{21}$?

13. Multiply the sum of 1, $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ by the difference of $\frac{4}{15}$ and $\frac{3}{20}$ and divide the product by the double of $21\frac{7}{8}$.

14. Of the fractions $\frac{3}{16}$, $\frac{5}{24}$, $\frac{7}{36}$, $\frac{11}{54}$, find how much the sum of the greatest and least exceeds the difference of the other two.

15. From 1 take its half, third, and twenty-fourth parts: add the product of those parts to the remainder; and multiply this sum by $7\frac{11}{13}$. What must $\frac{3}{4}$ be divided by to produce 2?

16. To $\frac{4}{15}$ of a dozen add $\frac{13}{24}$ of three hundred, and divide this sum by the difference of $3\frac{3}{4}$ of a hundred and $43\frac{3}{5}$.

17. Find the sum of the greatest and least of the fractions $\frac{3}{8}$, $\frac{7}{12}$, $\frac{4}{9}$ and $\frac{7}{20}$, the sum of the other two, and the difference of these sums.

18. Multiply the sum of $\frac{1}{2}$, $1\frac{2}{7}$ and $\frac{5}{6}$ by the difference of $\frac{4}{15}$ and $\frac{8}{25}$, and divide the product by $\frac{11}{18}$ of $1\frac{14}{15}$.

19. What fraction is that from which if $\frac{2}{3}$ of $\dfrac{3-1\frac{1}{4}}{2\frac{2}{5}}$ be subtracted and the remainder be divided by $5\frac{1}{9} \div 16\frac{2}{3}$, the result will be $\frac{1}{3}$?

20. Divide the sum of $2\frac{2}{3}$, $3\frac{3}{4}$ and $5\frac{5}{6}$ by the sum of $4\frac{4}{5}$ and $8\frac{8}{9}$, and to the quotient add the difference of $10\frac{9}{10}$ and $5\frac{7}{8}$.

21. To the sum of $3\frac{3}{4}$ and $4\frac{4}{5}$ add the difference between $4\frac{4}{5}$ and $5\frac{5}{6}$ and multiply the result by $11\frac{5}{11}$.

22. A merchant owned $\frac{5}{16}$ of a ship and sold $\frac{3}{8}$ of his share, what share has he remaining?

23. If I pay away $\frac{1}{2}$ of my money, then $\frac{1}{3}$ of what remains, then $\frac{1}{4}$ of what then remains and then $\frac{1}{5}$ of what still remains, what fraction of the whole will be left?

24. What is the least fraction which must be added to the sum of 4 and $\frac{1}{4}$ divided by their difference to make the result an integer?

25. The difference of two numbers is $15\frac{4}{35}$; the smaller number is $5\frac{13}{21}$; find the greater number.

26. Multiply $3\frac{3}{20}$ by $15\frac{5}{7}$, and divide $\dfrac{2}{3\frac{3}{4}}$ by $\dfrac{2\frac{3}{4}}{3}$; and add together the sum and difference of these results.

27. Divide 2 by the sum of $2\frac{2}{3}$, $\frac{4}{5}$ and 4; add $1\frac{2}{3}-\frac{7}{9}$ to the quotient; and multiply the result by the difference of $5\frac{1}{5}$ and $4\frac{1}{2}$.

28. If I pay away $\frac{1}{5}$ of my money, then $\frac{2}{3}$ of the remainder, then $\frac{1}{2}$ of what then remains and then $\frac{1}{20}$ of the original sum; what fractional part of my money have I left after the second, and also after the final payment?

29. What must be taken from

$$8\frac{4}{7} \text{ of } \frac{5\frac{4}{5}-2\frac{1}{8}}{3\frac{3}{4}+\frac{9}{20}} \div \frac{\frac{5\frac{5}{8}\div\frac{2}{3}}{1\frac{1}{5}\text{ of }\frac{5}{9}}}{1\frac{1}{3}} \text{ of } \frac{\frac{\frac{4}{5}}{3}}{1-\frac{3}{7}} \text{ to reduce its value to } \frac{16}{21}?$$

30. *A* has a certain sum of money in his pocket of which he loses $\frac{3}{5}$ths; he gives $\frac{1}{12}$th of what remains to *B*, and then $\frac{1}{4}$th of $(\frac{1}{2}-\frac{1}{3})$ of what then remains to *C*; find what fractional part of *A*'s original money *B* and *C* respectively receive; and compare these sums with the amount *A* has after his loss.

31. A man having $$\frac{17\frac{5}{6}-\left(\dfrac{3\frac{1}{7}}{4\frac{2}{5}}\div\dfrac{\frac{1}{6}}{8-5\frac{2}{3}}\right)}{7\frac{1}{4}\text{ of }\dfrac{\frac{4}{5}}{7}\div\dfrac{\frac{4}{7}}{\frac{5}{6}}}\div\frac{6\frac{2}{3}\text{ of }7}{5\frac{4}{5}}$$ of an estate, gives $\frac{1}{3}$ of his share to his son, and $\frac{3}{23}$ of the remainder to his daughter; what fraction of the estate has he still remaining?

32. If I cut half a cake into 5 equal parts, and the remainder into 7 equal parts, and then cut one of the 5 equal parts into 6 equal parts, and one of the 7 equal parts into 4 equal parts and then give 2 children each one of each of these small slices, what fractional part of the whole cake will they receive, and what part of the cake will be left?

XI. APPLICATION OF FRACTIONS TO COMPOUND QUANTITIES.

289. In the Fundamental Operations of Compound Quantities, if the lowest denominations of the given compound quantities be mixed numbers, we shall treat separately, first the fractional parts by the ordinary method for Fractions and then the integral parts.

Ex. 1. Add together £16. 2*s*. $1\frac{4}{9}d$, £4 18*s*. $1\frac{4}{5}d$. and £1. 0*s*. $9\frac{4}{15}d$

£.	s.	d.
16	2	$1\frac{4}{9}$
4	18	$1\frac{4}{5}$
1	0	$9\frac{4}{15}$
22	1	$0\frac{23}{45}$

Now $(\frac{4}{9}+\frac{4}{5}+\frac{4}{15})d. = \frac{20+36+12}{45}d. = \frac{68}{45}d. = 1\frac{23}{45}d.$; we therefore put down $\frac{23}{45}d.$, carry on 1*d*. to the column of pence, and proceed in the usual way.

Ex. 2. Subtract *Rs* 32. 14*a*. $9\frac{11}{12}p$. from *Rs*.87. 8*a*. $6\frac{3}{8}p$.

Rs.	a.	p.
87	8	$6\frac{3}{8}$
32	14	$9\frac{11}{12}$
54	9	$8\frac{11}{24}$

Here $\frac{11}{12}$ is greater than $\frac{3}{8}$, therefore we add 1 to $\frac{3}{8}$, which makes it $\frac{11}{8}$.

Now $\frac{11}{8} - \frac{11}{12} = \frac{33-22}{24} = \frac{11}{24}$. We must add 1*p*. to 9*p*., and proceed in the usual way.

Ex. 3. Multiply £6. 12*s*. $8\frac{5}{6}d$. by 57, and divide *Rs*.21. 14*a*. $5\frac{1}{4}p$. by 21.

(1)

£.	s.	d.	
6	12	$8\frac{5}{6}$	$57 = 11 \times 5 + 2.$
		11	
73	0	$1\frac{1}{6}$,	for $\frac{5}{6}d. \times 11 = \frac{55}{6}d. = 9\frac{1}{6}d.$
		5	
365	0	$5\frac{5}{6}$,	for $\frac{1}{6}d. \times 5 = \frac{5}{6}d.$
13	5	$5\frac{2}{3}$	$\times 2 = \frac{5}{3}d. = 1\frac{2}{3}d.$
378	5	$11\frac{1}{2}$	

(2)

	Rs.	a.	p.	
21 { 3	21	14	$5\frac{1}{4}$	$21 = 3 \times 7.$
7	7	4	$9\frac{3}{4}$	for $2\frac{1}{4} \div 3 = \frac{3}{4}$.
	1	0	$8\frac{1}{4}$	for $1\frac{3}{4} \div 7 = \frac{1}{4}$.

Examples LXXIX.

1. Add together :—

(1)

Rs.	a.	p.
3	15	$7\frac{1}{2}$
5	14	$2\frac{1}{3}$
7	6	$10\frac{5}{8}$
8	1	$11\frac{11}{16}$
2	4	$6\frac{2}{3}$
1	4	$5\frac{9}{10}$

(2)

Rs.	a.	p.
17	13	$5\frac{1}{3}$
32	6	$11\frac{1}{4}$
12	10	$9\frac{5}{6}$
7	0	$8\frac{1}{2}$
11	5	$4\frac{1}{8}$
6	10	$5\frac{7}{12}$

(3)

£.	s.	d.
7	13	$1\frac{1}{3}$
2	17	$4\frac{3}{7}$
5	2	$8\frac{4}{9}$
6	11	$2\frac{11}{12}$
4	5	$0\frac{2}{7}$
6	3	$4\frac{1}{9}$

(4)

£.	s.	d.
23	2	$6\frac{2}{9}$
14	1	$5\frac{1}{6}$
7	8	$11\frac{2}{3}$
4	9	$5\frac{11}{12}$
16	4	$2\frac{7}{8}$
5	4	$3\frac{3}{4}$

(5)

oz.	dwts.	grs.
5	16	$15\frac{2}{3}$
1	14	$23\frac{3}{8}$
	17	$0\frac{7}{12}$
2	4	$21\frac{5}{8}$
3	19	$8\frac{3}{4}$
6	18	$20\frac{1}{2}$

(6)

cwt.	qrs.	lbs.	oz.
13	0	21	$13\frac{5}{6}$
	3	18	$9\frac{7}{10}$
		25	$15\frac{3}{8}$
	1	13	$3\frac{13}{15}$
	2	12	$12\frac{4}{5}$
4	0	8	$15\frac{3}{5}$

(7)

poles	yds.	ft.	in.
25	4	2	$8\frac{3}{5}$
17	2	0	$6\frac{4}{7}$
2	0	1	$7\frac{7}{16}$
15	5	1	$11\frac{9}{14}$
6	4	2	$10\frac{5}{6}$
20	3	0	$9\frac{11}{15}$

2. Perform the following subtractions :—

(1) *Rs.*	*a.*	*p.*	(2) *Rs.*	*a.*	*p.*	(3) £.	*s.*	*d.*	(4) £.	*s.*	*d.*
15	0	$3\frac{3}{8}$	17	15	$7\frac{5}{12}$	48	13	$6\frac{3}{5}$	163	1	$7\frac{5}{12}$
9	14	$9\frac{11}{12}$	6	15	$9\frac{13}{18}$	34	19	$9\frac{2}{3}$	64	2	$5\frac{13}{18}$

(5) cwt.	qrs.	lbs.	(6) cwt.	qrs.	lbs.	(7) fur.	po.	yds.	in.	(8) hrs.	min.	sec.
15	1	$16\frac{3}{7}$	23	1	$7\frac{7}{11}$	5	15	0	0	23	45	$35\frac{5}{8}$
8	3	$25\frac{4}{5}$	14	0	$24\frac{8}{9}$	2	4	3	$8\frac{7}{13}$	15	50	$48\frac{11}{12}$

3. Multiply :—

(1) *Rs.*9. 4*a.* $2\frac{3}{22}$*p.* separately by 8, 11, 45 and 139.

(2) £75. 13*s.* $9\frac{1}{5}$*d.* separately by 4, 15, 88 and 96.

(3) 14 cwt. 3 qrs. 25 lbs. $13\frac{8}{21}$ oz. separately by 12, 24 and 96.

(4) 45 mds. 14 sr. $7\frac{7}{18}$ ch. separately by 9, 24 and 35.

(5) 3 fur. 34 po. 4 yds. 1 ft. $8\frac{17}{36}$ in. separately by 45 and 99.

4. Divide :—

(1) *Rs.*246. 13*a.* $8\frac{1}{4}$*p.* separately by 12, 14, 26 and 58.

(2) £997. 18*s.* $10\frac{2}{5}\frac{1}{2}$*d.* separately by 26, 53, 84 and 145.

(3) 789 lbs. 12 oz. $14\frac{2}{3}\frac{4}{5}$ drs. separately by 7, 15 and 67.

(4) 1994 mds. $20\frac{8}{9}$ sr. separately by 729 and 1521.

(5) *Rs.*7. 8*a.* $11\frac{5}{6}$*p.* separately by *Rs.*3. 2*a.* $7\frac{5}{8}$*p.*, and 15*a.* $9\frac{3}{8}$*p.*

(6) £282. 18*s.* $7\frac{1}{2}\frac{6}{7}$*d.* separately by £6. 18*s.* $0\frac{5}{27}$*d.*, and £27. 15*s.* $9\frac{2}{3}$*d.*

XII. REDUCTION OF FRACTIONS.

290. Our attention has hitherto been confined to fractions considered *generally*, without regard to the particular value of their **units**; and it remains to apply what has been said to such *concrete* quantities as constitute the principal subjects of practical computation.

291. We shall notice here, that while **times** denotes the multiplication of a quantity by an integer, **of** denotes its multiplication by a fraction, and either **times** or **of** its multiplication by a mixed number.

Thus, each of the expressions 5 *times Rs.*7, $\frac{4}{5}$ *of Rs.*7, and either $3\frac{4}{5}$ *times Rs.*7 or $3\frac{4}{5}$ *of Rs.*7 denotes the multiplication of *Rs.*7 by 5, by $\frac{4}{5}$ and by $3\frac{4}{5}$ respectively. Also the notation for 5 *times Rs.*7 is either $5 \times$ *Rs.* 7 or *Rs.* (5×7).

292. Reduction of Fractions can conveniently be divided into the two following cases :—

(1) To reduce a **fraction** of one denomination to a lower denomination; and conversely.

(2) To reduce a quantity of one denomination to a **fraction** of a higher denomination.

293. Case I. *To reduce a fraction of one denomination to a lower denomination.* **(Descending Reduction)**

RULE. Multiply the fraction of the given denomination by the number which connects the lower denomination with one (or unit) of the given denomination.

Ex. Reduce £$\frac{2}{7}$ to *pence*, and $\frac{8}{27}$ of a day to *seconds*.

(1) £$\frac{2}{7} = \frac{2}{7} \times (20 \times 12) d. = \frac{2 \times 20 \times 12}{7} d. = \frac{480}{7} d. = 68\frac{4}{7} d.$ *Ans.*

(2) $\frac{8}{27}$ of a day $= \frac{8}{27} \times (24 \times 60 \times 60)$ sec. $= \frac{8 \times 24 \times 60 \times 60}{27}$ sec.
$= \underline{25600 \text{ sec.}}$ *Ans.*

294. Case II. *To reduce a quantity of one denomination to a fraction of a higher denomination.* **(Ascending Reduction).**

RULE. Divide the number of the given denomination by the number which connects that denomination with one (or unit) of the higher denomination.

Ex. Reduce $5\frac{1}{4}d.$ to the fraction of a *pound*, and $18\frac{3}{4}$ grs. to the fraction of an *oz. Troy*.

(1) $5\frac{1}{4}d = £\,\frac{5\frac{1}{4}}{12 \times 20} = £\,\frac{21}{4} \times \frac{1}{12 \times 20} = £\,\frac{7}{320}.$ *Ans.*

(2) $18\frac{3}{4}$ grs. $= \frac{18\frac{3}{4}}{24 \times 20}$ oz. $= \frac{75}{4} \times \frac{1}{24 \times 20}$ oz. $= \frac{5}{128}$ oz. *Ans.*

295. Sometimes we employ both the *descending* and the *ascending* process in reducing a fraction of one denomination to a fraction of another denomination.

Ex. Reduce $\frac{3}{5}$ of a guinea to the fraction of £1.

$\frac{3}{5}$ of a guinea $= \frac{3 \times 21}{5} s. = \frac{63}{5} s. = £\,\frac{63}{5 \times 20} = £\,\frac{63}{100}.$

Examples LXXX.

1. Reduce $\frac{1}{5}$, $\frac{2}{7}$, $\frac{5}{8}$, $\frac{4}{15}$ and $\frac{1}{55}$ of a rupee to *annas*; and $\frac{8}{63}$ of *Re.* 1 to *gandas*.

2. Reduce $\frac{1}{7}$, $\frac{4}{9}$, $\frac{3}{64}$, and $\frac{545}{672}$ of a pound to *pence*.

3. Express $\frac{2}{5}$ of a shilling, $\frac{5}{7}$ of a penny, and $\frac{160}{11}$ of a farthing as fractions of a *pound*.

4. Reduce $\frac{2}{9}$ of a guinea, $\frac{3}{4}$ of a half-guinea, and $6\frac{6}{7}$ of a crown to fractions of £1.

5. Reduce $\frac{3}{32}$ of a cwt. to the fraction of 1 lb.; $\frac{4}{7}$ of an ounce to that of 1 *cwt.*; and $\frac{19}{25}$ of an ounce (Avoir.) to that of 1 *grain*.

6. Express $\frac{7}{342}$ of a yard as the fraction of an *inch*, and $\frac{108}{145}$ of an inch as that of a *pole*.

7. Find the fraction of a *yard* which expresses $\frac{3}{4}$ of an ell of 5 quarters; and that of a *day* which is equal to $\frac{5}{146}$ of a year of 365 days.

8 Reduce $\frac{7}{36}$ of a maund to the fraction of a *seer*; $\frac{84}{875}$ lb. to the fraction of 1 lb. Troy, and $\frac{21}{64}$ of a maund to *chhataks*.

9. Reduce $\frac{4}{279}$ of a barrel of beer to the fraction of a *quart*; and $\frac{4}{11}$ of a pint of wine to the fraction of a *hogshead*.

10. Reduce $\frac{487}{1056}$ of a mile to *poles*; $\frac{387}{704}$ of an acre to *sq. yards*.

11. Express $\frac{4}{7}$ of a guinea, $\frac{2}{3}$ of a shilling and $\frac{16}{15}$ of a farthing as fractions of £10.

12. Reduce $492\frac{12}{35}$ hours to the fraction of a *year* of $365\frac{1}{4}$ days.

13. Express $\frac{1}{3}$ of $2151\frac{1}{9}$ sq. yards in *acres*; $3\frac{487}{1056}$ miles in *yards*; and $\frac{1}{8}$ cubit as the fraction of an *anguli*.

14. What fraction expresses $\dfrac{5\frac{1}{12}}{7\frac{1}{14}}$ of 5940 seconds in *weeks*?

296. The preceding two cases in Art. 292 enable us

(1) To find the value of a given fraction of any **concrete** quantity in terms of its own or lower denominations; and

(2) To reduce a **compound** quantity to a fraction of a higher denomination.

297 **Case I.** *To find the value of a given fraction of any concrete quantity in terms of its own or lower denominations.*

(1) When the quantity is *simple* or can be easily reduced to a *simple quantity*.

RULE. Multiply the given quantity by the numerator of the fraction, divide the product (if possible) by the denominator; the quotient (if any) is the required number of parts of that denomination. If there be a remainder, multiply the numerator of the fraction which remains by the number of units of the next inferior denomination which are equivalent in value to the given denomination and divide the product by the denominator: the quotient will be the number of parts of that denomination. Proceed in the same way with the remainder (if any), and the parts of the next denomination will be found; repeat this process till the lowest denomination, to which the given quantity is capable of being reduced, is obtained.

Ex. 1. Find the value of $\frac{3}{8}$ of £1.

$$\tfrac{3}{8} \text{ of } £1 = \frac{3 \times 20}{8} s. = \frac{3 \times 5}{2} s. = \frac{15}{2} s. = 7\tfrac{1}{2} s.\ ;\ \tfrac{1}{2} s. = \frac{1 \times 12}{2} d. = 6d.$$

$\therefore$ the required value $= \underline{7s.\ 6d.}$

Ex. 2. Find the value of $\frac{5}{6}$ of *Rs.*4.

$\frac{5}{6}$ of $Rs.4 = Rs.\frac{5 \times 4}{6} = Rs.\frac{10}{3} = Rs.3\frac{1}{3}$; $Re\ \frac{1}{3} = \frac{1 \times 16}{3}a. = \frac{16}{3}a. = 5\frac{1}{3}a.$;

$\frac{1}{3}a. = \frac{1 \times 12}{3}p. = 4p.$ $\therefore$ the required value $= \underline{Rs.3.\ 5a.\ 4p.}$

Ex. 3. Find the value of $\frac{3}{5}$ of 13*s.* 4*d.*

$\frac{3}{5}$ of $13s.\ 4d. = \frac{3}{5}$ of $160d. = \frac{3 \times 160}{5}d. = (3 \times 32)d. = 96d. = \underline{8s.}$ *Ans*

298. When the given fraction is a mixed number,—(1) multiply separately by the integer and by the fraction and add the products so obtained; or (2) reduce the mixed number to a fraction and proceed as in Art. 297, Case 1.

Ex. Find the value of $3\frac{1}{12}$ of *Re.*1 4*a.*

$$\begin{aligned} \text{The required value} &= Re.1.\ 4a. \times 3 + Re.1.\ 4a. \times \tfrac{1}{12} \\ &= Rs\ 3.\ 12a. + 20a. \times \tfrac{1}{12} = Rs.3.\ 12a. + 1\tfrac{2}{3}a. \\ &= Rs.3.\ 12a. + 1a.\ 8p. = \underline{Rs.3.\ 13a.\ 8p.} \end{aligned}$$

299. Before applying the Rule, reduce compound and complex fractions to simple ones.

Ex. 1. Find the value of $2\frac{2}{3}$ of $\frac{5}{4}$ of 10*a.* 9*p.*

Here, $2\frac{2}{3}$ of $\frac{5}{4} = \frac{8}{3}$ of $\frac{5}{4} = \frac{10}{3}$.

$$\begin{aligned} \therefore \text{ the required value} = \tfrac{10}{3} \text{ of } 129p. &= \frac{10 \times 129}{3}p. = (10 \times 43)p. \\ &= 430p. = \underline{Rs.2.\ 3a.\ 10p.} \end{aligned}$$

Ex. 2. Find the value of $\frac{7}{8}$ of $7\frac{1}{3}$ of $\frac{8\frac{1}{2}}{4}$ of 3 maunds.

Here, $\frac{7}{8}$ of $7\frac{1}{3}$ of $\frac{8\frac{1}{2}}{4} = \frac{7}{8}$ of $\frac{22}{3}$ of $\frac{17}{2} \times \frac{1}{4} = \frac{1309}{96}$.

$\therefore \frac{1309}{96}$ of 3 mds $= \frac{1309}{32}$ mds. $= 40\frac{29}{32}$ mds. ;

$\frac{29}{32}$ mds. $= \frac{29}{32} \times 40$ sr. $= \frac{29 \times 40}{32}$ sr. $= \frac{145}{4}$ sr. $= 36\frac{1}{4}$ sr. ;

$\frac{1}{4}$ sr. $= \frac{1}{4} \times 16$ ch. $= \frac{16}{4}$ ch. $= 4$ ch.

$\therefore$ the required value $=$ <u>40 mds. 36 sr. 4 ch.</u>

300. The preceding Articles enable us to find the value of the sum or difference of fractional parts of magnitudes of the same kind.

Ex. 1. Find the value of $\frac{2}{3}$ of £1 + $\frac{4}{9}$ of a guinea − $\frac{3}{7}$ of 3*s.* 6*d.*

$$\left.\begin{array}{l} \tfrac{2}{3} \text{ of } £1 = \tfrac{2}{3} \times 20s. = \tfrac{40}{3}s. = 13s.\ 4d. \\ \tfrac{4}{9} \text{ of a gui} = \tfrac{4}{9} \times 21s. = \tfrac{28}{3}s. = 9s.\ 4d. \end{array}\right\} = £1.\ 2s.\ 8d.$$

$\frac{3}{7}$ of $3s.\ 6d = \frac{3}{7}$ of $42d. = (3 \times 6)d. = 18d. =$ 1*s.* 6*d.*

$\therefore$ the required value $= \underline{£1.\ 1s.\ 2d.}$

301. The following table, if carefully committed to memory, will greatly help a student in his calculations :

Re. $\frac{1}{16}$ = 1*a.* ;	*Re.* $\frac{1}{3}$ = 5*a.* 4*p.*	£$\frac{1}{20}$ = 1*s.* ;	£$\frac{1}{4}$ = 5*s.*
Re. $\frac{1}{12}$ = 1*a.* 4*p.* ;	*Re.* $\frac{1}{2}$ = 8*a.*	£$\frac{1}{12}$ = 1*s.* 8*d.*	£$\frac{1}{3}$ = 6*s.* 8*d.*
Re. $\frac{1}{8}$ = 2*a.* ;	*Re.* $\frac{2}{3}$ = 10*a.* 8*p.*	£$\frac{1}{10}$ = 2*s.*	£$\frac{1}{2}$ = 10*s.*
Re. $\frac{1}{6}$ = 2*a.* 8*p.* ;	*Re.* $\frac{3}{4}$ = 12*a.*	£$\frac{1}{8}$ = 2*s.* 6*d.* ;	£$\frac{2}{3}$ = 13*s.* 4*d.*
Re. $\frac{1}{4}$ = 4*a.*		£$\frac{1}{5}$ = 4*s.* ;	£$\frac{3}{4}$ = 15*s.*

Examples LXXXI.

1. Find the respective values of :—

(1) $\frac{5}{8}$ of *Re.*1 ; $\frac{7}{12}$ of *Re* 1 ; $\frac{9}{16}$ of *Rs.*30 ; $\frac{11}{15}$ of *Rs.*9 ; $\frac{17}{20}$ of *Rs.*8.

(2) $7\frac{7}{25}$ of *Rs.*50 ; $\frac{2}{3}$ of *Rs.*2 ; $8\frac{2}{3}$ of $\frac{4}{5}$ of 10*a.* 9*p.* ; $\frac{1}{12}$ of $\frac{3}{4}$ of 5*a.*

(3) $\frac{3}{5}$ of £1 ; $\frac{5}{9}$ of 1*s.* ; $\frac{7}{18}$ of a guinea ; $\frac{5}{6}$ of £1 ; $\frac{1}{15}$ of £1 ; $\frac{17}{32}$ of £1.

(4) $\frac{13}{20}$ of £5 ; $\frac{2}{5}$ of 6*s.* 8*d.* ; $3\frac{3}{4}$ of 2*s.* 6*d.* ; $2\frac{8}{9}$ of a guinea ; £$3\frac{5}{8}$.

(5) $\frac{3}{5}$ of $\frac{7}{9}$ of $\frac{20}{21}$ of 5 guineas ; $\frac{8}{21}$ of a moidore ; $\frac{3}{5}$ of 13*s.* 4*d* ; $\frac{13}{24}$ of £9.

(6) $\frac{5}{7}$ of a cwt. ; $\frac{5}{14}$ of 1 qr. ; $\frac{3}{8}$ of 1 lb. ; $\frac{47}{84}$ of a cwt. ; $2\frac{2}{3}$ of 8 cwt.

(7) $\frac{8}{35}$ of a ton ; $2\frac{1}{2}$ of 6*s.* 8*d.* ; $\frac{2}{21}$ of 5*s.* 3*d.* ; $\frac{7}{12}$ of a mile.

(8) $\frac{4}{7}$ of a lb. Troy ; $\frac{4}{7}$ of a lb. Avoir. ; $\frac{4}{7}$ of a lb. Apoth. ; $2\frac{17}{36}$ lbs. Tory.

(9) $\frac{13}{20}$ of $2\frac{1}{2}$ yds. ; $\frac{2}{9}$ of $\frac{1}{20}$ of $2\frac{6}{7}$ fur. ; $\frac{3}{5}$ of an acre ; $\frac{39}{160}$ of an acre.

(10) $2\frac{13}{16}$ of $\frac{4}{15}$ of a cwt. ; $\frac{5}{18}$ of a week . $\frac{2}{15}$ of 1 mo. of 28 days.

(11) $\frac{1}{32}$ of 1 qr. ; $\frac{3}{8}$ of a bus. ; $\frac{5}{7}$ of a peck . $\frac{8}{15}$ of $\frac{5}{6}$ of $2\frac{7}{8}$ of $2\frac{1}{2}$ loads.

(12) $8\frac{11}{15}$ of 17 cub. yds. ; $\frac{7}{8}$ of $3\frac{2}{3}$ of $4\frac{1}{4}$ of 2 mds. ; $\frac{5}{6}$ of 175 tons.

(13) $\frac{7}{8}$ of $\frac{3}{5}$ of $10\frac{2}{3}$ hrs. ; $\frac{3}{16}$ of a day ; $2\frac{1}{12}$ of a pipe of wine.

(14) $\dfrac{6\frac{5}{12}}{3\frac{2}{3}}$ of £4 ; $\dfrac{3\frac{13}{15}}{12\frac{15}{17}}$ of 365 days ; $\dfrac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\dfrac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\dfrac{77}{540}$ of a moidore.

(15) $\dfrac{3\dfrac{1\frac{3}{14}}{4}}{20}$ of a ton ; $\dfrac{6\dfrac{11\frac{13}{60}}{12}}{14}$ of a week ; $\dfrac{2\dfrac{7\frac{11}{24}}{20}}{25}$ of £50.

2. Find the respective values of :—

(1) $\frac{2}{3}$ of 5 guineas + $\frac{3}{4}$ of $\frac{7}{9}$ of £1 ; $\frac{2}{3}$ of 5 guineas − $\frac{3}{4}$ of $\frac{7}{9}$ of £1.

(2) $\frac{5}{9}$ of a guinea + $\frac{3}{10}$ of £1 + $\frac{7}{10}$ of a crown + $\frac{5}{8}$ of 1*s.*

(3) $\frac{3}{7}$ of a guinea + $\frac{3}{8}$ of a crown + $\frac{3}{5}$ of 7*s.* 6*d.* − $\frac{3}{4}$ of 2*d.*

(4) $\frac{3}{5}$ of a ton + $\frac{5}{8}$ of a cwt. + $\frac{2}{3}$ of a lb. ; $\frac{4}{7}$ cwt. + $8\frac{4}{5}$ lbs. + $3\frac{9}{10}$ oz.

(5) $\frac{2}{5}$ of a week + $\frac{3}{4}$ of a day + $\frac{5}{7}$ of an hour + $\frac{2}{3}$ of a minute.

(6) $3\frac{7}{11}$ miles − $7\frac{7}{10}$ fur. + $35\frac{23}{33}$ po. ; $\frac{3}{5}$ of 28mds. + $\frac{5}{8}$ of $1\frac{3}{5}$mds. + $\frac{2}{3}$ of 8ch.

(7) $\frac{3}{8}$ of *Rs.*$10\frac{1}{2}$ + $\frac{1}{2}$ of $\frac{3}{4}$ of *Rs.*10 − $\frac{1}{6}$ of $\frac{3}{4}$ of *Rs.*$2\frac{1}{2}$ + $\frac{1}{3}$ of $\frac{5}{8}$ of 8*a.*

(8) $\dfrac{15\frac{3}{5}}{7\frac{4}{5}}$ of £1 + $\frac{1}{3}$ of £$140\frac{31}{40}$ + $1\frac{13}{126}$ guineas.

(2) When the quantity is a *compound* one.

RULE. Multiply the quantity by the numerator and divide the product by the denominator of the fraction.

Ex. Find the value of $\frac{5}{8}$ of *Rs.*3. 9*a.* 4*p.*

The required value $=$ (*Rs.*3. 9*a.* 4*p.* $\times 5) \div 8$

$=$ *Rs.*17. 14*a.* 8*p.* $\div 8 =$ *Rs.*2. 3*a.* 10*p.*

302. To multiply a compound quantity by a mixed number, multiply separately by the integer and by the fraction and add the two products thus obtained.

Ex. Multiply £13. 15*s.* 4*d.* by $4\frac{5}{8}$.

	£.	s.	d.		£.	s.	d.
	13	15	4		13	15	4
			5				4
8)	68	16	8		55	1	4
	8	12	1				

∴ the required value
$=$ £55. 1*s.* 4*d.* $+$ £8. 12*s.* 1*d.*
$=$ £63. 13*s.* 5*d.*

303. To divide a compound quantity by a fraction, multiply by the denominator and divide the product by the numerator.

Ex. Divide *Rs.*600. 13*a.* 4*p.* by $\frac{7}{9}$.

The required value $=$ (*Rs.*600. 13*a.* 4*p.* $\times 9) \div 7$

$=$ *Rs.*5407. 8*a.* $\div 7 =$ *Rs.*772. 8*a.*

304. To divide a compound quantity by a mixed number, reduce the mixed number to an improper fraction and then proceed as in Art. 303.

Ex. Divide £5. 4*s.* $6\frac{1}{4}$*d.* by $1\frac{2}{3}$.

	£.	s.	d.	
	5	4	$6\frac{1}{4}$	
			3	for $1\frac{2}{3} = \frac{5}{3}$.
5)	15	13	$6\frac{3}{4}$	
	3	2	$8\frac{11}{20}$	for $2\frac{3}{4} \div 5 = \frac{11}{20}$.

Hence the required value
$=$ £3. 2*s.* $8\frac{11}{20}$*d.*

Note. Before applying the above Rules of both Multiplication and Division, the compound and complex fractions must first be reduced to simple ones.

Examples LXXXII.

1. Multiply :—

(1) £3. 16*s.* $8\frac{3}{4}$*d.* by $\frac{3}{7}$; £6. 18*s.* $7\frac{15}{16}$*d.* by $\frac{9}{13}$; £10. 11*s.* $2\frac{1}{4}$*d.* by $3\frac{3}{4}$.

(2) *Rs.*50. 5*a.* 6*p.* separately by $9\frac{2}{3}$, $18\frac{9}{10}$, $53\frac{5}{13}$ and $156\frac{23}{26}$.

(3) £12. 5*s.* $7\frac{3}{5}$*d.* by $6\frac{3}{4}$; £13. 5*s.* $7\frac{1}{4}$*d.* by $7\frac{5}{11}$; £34. 12*s.* $5\frac{1}{4}$*d.* by $11\frac{7}{11}$.

(4) 5 tons 3 cwt. 6 lbs. separately by $4\frac{8}{9}$, $20\frac{7}{11}$, $46\frac{29}{30}$ and $213\frac{4}{7}$.

(5) 19 hrs. 43 m. $56\frac{8}{13}$ sec. by $12\frac{7}{10}$; 10 ac. 3ro. 37po. $15\frac{8}{9}$yds. by $10\frac{6}{7}$.

2. Divide :—

(1) *Rs.*307. 4*a.* 4*p.* by $\frac{1}{23}$; *Rs.*76. 10*a.* 8*p.* by $\frac{4}{7}$; £5. 4*s.* $6\frac{1}{4}$*d.* by $3\frac{3}{8}$.
(2) £25. 8*s.* $4\frac{3}{4}$*d.* by $\frac{3}{5}$; £4 7*s.* $3\frac{3}{4}$*d.* by $\frac{12}{127}$; £34. 16*s.* $9\frac{1}{2}$*d.* by $9\frac{5}{8}$.
(3) *Rs.*173. 5*a.* 4*p.* separately by $2\frac{2}{3}$, $4\frac{3}{4}$, $8\frac{7}{9}$ and $20\frac{1}{5}$.
(4) 13 cwt. 3 qrs. 26 lbs. $15\frac{7}{8}$ oz. by $3\frac{4}{7}$; 15 ac. 3 ro. 25 po. by $1\frac{5}{18}$.
(5) 8 days 15hrs. 48m. $57\frac{13}{63}$sec. by $4\frac{7}{9}$; 12cub. yds. 20c.ft. 100in. by $1\frac{5}{7}$.
(6) 1 mi. 5 fur. 91 yds. 2 ft. by $2\frac{7}{8}$ of $1\frac{9}{11}$; 7 mds. 35 sr. by $1\frac{2}{3}$.

3. Find the respective values of :—

(1) $\frac{5}{16}$ of *Rs.*10. 8*a* ; $\frac{3}{4}$ of *Rs.*2. 6*a.* ; $\frac{5}{6}$ of *Rs.*31. 8*a.* ; $\frac{3}{7}$ of *Re.*1. 12*a.*
(2) $3\frac{5}{11}$ of *Rs.*7. 5*a.* 4*p.* ; $2\frac{2}{5}$ of *Rs.*51. 4*a.* ; $2\frac{2}{3}$ of $3\frac{3}{8}$ of *Rs.*173. 12*a.*
(3) $11\frac{7}{9}$ of 6*s.* $11\frac{1}{4}$*d.* ; $\frac{1}{5}$ of $5\frac{1}{3}$ of 2*s.* $9\frac{3}{4}$*d.* ; $\frac{3}{8}$ of $\frac{4}{11}$ of 16*s.* 6*d.*
(4) $\frac{3\frac{1}{7}}{1\frac{2}{9}}$ of £4. 14*s.* 6*d.* ; $\frac{1\frac{8}{11}}{1\frac{2}{3}}$ of £8. 8*s.* $5\frac{1}{4}$*d.* ; $\frac{3\frac{1}{7}}{1\frac{2}{9}}$ of *Rs.*15. 12*a.*
(5) $3\frac{11}{21}$ of 3 mds. 10 sr. 8 ch. ; $2\frac{22}{35}$ of 3 cwt. 3 qrs. 20 lbs.
(6) $\frac{3\frac{1}{4}}{7\frac{1}{2}}$ of 10 ft. $6\frac{1}{2}$ in. ; $\frac{5}{6}$ of $\frac{\frac{4}{7}}{4\frac{2}{7}}$ of $3\frac{1}{3}$ sq. yds. ; $\frac{3\frac{5}{8}}{7\frac{3}{5}}$ of $\frac{4}{7\frac{1}{4}}$ of $4\frac{25}{27}$ cub. ft.
(7) $4\frac{1}{12}$ of $\frac{8}{1\frac{3}{4}}$ of 5 mi. 3 fur. 37 po. $4\frac{1}{5}$ yds. ; $\frac{2}{7}$ of $\frac{1}{3\frac{1}{5}}$ of $\frac{1}{17}$ of £$6304\frac{2}{3}$.
(8) $\frac{5\frac{9}{11}}{3\frac{5}{7}+\frac{7}{8}}$ of $(3\frac{5}{7}-\frac{2}{5})$ of 5 cwt. 2 qrs. 10 lbs. $7\frac{3}{4}$ oz. ; $\frac{3}{20}$ of $\frac{7}{9\frac{1}{3}}$ of 1 mile.
(9) $\frac{3\frac{1}{2}}{3\frac{1}{3}}$ of $(3\frac{5}{7}+1\frac{2}{3})$ of 5 days $17\frac{2}{3}$ hrs. ; $\frac{4}{3\frac{1}{2}}$ of $\frac{4}{\frac{5}{7}-\frac{1}{9}}$ of *Rs.*10. 8*a.*
(10) $\frac{7\frac{3}{4}-3\frac{11}{12}}{18\frac{1}{2}\div\frac{6}{7}}$ of 3 ac. 1 ro. 35 po. ; $\frac{\frac{5}{8}+\frac{1}{2}(\frac{2}{3}-\frac{1}{2})-\frac{6}{7}(\frac{4}{5}+\frac{1}{9})}{\frac{3}{4}(\frac{7}{8}-\frac{1}{2})-\frac{1}{3}(\frac{1}{7}-\frac{1}{10})}$ of £44. 17*s.*

4. Find the values of :—

(1) $\frac{3}{5}$ of *Rs.*3. 5*a.* 4*p.* + $\frac{5}{7}$ of *Rs.*21. 14*a.* + $\frac{9}{11}$ of *Rs.*47. 3*a.* 4*p.*
(2) $1\frac{3}{4}$ of $3\frac{1}{2}$ of *Rs.*13. 8*a.* + $\frac{5}{8}$ of *Rs.*6. 10*a.* 8*p.* − $\frac{2}{7}$ of $1\frac{5}{9}$ of *Rs.*3. 5*a.* 4*p.*
(3) $\frac{3}{8}$ of 6*s.* 8*d.* + $\frac{4}{7}$ of £2. 3*s.* 9*d.* + $\frac{9}{11}$ of £4. 14*s.* 5*d.*
(4) $\frac{3}{7}$ of £15 + $\frac{1}{4}$ of $\frac{1}{2\frac{1}{5}}$ of $\frac{4}{5}$ of £1. 2*s.* + $\frac{4}{7}$ of 3*d.*
(5) $\frac{15\frac{3}{5}}{7\frac{4}{5}}$ of £1 + $\frac{1}{3}$ of £140. 10*s.* 6*d.* + $2\frac{13}{63}$ of half-a-guinea.
(6) $\frac{7}{8}$ of £5. 10*s.* 6*d.* − $\frac{4}{7}$ of 2 guineas + $\frac{2}{3\frac{1}{2}}$ of $\frac{4}{\frac{8}{7}-\frac{2}{9}}$ of £$\frac{5}{9}$.
(7) $\frac{1\frac{8}{11}}{1\frac{2}{3}}$ of *Rs.*84. 3*a.* 6*p.* − $\frac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\frac{10\frac{5}{7}}{7\frac{1}{2}}$ of *Rs.*20. 4*a.*
(8) $\frac{3}{4}$ of 3 mds. 34 sr. + $\frac{3}{7}$ of 8 mds. 9 sr. + $\frac{2}{5}$ of 3 sr. 12 ch.
(9) $7\frac{5}{8}$ of a year of $365\frac{1}{4}$ days + $3\frac{9}{10}$ of $\frac{5}{6}$ of a week + $\frac{3}{4}$ of $5\frac{5}{9}$ hrs.
(10) 5 yds. 2 ft. $5\frac{5}{8}$ in. × $7\frac{3}{5}$ − 9 yds. 2 ft. $7\frac{21}{25}$ in. − 15 yds. 1 ft. $9\frac{5}{8}$ in.
÷ $3\frac{8}{9}$ + 3 po. 3 yds. 2 ft. 3 in. ÷ $2\frac{1}{10}$.

305. Case II. *To reduce a compound quantity to a fraction of a higher denomination.*

Proceed as in the following Examples.

Ex. 1. Reduce $8p.$, $6a.\ 10p.$ and $14a.\ 4p.$ to the fraction of a rupee.

$8p. = \frac{8}{12}a. = \frac{2}{3}a. \times Re.\frac{1}{16} = Re.\frac{1}{24}$.

$6a.\ 10p. = 6a. + \frac{10}{12}a. = 6\frac{5}{6}a. = \frac{41}{6} \times Re.\frac{1}{16} = Re.\frac{41}{96}$.

$14a.\ 4p. = 14a. + \frac{4}{12}a. = 14\frac{1}{3}a. = \frac{43}{3} \times Re.\frac{1}{16} = Re.\frac{43}{48}$.

Ex. 2. Express $9d.$, $2s.\ 4d.$ and $18s.\ 11\frac{3}{4}d.$ in pounds.

$9d. = \frac{9}{12}s. = \frac{3}{4}s. = \frac{3}{4} \times £\frac{1}{20} = £\frac{3}{80}$.

$2s.\ 4d. = 2s. + \frac{4}{12}s. = 2\frac{1}{3}s. = \frac{7}{3} \times £\frac{1}{20} = £\frac{7}{60}$.

$18s.\ 11\frac{3}{4}d. = 18s. + \frac{47}{48}s. = \frac{911}{48}s. = \frac{911}{48} \times £\frac{1}{20} = £\frac{911}{960}$.

Ex. 3. Reduce $£4.\ 9s.\ 2\frac{1}{4}d.$ to pounds.

$9s.\ 2\frac{1}{4}d. = 9s. + \frac{9}{48}s. = 9\frac{3}{16}s. = \frac{147}{16} \times £\frac{1}{20} = £\frac{147}{320}$.

$\therefore £4.\ 9s.\ 2\frac{1}{4}d. = £4\frac{147}{320}$. *Ans.*

Ex. 4. Reduce 5 cwt. 3 qrs. 24 lbs. to the fraction of a ton.

3 qrs. 24 lbs. $= 3$ qrs. $+ \frac{24}{28}$ qrs. $= 3\frac{6}{7}$ qrs. $= \frac{1}{4} \times 3\frac{6}{7}$ cwt.

$= \frac{1}{4} \times \frac{27}{7}$ cwt. $= \frac{27}{28}$ cwt.

$\therefore$ 5 cwt. $+ \frac{27}{28}$ cwt. $= 5\frac{27}{28}$ cwt. $= \frac{1}{20} \times 5\frac{27}{28}$ ton.

Hence 5 cwt. 3 qrs. 24 lbs. $= \frac{1}{20} \times \frac{167}{28}$ ton $= \frac{167}{560}$ ton. *Ans.*

Examples LXXXIII.

1. Reduce $3a.\ 6p.$; $5a.$; $5a.\ 10p.$; $6a.\ 10p.$; $7a.\ 8p.$; $13a.\ 7\frac{1}{5}p.$; $15a.\ 7\frac{3}{8}p.$; each to the fraction of a rupee.

2. Express $4s.\ 11d.$; $17s.\ 11\frac{1}{2}d.$; $19s.\ 10\frac{3}{4}d.$; $6s.\ 11\frac{3}{4}d.$; $14s.\ 4\frac{5}{7}d.$; $16s.\ 9\frac{3}{4}d.$; each as the fraction of a pound.

3. Express $£1.\ 13s.\ 11\frac{1}{2}d.\ \frac{10}{21}q.$; $£3.\ 19s.\ 8\frac{1}{4}d.$; $£37.\ 16s.\ 6\frac{3}{4}d.$; $£5.\ 16s.\ 11\frac{3}{4}d.\ \frac{5}{8}q.$; each in pounds.

4. Reduce $Rs.3.\ 10a.\ 8p.$; $Rs.8.\ 5a.\ 4p.$; $Rs.15.\ 10a.\ 7p.$; $Rs.81.\ 7a.\ 3\frac{1}{2}ps.$ to rupees.

5. Reduce 2 cwt. 1 qr. 16 lbs. to the fraction of a ton ; 3 qrs. 27 lbs. 9 oz. $12\frac{4}{9}$ drs. to the fraction of a cwt. ; 2 sr. 15 ch. 2 kan. to the fraction of a maund.

6. What fraction is 2 ft. 9 in. of a pole ; 23 po. 4 yds. of a mile ; and 3 ro. 26 po. of an acre ?

7. Reduce 3 fur. 29 po. 4 yds. 1 ft. 9 in. to the fraction of a mile, and 1 sq. ft. $\frac{7}{8}$ sq. in. to the fraction of a sq. yd.

8. Reduce 4 mds. 37 sr. 8 ch. to maunds ; 2 bi. 15 kat. 5 ch. to bighas ; and 2 qts. $1\frac{1}{2}$ pt. to the fraction of a barrel.

9. Express 5 bus. 3 pks. 1 gal. as the fraction of a quarter.

10. Express 2 wks. 5 days 18 hrs. as the fraction of a year of 365 days, and 3 ro. $27\frac{1}{2}$ po. as the fraction of an acre.

11. Reduce 72 days 6 hrs. 56 m. 15 sec. to the fraction of a year of $365\frac{1}{4}$ days, and 1 sc. 13 grs. to the fraction of a lb.

12. Express 1 day 1 hr. 4 dan. 30 pals and 3 mo. 12 days as fractions of a year.

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306. *To find what fraction one concrete quantity is of any other of the same kind.*

RULE. Reduce both the quantities to the same denomination; then the fraction whose numerator is the first and denominator the second of these results, will be the one required.

Ex. 1. Reduce *Rs.*2. 8*a.* 2*p.* to the fraction of *Rs.*3. 12*a.*

*Rs.*2. 8*a.* 2*p.* = 482*p.*; and *Rs.*3. 12*a.* = 720*p.*

$\therefore$ the required fraction $= \dfrac{482p}{720p.} = \dfrac{241}{360}$. *Ans.*

Ex. 2. What part of $4\frac{3}{8}$ of £1 is $3\frac{3}{4}$ of a guinea?

$3\frac{3}{4}$ of a gui. $= \frac{15}{4} \times 21s. = \frac{315}{4}s.$, and $4\frac{3}{8}$ of £1 $= \frac{35}{8}$ of $20s. = \frac{175}{2}s.$

$\therefore$ the required fraction $= \frac{315}{4}s. \div \frac{175}{2}s. = \frac{9}{10}$. *Ans.*

Ex. 3. What fraction is 1 md. 4 sr. of 2 mds. 32 sr.?

1 md. 4 sr. = 44 sr.; and 2 mds. 32 sr. = 112 sr.

$\therefore$ the required fraction $= \dfrac{44 \text{ sr.}}{112 \text{ sr.}} = \dfrac{11}{28}$. *Ans.*

307. By means of the preceding Articles, magnitudes of the same kind, consisting of fractions of simple or compound quantities, and connected by the operations of Addition or Subtraction, may be reduced to simple fractions of a given denomination.

Ex. 1. Express $\frac{2}{3}$ of a guinea $-$ $\frac{3}{4}$ of a shilling $-$ $\frac{4}{9}$ of 7*s.* 6*d.* as the fraction of £2. 19*s.* 6*d.*

Here, $\frac{2}{3}$ of a guinea $= \frac{2}{3} \times £\frac{21}{20} = £\frac{7}{10}$; $\frac{3}{4}s. = £\frac{3}{4} \times \frac{1}{20} = £\frac{3}{80}$;

and $\frac{4}{9}$ of 7*s.* 6*d.* $= \frac{4}{9}$ of $7\frac{1}{2}s. = £\frac{4}{9} \times \frac{15}{2} \times \frac{1}{20} = £\frac{1}{6}$.

$\therefore$ the exp. $= £(\frac{7}{10} - \frac{3}{80} - \frac{1}{6}) = £\frac{119}{240}$; also £2. 19*s.* 6*d.* $= £2\frac{39}{40} = £\frac{119}{40}$.

$\therefore$ the required fraction $= £\frac{119}{240} \div £\frac{119}{40} = \frac{119}{240} \times \frac{40}{119} = \frac{1}{6}$. *Ans.*

Ex. 2. Reduce $\frac{7}{9}$ of *Rs.*10 $-$ $\frac{2}{5}$ of *Rs.*10. 8*a.* to the fraction of *Rs.*5. 4*a.*

$\frac{7}{9}$ of *Rs.*10 $=$ *Rs.*$\frac{70}{9}$; $\frac{2}{5}$ of *Rs.*10. 8*a.* $= \frac{2}{5}$ of *Rs.*$10\frac{1}{2}$ $=$ *Rs.*$\frac{21}{5}$.

$\therefore$ the difference $=$ *Rs.*$(\frac{70}{9} - \frac{21}{5})$ $=$ *Rs.*$\frac{161}{45}$; also *Rs.*5. 4*a.* $=$ *Rs.*$\frac{21}{4}$.

$\therefore$ the required fraction $=$ *Rs.*$\frac{161}{45} \div$ *Rs.*$\frac{21}{4} = \frac{161}{45} \times \frac{4}{21} = \frac{92}{135}$. *Ans.*

Ex. 3. What fraction of *Rs.*10. 8*a.* together with *Rs.*2. 4*a.* is equivalent to *Rs.*7. 8*a.*?

Here, *Rs.*7. 8*a.* − *Rs.*2. 4*a.* = *Rs.*5. 4*a.*

Now, the question reduces itself to finding—What fraction of *Rs.*10. 8*a.* is *Rs.*5. 4*a.*?

*Rs.*5. 4*a.* = 84*a.*; and *Rs.*10 8*a.* = 168*a.*

$\therefore$ the fraction required $= \dfrac{84a.}{168a.} = \frac{1}{2}$. *Ans.*

Ex. 4. What fraction of £2. 10*s.* is the sum which being diminished by 10*s.* 6*d.* is equal to £2. 2*s.*?

The meaning is—What fraction of £2. 10*s.* is £2. 2*s.* + 10*s.* 6*d.*?

£2. 2*s.* + 10*s.* 6*d.* = £2. 12*s.* 6*d.* = £$2\frac{5}{8}$ = £$\frac{21}{8}$;

also £2. 10*s.* = £$2\frac{1}{2}$ = £$\frac{5}{2}$.

$\therefore$ the required fraction = £$\frac{21}{8}$ ÷ £$\frac{5}{2}$ $= \frac{21}{8} \times \frac{2}{5} = \frac{21}{20} = 1\frac{1}{20}$. *Ans.*

Ex. 5. Compare the values of $\frac{7}{8}$ of £1, $\frac{5}{9}$ of a guinea, and $\frac{2}{3}$ of 15*s.* $7\frac{1}{2}d.$

$\frac{7}{8}$ of £1 = £$\frac{7}{8}$; $\frac{5}{9}$ of a guinea $= \frac{5}{9} \times$ £$\frac{21}{20}$ = £$\frac{7}{12}$; $\frac{2}{3}$ of 15*s.* $7\frac{1}{2}d. = \frac{25}{48}$.

Now, to compare £$\frac{7}{8}$, £$\frac{7}{12}$, £$\frac{25}{48}$, reduce them to equivalent fractions with the same denominator, and proceed as in Art. 257.

The L. C. M. of the denominators 8, 12, 48 is 48;

$$\therefore \pounds\frac{7}{8} = \pounds\frac{7\times 6}{8\times 6} = \pounds\frac{42}{48}\,;\ \pounds\frac{7}{12} = \pounds\frac{7\times 4}{12\times 4} = \pounds\frac{28}{48};\ \pounds\frac{25}{48} = \pounds\frac{25\times 1}{48\times 1} = \pounds\frac{25}{48}.$$

$\therefore$ $\frac{7}{8}$ of £1 is the *greatest* and $\frac{2}{3}$ of 15*s.* $7\frac{1}{2}d.$ is the *least*. *Ans.*

Ex. 6. What sum is that $\frac{2}{3}$ of which is *Rs.*2. 10*a.* 8*p.*?

*Rs.*2. 10*a.* 8*p.* = *Rs.*$2\frac{2}{3}$ = *Rs.*$\frac{8}{3}$.

$\therefore$ the required sum = *Rs.*$\frac{8}{3} \div \frac{2}{3}$ = *Rs.*$\frac{8}{3} \times \frac{3}{2}$ = *Rs.*6. 10*a.* 8*p.* *Ans.*

Examples LXXXIV.

1. Express :—

(1) *Rs.*4. 15*a.* 4*p.* as the fraction of *Rs.*6. 11*a.* 4*p.*; and *Rs.*7. 7*a.* 2*p.* as the fraction of *Rs.*31. 4*a.* 6*p.*

(2) *Rs.*25. 0*a.* 6*p.* as the fraction of *Rs.*29. 8*a.* 6*p.*

(3) £1. 7*s.* $8\frac{1}{2}d.$ as the fraction of £2. 7*s.* 6*d.*; and £3. 4*s.* $0\frac{3}{4}d.$ as the fraction of £7. 7*s.* 11*d.*

(4) 13*s.* $10\frac{1}{2}d.$ $\frac{2}{3}q.$ as the fraction of £2. 9*s.* 7*d.*

(5) $\frac{5}{11}$ of £5. 17*s.* 4*d.* and $\frac{3}{7}$ of $\dfrac{6\frac{2}{3}}{9\frac{1}{10}}$ of £1. 12*s.* $1\frac{1}{2}d.$ as fractions of £10.

(6) 2 fur. 29 po. 2 ft. 10 in. as the fraction of 1 mi. 5 fur. $16\frac{2}{3}$ po.

(7) £22. 13*s.* $8\frac{1}{4}d.$ as the fraction of $3\frac{1}{2}$ guineas.

(8) $\dfrac{7\frac{3}{4}-3\frac{11}{12}}{18\frac{1}{2}\div\frac{9}{7}}$ of £33. 14*s.* $5\frac{3}{4}$*d.* as the fraction of £157. 17*s.* $8\frac{1}{2}$*d.*

2. Reduce :—

(1) $\frac{2}{5}$ of 2*s.* $4\frac{1}{2}$*d.* to the fraction of a half crown ; and 9*s.* $10\frac{1}{2}$*d.* to the fraction of 13*s.* $2\frac{1}{4}$*d.*

(2) $6\frac{3}{8}\frac{2}{5}$ of *Rs.*15. 3*a.* 10*p.* to the fraction of *Rs.*31. 8*a.* 2*p.*

(3) $33\frac{1}{4}$ of 1 md. 2 sr. to the fraction of $3\frac{3}{4}$ of 28 mds. ; and 32 seers *to the fraction of* 3 mds. 22 sr. 2 ch.

(4) $2\frac{2}{3}$ of 2 bi. 7 kat. 4 ch. to the fraction of 4 bi. 11 kat.

(5) 1 md. 11 sr. 8 ch. to the fraction of 28 mds. ; and $12\frac{6}{9}$ of 15 sr. 12 ch. to the fraction of 30 mds. 32 sr.

(6) 3 qts. 1 pt. $2\frac{2}{3}$ gills to the fraction of 5 gals. 2 qts. 1 pt.

(7) 2 sq. yds. 2 ft. 120 in. to the fraction of 3 sq. po. $13\frac{3}{4}$yds. 1ft. 72 in.

(8) 12 oz. $12\frac{1}{5}$ drs. Avoir. to the fraction of 1 lb. Troy ; and 35 lbs. $8\frac{3}{4}$ oz. Troy to the fraction of a cwt.

(9) $2\frac{2}{3}$ half-guineas to the fraction of 10*s.* $11\frac{1}{4}$*d.*

(10) $7\frac{1}{2}\frac{3}{1}$ of 10 oz. 18 dwts. 11 grs. to the fraction of 8 lbs. $8\frac{6}{15}$oz. Avoir.

3. What part of $\frac{2}{5}$ of $\frac{6}{7}$ of 3 guineas is $\frac{3}{4}$ of $\frac{8}{9}$ of 15*s.* 9*d.* ?

4. What part of 13 cwt. 2 qrs. 21 lbs. is 11 cwt. 1 qr. 14 lbs. 15 oz.?

5. What part is 6 ft. $3\frac{2}{8}$ in. of 13 ft. $8\frac{8}{10}$ in.?

6. What part of a maund is 10 sr. 13 ch. 2 kan.?

7. What fraction of $\frac{1}{1}\frac{1}{2}$ of *Rs.*2. 5*a.* 8*p.* is $3\frac{1}{2}$ of $3\frac{1}{3}$ of *Rs.*12. 9*a.* 3*p.*, and of 7 guineas is $\frac{2}{3}$ of a moidore?

8 What fraction of 3 cwt. 2 qrs. 14 lbs. is 3 cwt. 19 lbs. 2 oz.?

9. What fraction of a year of $365\frac{1}{4}$ days is 27 days 16 hrs. 29 min. 4 sec., and of 1 oz. Avoir. is 1 oz. Troy?

10 What fraction of $19\frac{6}{8}\frac{1}{5}$ of 4 cub. yds. 18 ft. 1127 in. is $\frac{7}{12}$ of 200 cub. yds., and of $2\frac{7}{8}$ miles is $3\frac{7}{8}$ furlongs?

11. What fraction of $\dfrac{7\frac{4}{5}}{4\frac{2}{3}}$ of *Rs.*306. 9*a.* 10*p.* is $(8\frac{4}{7}-3\frac{3}{8})$ of *Rs.*54. 15*a.* 8*p.*, and of $2\frac{1}{2}$ tons is $\frac{3}{5}$ of 2 lbs.?

12. What fraction of 8 lbs. $12\frac{1}{3}$ oz. is 3 lbs. 9 oz. $62\frac{1}{2}$ grs.?

13. How many times is—

(1) *Rs.*9. 12*a.* $4\frac{1}{2}$*p.* contained in *Rs.* 7. 9*a.* $7\frac{1}{2}$*p.*?

(2) £24. 16*s.* $4\frac{1}{2}$*d.* contained in £335. 1*s.* $0\frac{3}{4}$*d.*?

(3) 2 tons 2 cwt. 2 qrs. contained in 3 cwt. 14 lbs.?

(4) 7 kathas 9 ch. contained in a bigha?

14. Express $\dfrac{3\frac{5}{8}}{7\frac{4}{5}}$ of $\dfrac{4}{7\frac{1}{4}}$ of *Rs.*33. 11*a.* 6*p.*, and $\frac{5}{2}$ of $\dfrac{7}{3\frac{4}{5}}$ of *Rs.*19. 1*a.* 7*p.* in terms of *Rs.*70. 5*a.* 10*p.* as unit.

15. What is the measure of $7\frac{3}{5}$ of $\dfrac{3\frac{3}{4}}{3\frac{17}{19}}$ of 5 cwt. 3 qrs. $3\frac{1}{2}$ lbs. when the unit is $(5\frac{8}{9}-3\frac{1}{4})$ of 3 tons 16 cwt. 3 qrs. $22\frac{4}{5}$ lbs. ?

16. Express :—

(1) $\frac{1}{3}$ of $\frac{7}{10}$ of 13*s*. 4*d*. + $\frac{3}{4}$ of $\frac{4}{7}$ of 10*s*. 6*d*. as the fraction of £1.

(2) $\frac{1}{9}$ of a guinea + $\frac{3}{8}$ of £1 + $\frac{5}{24}$ of 1*s*. + $\frac{1}{4}$ of 1*d*. as the fraction of a guinea, and of £24. 3*s*.

(3) $\frac{3}{8}$ of *Rs*.2. 8*a*. + $\frac{4}{5}$ of 8*a*. as the fraction of *Rs*.10. 8*a*.

(4) $\frac{2}{5}$ of *Rs*.3. 8*a*. + $\frac{2}{7}$ of *Rs*.5. 4*a*. − $\frac{2}{9}$ of *Rs*.10. 8*a*. as the fraction of *Rs*.13. 8*a*., and of *Rs*.39. 8*a*.

(5) $\frac{2}{11}$ of £13. 10*s*. $10\frac{1}{2}$*d*. − $\frac{5}{7}$ of £1. 2*s*. 9*d*. as the fraction of £6.

(6) *Rs*.$7\frac{3}{4}$ − $\frac{2}{7}$ of *Rs*.7 as the fraction of *Rs*.103. 5*a*. 4*p*.

(7) $\dfrac{3\frac{1}{9}}{1\frac{1}{13}}$ of $\left\{\dfrac{19}{120}\text{ of £1} - \dfrac{17}{48}\text{ of 1}s.\right\}$ as the fraction of 27*s*.

17. Compare the values of :—

(1) $\frac{1}{19}$ of £1, $\frac{1}{20}$ of a guinea and $\frac{8}{35}$ of a crown.

(2) $\frac{1}{21}$ of £1, $\frac{1}{22}$ of £1. 1*s*. and $\frac{1}{4}$ of 3*s*. $9\frac{1}{2}$*d*.

(3) $\frac{7}{8}$ of *Rs*.10, $\frac{5}{6}$ of *Rs*.10. 8*a*. and $\frac{2}{3}$ of *Rs*.7. 13*a*.

(4) $\frac{1}{5}$ of a maund, $\frac{3}{8}$ of 14 sr. and $\frac{4}{5}$ of 3 sr. 6 ch.

(5) $\frac{2}{3}$ of 5 days, $\frac{6}{7}$ of 20 hours, and $\frac{4}{5}$ of 59 min.

18. What fraction of *Rs*.100 together with *Rs*,36. 12*a*. is equivalent to *Rs*.52. 8*a*. ?

19. What fraction of 3 mds. 20 sr. together with 1 md. 9 sr. will give 42 mds. ?

20. What fraction of a ton added to $\frac{11}{28}$ of 2 cwt. will make it equal to 1 cwt. 2 qrs. 11 lbs. ?

21. What fraction of 2 tons 12 lbs. is the weight which being diminished by 1 cwt. 20 lbs. is equal to 1 cwt. 1 qr. 8 lbs. ?

22. What fraction of *Rs*.29. 12*a*. must be added to $\dfrac{3\frac{1}{2}}{4\frac{1}{5}}$ of $(3\frac{5}{7}+1\frac{2}{7})$ of *Rs*.6. 9*a*. to make the sum equal to *Rs*.32. 8*a*. ?

23. What fraction of a mile diminished by 39 yds. 1 ft. 9 in. is equal to 87 yds. 9 in. ?

24. What fraction of 2 lbs. 10 oz. Avoir. must be added to 1 lb. 8 oz. Troy to give 3 lbs. 7 oz. 10 dwts. ?

25. What sum is that $\frac{2}{3}$ of $\frac{1}{5}$ of which is $\frac{1}{7}$ of $\frac{6}{11}$ of *Rs*.5. 10*a*. ?

26. What length is that $\frac{27}{80}$ of which is $\frac{3}{5}$ of $7\frac{1}{2}$ of $16\frac{1}{2}$ yards?

27. What is the sum $\frac{13}{135}$ of which is $(4\frac{13}{21}-10\frac{4}{45}+9\frac{8}{45}-\frac{52}{117})$ of 8*p*., and what fraction is it of $\frac{8}{75}$ of *Rs*.6. 8*a*. ?

28. What weight is the same fraction of 15 cwt. 2 qrs. 13 lbs. that £1. 11*s.* $10\frac{1}{2}$*d.* is of £3. 10*s.* $1\frac{1}{2}$*d.*?

XIII. SIMPLIFICATION OF CONCRETE FRACTIONS.

308. It should be borne in mind what has already been said that when a concrete number is divided by another concrete number of the same kind, the quotient is an abstract number.

Ex. 1. Simplify $\dfrac{\frac{6}{7} \text{ of } 1\frac{3}{4} \text{ of } Rs.37.\ 6a.}{Rs.65.\ 6a.\ 6p.} \times \dfrac{1\frac{9}{16} \text{ of } 16 \text{ cwt. } 3 \text{ qrs.}}{41 \text{ cwt. } 3\frac{1}{2} \text{ qrs.}}$.

$$\text{The result} = \frac{\frac{6}{7} \text{ of } \frac{7}{4} \text{ of } Rs.37\frac{3}{8}}{Rs.65\frac{13}{32}} \times \frac{1\frac{9}{16} \text{ of } 67 \text{ qrs.}}{167\frac{1}{2} \text{ qrs.}} = \frac{\frac{6}{7} \text{ of } \frac{7}{4} \text{ of } \frac{299}{8}}{\frac{2093}{32}}$$

$$\times \frac{\frac{25}{16} \text{ of } 67}{\frac{335}{2}} = \left(\frac{6 \times 7 \times 299}{7 \times 4 \times 8} \times \frac{32}{2093}\right) \times \left(\frac{25 \times 67}{16} \times \frac{2}{335}\right)$$

$$= \tfrac{6}{7} \times \tfrac{5}{8} = \tfrac{15}{28}. \quad \textit{Ans.}$$

Ex. 2. Simplify $\dfrac{£1.\ 11s.\ 8d}{£2\ \ 17s.\ 0d.}$ of $\dfrac{142 \text{ yds. } 0\frac{4}{5} \text{ ft.}}{2 \text{ yds. } 1\frac{7}{10} \text{ ft.}}$ of 13 days 3 hrs.

$$\text{The result} = \frac{31\frac{2}{3}s}{57s.} \text{ of } \frac{426\frac{4}{5} \text{ ft.}}{7\frac{7}{10} \text{ ft.}} \text{ of } 13\frac{1}{8} \text{ days} = \left(\frac{95}{3} \times \frac{1}{57}\right) \text{ of}$$

$$\left(\frac{2134}{5} \times \frac{10}{77}\right) \text{ of } \frac{105}{8} \text{ days} = \frac{95}{3 \times 57} \times \frac{2134 \times 10}{5 \times 77} \times \frac{105}{8} \text{ days}$$

$$= \frac{5}{9} \times \frac{388}{7} \times \frac{105}{8} \text{ days} = \frac{2425}{6} \text{ days} = 404\tfrac{1}{6} \text{ days} = \underline{404 \text{ days } 4 \text{ hrs.}} \quad \textit{Ans.}$$

Examples LXXXV.

Simplify the following :—

1. $1\frac{1}{2}$ of $\dfrac{4\frac{1}{5}}{5\frac{1}{4}}$ of $\dfrac{18s.\ 6\frac{3}{4}d.}{£1}$ of 3 days 2 hrs. **2.** $\dfrac{£3\frac{9\frac{1}{2}}{14}}{8s.}$ of 2 mds. 32 sr.

3. $\dfrac{14 \text{ lbs. } 8 \text{ oz. } 18 \text{ dwts.}}{1 \text{ lb. } 0 \text{ oz. } 10 \text{ dwts.}}$ of 5*s.* $2\frac{1}{2}$*d.* **4.** $\dfrac{7 \text{ lbs. } 3 \text{ oz.}}{11 \text{ lbs. } 8 \text{ oz.}}$ of 8 po. 1 ft. 4 in.

5. $\dfrac{15s.\ 10d.}{£1.\ 18s.\ 6d.}$ of $\dfrac{71 \text{ yds. } 0\frac{2}{5} \text{ ft.}}{5 \text{ yds. } 0\frac{2}{5} \text{ ft.}}$ of $2021\frac{1}{4}$ days.

6. $\dfrac{\frac{1}{2} - \frac{2}{5}}{6\frac{1}{6} - 1\frac{3}{7} \text{ of } 4\frac{1}{5}}$ of $\dfrac{£1.\ 11s.\ 8d.}{£2.\ 17s.}$ of $\dfrac{142 \text{ yds. } 0\frac{4}{5} \text{ ft.}}{2 \text{ yds. } 1\frac{7}{10} \text{ ft.}}$ of 13 days 3 hrs.

7. $\dfrac{7 \text{ tons } 13 \text{ cwt. } 1\frac{1}{3} \text{ qrs. } 14 \text{ lbs.}}{15 \text{ cwt. } 3 \text{ qrs. } 7 \text{ lbs. } 12 \text{ oz.}}$ of $\dfrac{£15.\ 10s.\ 6\frac{3}{4}d.}{8s.\ 9d.}$.

8. $\dfrac{£10.\ 17s.\ 6d.}{£19.\ 6s.\ 8d.} + \dfrac{5 \text{ years } 73 \text{ days}}{18 \text{ hrs. } 40 \text{ min}} - \dfrac{Rs.4.\ 6a.\ 8p.}{Rs.5.\ 8a.}$.

9. $\left(\dfrac{3\frac{1}{2} \text{ of } 5\frac{5}{8}}{2\frac{5}{6} \text{ of } 3\frac{1}{3}} \div \dfrac{2\frac{2}{11} \text{ of } 1\frac{1}{12}}{3\frac{1}{9} \text{ of } 7\frac{5}{7}}\right)$ of $\dfrac{1s.\ 5d.}{4s.\ 7d.}$ of $\dfrac{2 \text{ ft. } 3 \text{ in.}}{5 \text{ ft. } 5 \text{ in.}}$ of 24 weeks 4 days 19 hrs.

10. Reduce $\dfrac{£2.\ 3s.\ 4d.}{£5.\ 6s.\ 8d.}$ of $\dfrac{2 \text{ tons } 4 \text{ cwt.}}{5 \text{ tons } 10 \text{ cwt.}}$ to a complex fraction having $12\frac{1}{2}$ for its numerator, and also to a complex fraction having $5\frac{1}{3}$ for its denominator.

XIV. MISCELLANEOUS PROPOSITIONS.

(ON VULGAR FRACTIONS.)

309. The Unitary Method. We have in Art. 171 given an outline of this method and treated it in the case of *integers*. We now propose to extend the method to fractional quantities. The following solutions, we hope, will serve as a guide to the students.

If the value, weight, length, &c. of **one** thing be given, the value, weight, length, &c. of **any number** of them (whether *integral* or *fractional* or *mixed*) may always be found by Multiplication; and *conversely*, if the value, weight, length, &c. of **any number** of things (whether *integral* or *fractional* or *mixed*) be given, the value, weight, length, &c. of **one** of them may always be found by Division.

Ex. 1. If a yard of lace cost *Re.*1. 6*a.* 6*p.*, what will 7 yds. 4 in. cost?

Here, *Re.*1. 6*a.* 6*p.* = *Rs.*$1\frac{13}{32}$; and 7 yds. 4 in. = $7\frac{1}{9}$ yds.

The cost of 1 yard = *Rs.*$1\frac{13}{32}$;

∴ the cost of $7\frac{1}{9}$ yds. = *Rs.*$1\frac{13}{32} \times 7\frac{1}{9}$ = *Rs.*$\frac{45}{32} \times \frac{64}{9}$ = *Rs.*10. *Ans.*

Ex. 2. If the cost of $20\frac{4}{5}$ yds. of cloth be *Rs.*173. 5*a.* 4*p.*, find the cost per yard of the same quality.

The cost per yard = *Rs.*173. 5*a.* 4*p.* $\div\ 20\frac{4}{5}$ = $\dfrac{Rs.173.\ 5a.\ 4p. \times 5}{104}$

= *Re.*1. 10*a.* 8*p.* × 5 = *Rs.*8. 5*a.* 4*p.* *Ans.*

Ex. 3. If $3\frac{2}{5}$ lbs. of tea cost *Rs.*7. 10*a.*, how much can I buy for *Rs.*41. 15*a.*?

Here, *Rs.*7. 10*a.* = *Rs.*$7\frac{5}{8}$; and *Rs.*41. 15*a.* = *Rs.*$41\frac{15}{16}$.

The cost of $3\frac{2}{5}$ lbs. = *Rs.*$7\frac{5}{8}$;

∴ the cost of 1 lb. = *Rs.*$7\frac{5}{8} \div 3\frac{2}{5}$ = *Rs.*$(\frac{61}{8} \times \frac{5}{17})$;

∴ the reqd. no. of lbs. = *Rs.*$41\frac{15}{16} \div$ *Rs.* $(\frac{61}{8} \times \frac{5}{17})$

$= \frac{671}{16} \times \frac{8}{61} \times \frac{17}{5} = \frac{187}{10} = 18\frac{7}{10}$. *Ans.*

Ex. 4. If $\frac{2}{3}$ of an estate be worth *Rs.*2200, find the value of $\frac{8}{11}$ of it.

$\frac{2}{3}$ of the estate is worth *Rs.*2200;

$\because$ the whole estate is worth $Rs.2200 \div \frac{2}{3} = Rs.3300$;

$\therefore \frac{3}{11}$ of the estate is worth $Rs.3300 \times \frac{3}{11} = \underline{Rs.900}$. *Ans.*

Ex. 5. A person, possessed of $\frac{2}{5}$ths of a coal mine, sells $\frac{3}{4}$ths of his share for £2000 ; what is the whole mine worth ?

Here, the part sold $= \frac{3}{4}$ of $\frac{2}{5}$ of the whole mine $= \frac{3}{10}$ of the mine.

The cost of $\frac{3}{10}$ of the mine $=$ £2000 ;

$\therefore$ the cost of the whole mine $=$ £$2000 \times \frac{10}{3} = \frac{1}{3} \times$ £20000

$=$ £6666. 13*s.* 4*d.* *Ans.*

Ex. 6. Express $\frac{2}{3}$ of $1\frac{5}{9}$ of a mile in terms of a metre, supposing 32 metres $=$ 35 yards.

35 yards $=$ 32 metres ; $\therefore$ 1 yard $= (32 \div 35)$ metres ;

$\therefore \frac{2}{3}$ of $1\frac{5}{9}$ of a mile $= (\frac{2}{3} \times \frac{14}{9} \times 1760)$ yds.

$= (\frac{2}{3} \times \frac{14}{9} \times 1760 \times 32 \div 35)$ metres

$= \frac{45056}{27}$ metres $= 1668\frac{20}{27}$ metres.

Ex. 7. If 5 men or 7 women can do a piece of work in 37 days ; in what time will 7 men and 5 women do the same piece of work ?

The work of 5 men $=$ that of 7 women ;

$\therefore$ the work of 1 man $=$ that of $\frac{7}{5}$ women,

$\therefore$ the work of 7 men $=$ that of $\frac{49}{5}$ women ;

$\therefore$ the work of 7 men $+$ 5 women $=$ that of $(\frac{49}{5} + 5)$ women

$=$ that of $\frac{74}{5}$ women.

Now, 7 women do the work in 37 days,

$\therefore$ 1 woman does the work in (37×7) days,

$\therefore \frac{74}{5}$ women do..... ....in $(37 \times 7 \div \frac{74}{5})$ days.

Hence the required time $= 37 \times 7 \times \frac{5}{74}$ days $= \underline{17\frac{1}{2} \text{ days}}$. *Ans.*

Ex. 8. If the six penny loaf weigh $4\frac{2}{3}$ lbs. when wheat is 6*s.* 9*d.* a bushel, what is the price of wheat per bushel when the same loaf weighs $3\frac{1}{2}$ lbs. ?

The loaf weighs $4\frac{2}{3}$ lbs. when wheat is $6\frac{3}{4}$*s.* a bus. ;

$\therefore$ ... ... 1 lb. ... ... $(6\frac{3}{4} \times 4\frac{2}{3})$*s.* a bus. ;

$\therefore$ ... ... $3\frac{1}{2}$ lbs. ... ... $(6\frac{3}{4} \times 4\frac{2}{3} \div 3\frac{1}{2})$*s.* a bus.

Hence the required price $= \frac{27}{4} \times \frac{14}{3} \times \frac{2}{7}$*s.* $=$ 9*s.* *Ans.*

Ex. 9. If 1000 men have provisions for 85 days, and if after 17 days, 150 of the men go away, find how long the remaining provisions will serve the number left.

Here $85 - 17 = 68$; and $1000 - 150 = 850$.

After 17 days, 1000 men have provisions for 68 days.

$\therefore$ 10 men ... ... for (68×100) days.

$\therefore$ 850 men ... ... for $(68 \times 100 \div 85)$ days,

or 80 days. *Ans.*

Ex. 10. If the cost of maintaining a family be *Rs.*50 a month, when rice is 12 seers a rupee, and *Rs.*48 when rice is 14 seers a rupee; what will be the cost when rice is 16 seers a rupee?

The price of 1 sr. is first reduced from *Re.*$\frac{1}{12}$ to *Re.*$\frac{1}{14}$ and lastly to *Re.*$\frac{1}{16}$.

Now, $\frac{1}{12}-\frac{1}{14}=\frac{1}{84}$ and $\frac{1}{12}-\frac{1}{16}=\frac{1}{48}$; also $50-48=2$.

Since a reduction of *Re.*$\frac{1}{84}$ in price causes a diff. of *Rs.*2 in expenses

$\therefore$ ... of *Re.*1 ... ... *Rs.*(2×84)...........

$\therefore$ ... ... of *Re.*$\frac{1}{48}$ ... ... *Rs.*$\frac{2\times 84}{48}$...........

or *Rs.*3. 8*a.*...............

Hence the required expenses $=$ (*Rs.*50 $-$ *Rs.*3. 8*a.*) $=$ *Rs.*46. 8*a.* *Ans.*

Examples LXXXVI.

1. Find the value of $5\frac{3}{7}$yds. of silk, when $3\frac{1}{7}$yds. cost *Rs.*21. 14*a.*

2. If $12\frac{1}{3}$ articles cost *Rs.*26. 3*a.* 4*p.*, how many can be bought for *Rs.*117. 3*a.* 8*p.*?

3. If 3 cwt. 3 qrs. 21 lbs. $12\frac{4}{9}$ oz. cost £4. 8*s.* 9*d.*, what is the price per cwt.?

4. If a silver cup weighing 20 oz. 19 dwts. $2\frac{2}{11}$ grs. cost *Rs.*57. 10*a.*, what is the price per oz.?

5. If $4\frac{3}{8}$ oz. of tea cost $8\frac{2}{3}$*s.*, what will $30\frac{2}{7}$ lbs. cost?

6. If $\frac{3}{16}$ of a lottery ticket cost £4. 10*s.* what is the price of $\frac{1}{3}$ of a ticket?

7. The owner of $\frac{4}{17}$ of a ship sold $\frac{5}{11}$ of $\frac{3}{5}$ of his share for £$12\frac{4}{33}$; what would $\frac{2\frac{1}{2}}{4\frac{1}{4}}$ of $\frac{2}{5}$ of it cost, at the same rate?

8. Express a degree of $69\frac{1}{4}$ miles in metres, where 32 metres are equal to 35 yards

9. If the sum paid for 247 bottles of wine amount, together with the duty, to *Rs.*774. 7*a.* 2*p.*; and the duty on each bottle be $\frac{1}{7}$th part of its original cost; what is the duty per bottle?

10. If the rent of 39 ac. 2 ro. 20 po. be *Rs.*1485. 15*a.*, what is the rent of 6 acres?

11. If $\frac{1}{8}$ of a ship be worth *Rs.*365. 5*a.*, what share of it will cost *Rs.*1252. 8*a.*?

12. A ship is worth *Rs.*160000 and a person possessed of $\frac{5}{16}$ of it, sells $\frac{3}{8}$ of his share; what share has he remaining, and what is it worth?

13. A party having a bill to pay of *Rs.*123. 9*a.*, one of them

pays for himself and three friends the sum of *Rs.*54. 14*a.* 8*p.*; how many were there?

14. If 7 men or 11 women can finish a piece of work in 17 days, how many days will it take 11 men and 7 women to finish it?

15. If 74 men had provisions for 35 days, and if after 5 days, 20 men were sent away; how long will the provisions last the remaining men?

16. If 6 men or 10 women can do a piece of work in 12 days, in what time will 5 men and 7 women do a piece of work twice as great?

17. If $3\frac{3}{4}$ tons of goods are carried 49 miles for *Rs.*19. 6*a.*, how far ought 26 tons 5 cwt. to be carried for the same money?

18. If $22\frac{1}{2}$ cwt. be carried 20 miles for *Rs.*5. 7*a*; what weight can be carried the same distance for *Rs.*14. 8*a.*?

19. The four-penny loaf weighs 1 lb. $15\frac{1}{2}$ oz., when wheat is at 7*s.* 11*d.* per bushel: find what its weight should be when wheat is at 7*s.* $1\frac{1}{2}$*d.* per bushel.

20. A fortress is provisioned for 3 weeks at the rate of 15 ch. a day for each man; if only $10\frac{1}{2}$ ch. be served out daily to each man, how long can the place hold out?

21. *A* borrowed of *B* *Rs.*1752. 8*a.* for 102 days, and afterwards would return the favour by lending *B* the sum of *Rs.*2103; for how long should he lend it?

22. A besieged town, containing 22400 inhabitants, has provisions to last 3 weeks; how many must be sent away that they may be able to hold out 7 weeks?

23. If the two-anna loaf weighs 4 ch., when wheat is *Rs.*3. 6*a.* a maund; what would be the price of wheat per maund when the same loaf weighs 3 ch.?

24. When rice is *Rs.*3 a maund, how many people can be fed for the same sum that would feed 90 people when rice is *Rs.*2. 8*a.* a maund?

25. If 2000 men have provisions for 95 days, and if after 15 days 400 men go away, find how long the remaining provisions will serve the number left.

26. The monthly expenditure of a shop in oil is *Rs.*40. 8*a.* when oil is sold at $3\frac{3}{8}$ seers a rupee; what will it amount to when the price of oil has risen to 4*a.* 10*p.* per seer?

27. A piece of cloth, measured with a yard measure which is $\frac{3}{4}$ of an inch too short, appears to be $10\frac{1}{2}$ yards long; what is its true length?

28. The expenses of a family when rice is sold at 20 seers a rupee are *Rs.*50 a month; when rice is sold at 25 seers a rupee the

expenses are *Rs.*48 a month; what will they be when rice is sold at 30 seers a rupee?

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310. Bankruptcy or Insolvency.

A tradesman becomes **bankrupt** or **insolvent**, when the money that he owes is more than that which he has in his possession. What he owes is called his **liabilities** or **debts**; his property or what he possesses is called his **effects** or **assets**. He is the **debtor**; those to whom he owes anything are his **creditors**. The amount paid by bankrupts is generally reckoned at so much in the rupee or pound, called **a dividend**, and each creditor receives the same fraction of the assets that the money due to him is of the bankrupt's whole debts.

Thus, if the assets amount to $\frac{2}{3}$ of the debts, each creditor receives $\frac{2}{3}$ of a rupee for each rupee due to him; and the bankrupt is said to pay *a dividend* of 10*a.* 8*p.* in the rupee.

Book-debts are moneys which other men owe to the bankrupt; they are, therefore, considered a part of his assets. Book-debts may be **good** or **bad**, as the whole or part can be recovered or realized.

Ex. 1. A bankrupt's estates amount to *Rs.*3780 and his debts to *Rs.*5040; how much can he pay in the rupee?

On *Rs.*5040 he can pay *Rs.*3780;

$\therefore$ in one rupee he can pay *Re.*$\frac{3780}{5040}$ or *Re.*$\frac{3}{4}$.

Hence he can pay *Re.*$\frac{3}{4}$ or 12*a.* *Ans.*

Ex. 2. A bankrupt's debts amount to *Rs.*3240, and he can pay 5*a.* 4*p.* in the rupee; find the amount of his assets.

On every rupee of debts he can pay $5\frac{1}{3}$*a.* or *Re.*$\frac{1}{3}$;

$\therefore$ on *Rs.*3240 of debts.................. $\frac{1}{3}\times$ *Rs.*3240;

Hence assets $=\frac{1}{3}\times$ *Rs.*3240 = *Rs.*1080. *Ans.*

Ex. 3. A bankrupt can pay 10*a.* 8*p.* in the rupee; had he *Rs.*4250 more he could have paid 12*a.* in the rupee. Find the amount of his debts and assets.

Here, 12*a.* − 10*a.* 8*p.* = 1*a.* 4*p.* = *Re.*$\frac{1}{12}$.

He could have paid *Re.*$\frac{1}{12}$ more on *Re.*1 of his debts;

$\therefore$ he could have paid *Re.*1 more on *Rs.*12 of his debts.

$\therefore$ ....*Rs.*4250......*Rs.*12 × 4250 of his debts.

Hence his debts = *Rs.*12 × 4250 = *Rs.*51000; } *Ans.*

also his assets = 10*a.* 8*p.* × 51000 = *Rs.*34000. }

Ex. 4. A creditor receives on a debt of £296 a dividend of 12*s.* 4*d.* in the £, and he receives a further dividend of 3*s.* 9*d.* in the £ upon the deficiency; find how much the creditor receives in all.

The first payment $= £(12\frac{1}{3} \div 20)$ or $£\frac{37}{60}$ on £1 of debt;

$\therefore$ the deficiency $= £(1 - \frac{37}{60})$ or $£\frac{23}{60}$ on £1 of debt.

Also the second payment $= £(3\frac{3}{4} \div 20)$ or $£\frac{3}{16}$ on £1 of deficiency;

$\therefore$ the second payment $= £\frac{3}{16} \times \frac{23}{60}$ or $£\frac{23}{320}$ on £1 of debt.

$\therefore$ first payment + second payment $= £(\frac{37}{60} + \frac{23}{320})$ on £1 of debt

$= \frac{661}{960}$ on £1 of debt.

Now, in £1 of debt the creditor receives $£\frac{661}{960}$;

$\therefore$ in £296 of debt... .... $£\frac{661}{960} \times 296$.

Hence the creditor receives $£\frac{661}{960} \times 296 = $ £203. 16*s.* 2*d.* *Ans.*

Ex. 5. A bankrupt has book-debts equal in amount to his liabilities but on £3000 of them he can only recover 6*s.* 8*d.* in the £, and the expenses of the bankruptcy are £5 for every £100 of the book-debts; if he pay 15*s.* in the £, what is the amount of his liabilities?

As he can recover 6*s.* 8*d.* or $£\frac{1}{3}$ in the £, he recovers $£\frac{1}{3} \times 3000$ or £1000 out of £3000; therefore his *loss* amounts to £2000. Again, he pays £5 for £100, or 1*s.* in the £ for expenses. Therefore he recovers $(15+1)$*s.* or 16*s.* in the £, and his *loss* per £ = 4*s.* or $£\frac{1}{5}$.

Now, $£\frac{1}{5}$ is the loss on £1 of liabilities.

$\therefore$ £1.................£5..................

$\therefore$ £2000...............$£5 \times 2000$.........

Hence liabilities $= £5 \times 2000 = $ £10000. *Ans.*

Examples LXXXVII

1. A bankrupt's estates amount to *Rs.*950 and his debts to *Rs.*1200; how much can he pay in the rupee?

2. A bankrupt's debts amount to £5069. 10*s.*, and he can pay 14*s.* $11\frac{1}{2}$*d.* in the £; find the value of his assets.

3. A bankrupt's debts amount to *Rs.*35000, and his assets to *Rs.*13708. 5*a.* 4*p.*; find how much his estate will pay in the rupee.

4. A bankrupt's effects amount to *Rs.*1980, and he pays his creditors 13*a.* 4*p.* in the rupee; what do his debts amount to?

5. A bankrupt's debts amount to *Rs.*53422. 8*a.* and his creditors lose *Rs.*17362. 5*a.*; find how much in the rupee the bankrupt pays.

6. A bankrupt owes *A Rs.*5156. 4*a.*, *B Rs.*4070 and *C Rs.*2933. 5*a.* 4*p.*; his estate is worth *Rs.*9119. 11*a.*; how much can he pay in the rupee, and what will *A*, *B* and *C* each receive?

7. A bankrupt owes *Rs.*9000 to his three creditors; and his whole property amounts to *Rs.*6750; the claims of two of his creditors are *Rs.*1250 and *Rs.*3750 respectively; what sum will the remaining creditor receive for his dividend?

8. A creditor received 16*s.* 3*d.* in the £, and thereby lost £135. 10*s.*; how much was due to him?

9. A bankrupt's debts amount to £1700, and his assets to £900 15*s*; after paying costs his creditors receive 5*s.* 9*d.* in the £; find the amount of the costs.

10. A bankrupt has good debts to the amount of £456. 18*s.* 1*d.*, and the following bad debts, £360. 7*s.* 10*d.*, £120. 13*s.* and £19. 18*s.* for which he receives respectively 4, 5 and 9 shillings in the £; his own liabilities amount to £3408. 12*s.*; how much can he pay in the £?

11. A creditor received on a debt of *Rs.*3600 a dividend of 9*a.* 10*p.* in the rupee; and a further dividend of 6*a.* 8*p.* upon the remainder. What did he receive altogether?

12. A bankrupt can pay 12*s.* 4*d.* in the £: if his assets were £4205 more, he could pay 15*s.* 8*d.* in the £. Find his debts and assets.

13. A bankrupt has book-debts equal in amount to his liabilities; but on *Rs.*8640 of such debts he can recover only 8*a.* 6*p.* in the rupee, and on *Rs.*6300 only 5*a.* 3*p.* in the rupee. After allowing *Rs.*1054. 11*a.* for the expenses of bankruptcy, he finds he can pay his creditors 12*a.* in the rupee. Find the total amount of his debts.

14. A bankrupt pays £5850 on the whole liabilities, at the rate of 13*s.* 6*d.* in the £ on half his debts and 15*s.* 9*d.* in the £ on the other half; find the amount of his debts.

15. A bankrupt can pay 11*a.* in the rupee; had he *Rs.*2550 more, he could have paid 14*a* in the rupee. Find the amount of his debts and assets.

16. A bankrupt has book-debts equal in amount to his liabilities; but on £6000 of them he can only recover 13*s.* 4*d.* in the pound, and the expenses of the bankruptcy are £5 on every £100 on the book-debts; if he pay 13*s.* in the pound, what is the amount of his liabilities?

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311. Incomes, Taxes and Rates.

Proceed as in the following Examples.

Ex. 1. If the income-tax be at the rate of 4*p.* in the rupee, and a man has to pay *Rs.*13. 6*a.* 8*p.*, what is the amount of his income?

Here, *Rs.*13. 6*a.* 8*p.* $=$ *Rs.*$13\frac{5}{12}$.

He pays *Re.*$\frac{4}{192}$ or *Re.*$\frac{1}{48}$ income-tax on every *Re.*1 of income;

$\therefore$ he pays *Re.*1 income-tax on every *Rs.*48 of income;

$\therefore$ ....*Rs.*$13\frac{5}{12}$ .... *Rs.*$48 \times 13\frac{5}{12}$.......;

Hence income required $=$ *Rs.*$48 \times \frac{161}{12}$ $=$ *Rs.*644. *Ans.*

Ex. 2. After paying an income-tax of $8p.$ in the rupee, a man has $Rs.7283.\ 5a.\ 4p.$ left; find his gross income.

Here, $Re.1-8p.=15a.\ 4p.=Re.\frac{23}{24}$; and $Rs.7283\ 5a.\ 4p.=Rs.7283\frac{1}{3}$.

Since $Re\frac{23}{24}$ is left out of $Re.1$ of income;

$\therefore\ Re.1 \ldots\ldots\ldots\ldots\ Rs.\frac{24}{23}\ldots\ldots\ldots\ldots$

$\therefore\ Rs.7283\frac{1}{3}\ldots\ldots\ Rs.\frac{24}{23}\times 7283\frac{1}{3}.$

Hence income required $=Rs.\frac{24}{23}\times\frac{21850}{3}=\underline{Rs.7600.}$ *Ans.*

Ex. 3. Find a man's gross rental, if after paying an income-tax of $6d.$ in the £ on the whole, and $3s.\ 6d.$ in the £ on $\frac{3}{4}$ of his rental, his net income is £2700.

Tax on $£\frac{3}{4}$ at $3s.\ 6d.=\frac{3}{4}\times 42d.=31\frac{1}{2}d.$

$\therefore$ total amount paid in taxes $=(6+31\frac{1}{2})d.$ or $37\frac{1}{2}d.$ in the £.

$\therefore$ he has $(240-37\frac{1}{2})$ or $202\frac{1}{2}d.$, or $£\frac{27}{32}$ left out of £1.

Since $£\frac{27}{32}$ is left out of £1 of gross income;

$\therefore\ £1\ldots\ldots\ldots\ldots\ £\frac{32}{27}\ldots\ldots\ldots\ldots$

$\therefore\ £2700\ldots\ldots\ldots\ldots\ £\frac{32}{27}\times 2700\ldots\ldots\ldots$

Hence gross rental $=£\frac{32}{27}\times 2700=\underline{£3200.}$ *Ans.*

Ex. 4. When the income-tax is $7d.$ in the £, a person has to pay £63 less than when the tax was $11d.$ in the £; find his income.

On the diminution of tax from $11d.$ to $7d.$ in the £, the man has to pay $4d.$ or $£\frac{4}{240}$ or $£\frac{1}{60}$ less on £1.

In every $£\frac{1}{60}$ less of income-tax the man has £1;

$\therefore\ \ldots\ldots £1\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots £60$

$\therefore\ \ldots\ldots £63\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots £60\times 63=\underline{£3780.}$ *Ans.*

Ex. 5. The rent of a man's house is £120 per annum. It is assessed to the rates at $\frac{2}{3}$ of this; the poor-rate is $7s.\ 6d$ in the £, the paving rate is $1s.\ 9d.$, and the church rate $4d.$; how much does he pay altogether for his residence?

Assessed value $=\frac{2}{3}$ of £120 = £80.

Amount of rates on £1 is $(7s.\ 6d.+1s.\ 9d.+4d.)=9s.\ 7d.$

$\therefore$ rates on £80 is $9s.\ 7d.\times 80=£38.\ 6s.\ 8d.$

Hence the annual cost of the house $=£120+£38.\ 6s.\ 8d.$
$=\underline{£158.\ 6s.\ 8d.}$ *Ans.*

Examples LXXXVIII.

1. A man pays an income-tax of $Rs.63.\ 14a.\ 5\frac{1}{4}p.$ at the rate of $7p.$ in the rupee; find his income.

2. How much will a poor-rate of 2*s.* 8*d.* in the £ produce in a parish in which the whole property is rated at £4736. 5*s.*?

3. A person after paying 7*p.* in the rupee for income-tax has *Rs.*346. 14*a.* left. What was his gross income?

4. After paying an income-tax of 3*d.* in the £, a person has a net income of £590. 10*s.* 6*d.*; find his gross income.

5. Find a man's gross rental if after paying an income-tax of 8*d.* in the £ on the whole, and 2*s.* 6*d.* in the £ on two-thirds of his rental, he has a net income of £398. 16*s.* 6*d.*

6. After deducting 4*p.* in the rupee for income-tax and $\frac{1}{25}$ of the value of the whole estate for collecting expenses, the value of the remainder is *Rs* 11270; what is the value of the whole estate?

7. The net rental of an estate, after deducting 7*d.* in the £ for income-tax and $\frac{1}{20}$ of the remainder for cost of collecting, is £959. 3*s.* 8*d.*; find the gross rental.

8. A reduction in the income-tax diminishes a tax which is *Rs.*15 when the tax is 8 pies in the rupee by *Rs.*3. 12*a*; what is the diminished rate of tax in the rupee?

9. I hire a house at £90 a year, which is assessed in the rate-book at $\frac{4}{5}$ths of its rent; I agree to pay the rates upon it, *viz.*, 3 poor-rates of 9*d*, 10*d.* and 1*s.* 2*d.* respectively in the £, a church rate of 8*d.* in the £, and a paving rate of 1*s.* 7*d.* in the £: what is the whole annual cost of the house?

10. A man allows his agent $\frac{1}{8}$ of one anna in the rupee on his gross income for the expense of collecting his rents. He spends $\frac{1}{7}$ of his net income in assuring his own life, and this part of his income is in consequence exempt from income-tax. The income-tax being 8*p.* in the rupee, and his income-tax amounting to *Rs.*389. 8*a.*, find his gross income.

11. A man pays a house rate of 1*s.* 6*d.* per £ on his rental; a water-rate of 1*s.* per £; a poor-rate of 1*s.* 10$\frac{1}{2}$*d.* per £. If the rent and rates amount to £85. 6*s.* 3*d.*, what is the rent?

12. An occupier pays house-rate of 3*a.*, police-rate of 9*p.*, water-rate of 2*a.* 6*p.* and a lighting-rate of 1*a.* 9*p.* in the rupee. If the rent and rates amount to *Rs.*1440, what is the assessed annual value of the house?

13. The income-tax having been raised to 10*d.* in the pound, a man has to pay £45. 10*s.* 6*d.* more than when it was 7*d.* in the pound. Find his income.

14. If a person's net income after paying an income-tax of 7*d.* in the £ be £291. 5*s.*, find his net income after paying an income-tax of 1*s.* 4*d.* in the £.

312. Division into Parts and Shares.

Proceed as follows :—

Ex. 1. A post has $\frac{1}{4}$ of its length in the mud, $\frac{1}{3}$ in water and 10 ft. above the water. Find its whole length.

Let unity or 1 represent the length of the post.

Then the part in the mud $= \frac{1}{4}$, and..................water $= \frac{1}{3}$; $\Big\}$ Now, $\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$. $\therefore$ the part above water $= 1-\frac{7}{12}=\frac{5}{12}$.

Hence, by question, $\frac{5}{12}$ of the post $=$ 10 ft.

$\therefore$ the length of the post $=$ 10 ft. $\times \frac{12}{5} =$ 24 ft *Ans.*

Ex. 2. One-half of the trees in an orchard are apple trees, one-fourth are pear trees, one-sixth plum trees, and there are 50 cherry trees ; what number of trees does it contain ?

Representing the number of trees in the orchard by the unit or 1, we have

$\frac{1}{2}=$ number of apple trees ;
$\frac{1}{4}=$ number of pear trees ;
$\frac{1}{6}=$ number of plum trees. $\Big\}$ Now, $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$; $\therefore$ the no. of other trees $= 1-\frac{11}{12}=\frac{1}{12}$.

Hence, by question, $\frac{1}{12}$ of the whole no. of trees $= 50$,

$\therefore$ the whole no. of trees $= 50 \times 12 = 600$. *Ans.*

Ex. 3. After paying away one-half of a sum of money, and then $\frac{3}{5}$ of what was left, *Rs.*5. 4*a.* remained ; what was the sum ?

Let 1 represent the sum of money.

Then $\frac{1}{2}$ of the sum $= \frac{1}{2}$, the first paid-up part ;

$\therefore 1-\frac{1}{2}=\frac{1}{2}$,..............remaining...... ;

Again $\frac{3}{5}$ of $\frac{1}{2}=\frac{3}{10}$, the second paid-up...... ;

$\therefore \frac{1}{2}-\frac{3}{10}=\frac{1}{5}$,...........remaining...... ;

Hence, by question, $\frac{1}{5}$ of the sum $=$ *Rs.*5. 4*a.*

$\therefore$ the whole sum $=$ *Rs.*5. 4*a.* $\times 5 =$ *Rs.*26. 4*a.* *Ans.*

Ex. 4. *A* met two beggars, *B* and *C* ; and having $\frac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\frac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\frac{77}{540}$ of a moidore of 27*s.* in his pocket, gave $\frac{1}{7}$ of $\frac{3}{4}$ of it to *B* and $\frac{3}{5}$ of the remainder to *C* ; what did each receive ?

$$A \text{ had at first } \frac{\frac{40}{11}}{\frac{30}{7}} \text{ of } \frac{\frac{75}{7}}{\frac{15}{2}} \text{ of } \frac{77}{540} \text{ of } 27s.$$

$$= \frac{40\times 7}{30\times 11} \text{ of } \frac{75\times 2}{7\times 15} \text{ of } \frac{77\times 27}{540}s. = \frac{14}{3}s.$$

$\therefore$ *B* received $\frac{1}{7}$ of $\frac{3}{4}$ of $\frac{14}{3}s. = \frac{1}{2}s. = 6d.$,

and *A* had afterwards left $(\frac{14}{3}-\frac{1}{2})s. = \frac{25}{6}s.$

$\therefore$ *C* received $\frac{3}{5}$ of $\frac{25}{6}s. = \frac{5}{2}s. =$ 2*s.* 6*d.*

Ex. 5. A person left $\frac{7}{18}$ of his property to his elder son and $\frac{7}{18}$ of the remainder to his younger son and the rest to his widow. The elder son received £1029. 16*s.* 4*d.* more than the younger; how much did the widow receive?

Let 1 represent the whole property.

Then the elder son received $\frac{7}{18}$, and the part left is $(1-\frac{7}{18})=\frac{11}{18}$.

The younger son received $\frac{7}{18}$ of $\frac{11}{18}=\frac{77}{324}$, and the part left is $\frac{11}{18}-\frac{77}{324}=\frac{121}{324}$.

$\therefore$ the widow's share is $\frac{121}{324}$ of the property.

The sons' shares differ by $\frac{7}{18}-\frac{77}{324}=\frac{49}{324}$ of the whole.

Hence, by question, $\frac{49}{324}$ of the whole estate = £1029. 16*s.* 4*d.*

$\therefore$ the whole estate = £1029. 16*s.* 4*d.* $\times\frac{324}{49}$

= £21. 0*s.* 4*d.* $\times$ 324 = £6809. 8*s.*

$\therefore$ the widow's share = £6809. 8*s.* $\times\frac{121}{324}$ = £21. 0*s.* 4*d.* $\times$ 121

= £2543. 0*s* 4*d.* *Ans.*

Ex. 6. Gunpowder being composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts; find how much of each is required for 18 maunds of powder.

The whole number of parts = (15 + 3 + 2) = 20.

$\therefore$ of every 20 parts, $\frac{15}{20}$ or $\frac{3}{4}$ is nitre, $\frac{3}{20}$ is charcoal, $\frac{2}{20}$ or $\frac{1}{10}$ is sulphur.

Hence, required nitre = $\frac{3}{4}$ of 18 mds. = 13 mds. 20 sr.

...........charcoal = $\frac{3}{20}$ of 18 mds. = 2 mds. 28 sr.

........sulphur = $\frac{1}{10}$ of 18 mds. = 1 md. 32 sr.

Ans.

Examples LXXXIX.

1. After detaching $\frac{11}{28}$ and $\frac{12}{35}$ of a company of soldiers, the general had 1110 left; required his original force.

2. If a person lay out $\frac{5}{9}$ of his income in board and lodging, $\frac{1}{6}$ in clothes and save *Rs.*600 a year; what is his income?

3. What is the capacity of a vessel, out of which when a third of it is empty, 35 gallons being drawn, there remains $\frac{3}{8}$ of the whole content?

4. In an orchard, $\frac{1}{3}$ are apple trees, $\frac{1}{5}$ peach trees, $\frac{2}{7}$ pear trees, and the remainder which is 38, cherry trees. How many trees are there in the orchard?

5. After taking out of a purse $\frac{2}{5}$ of its contents, $\frac{2}{3}$ of the remainder was found to be *Rs.*6. 11*a.* 8*p.*; what sum did it contain at first?

6. If $\frac{11}{20}$ of an estate be left to the elder and the remainder to the younger of two children, and the difference of their legacies be *Rs.*22[illegible]d the value of the estate.

7. Of a field $\frac{1}{5}$ is meadow, $\frac{3}{8}$ is arable and the remainder is 1 ac. 3 ro. 26 po. ; find the quantities of meadow and arable land.

8. *A* had at first £1. 8*s*. , and *B*, when he had paid $2\frac{3\frac{1}{7}}{1\frac{2}{9}}$ of £1. 11*s*. 6*d*. to *A*, found that he had remaining $\frac{1}{43}$ of what *A* then had ; what had *B* at first ?

9. A man pays away $\frac{1}{4}$ of his money, then $\frac{1}{3}$ of what remains, and then $\frac{2}{5}$ of the second remainder ; after which he has 7*s*. 6*d*. left ; how much had he at first ?

10. A post is divided into 4 parts ; the first part is $\frac{3}{7}$ of the whole length, the second part is $\frac{2}{3}$ of the first, the third $\frac{7}{9}$ of the second, and the fourth is 2 yds. 1 ft. 4 in. ; find the length of the post.

11. Out of *Rs*.43 12*a*., $\frac{1}{3}$ is paid to *A* and $\frac{1}{7}$ to *B* ; after this $\frac{4}{11}$ of the remainder is paid to *A* and the rest to *B* ; find the sums respectively received by *A* and *B*.

12. A gentleman left his eldest son $\frac{5}{9}$ of his money, to the younger $\frac{3}{8}$ of the remainder, and the rest to his wife ; upon dividing the money, it was found that the eldest son had *Rs*.7500 more than the younger ; how much was left to each ?

13. *A* and *B* have *Rs*.18 and *Rs*.12 respectively ; and if *A* gives *B* $\frac{2\frac{3}{8}}{4\frac{3}{4}}$ of the difference of $\frac{2\frac{3}{14}}{13\frac{5}{13}}$ of their respective sums, and $\frac{1}{7}$ of $2\frac{1}{2}$ of *A*'s present sum be added to $\frac{11}{15}$ of $\frac{1}{2}$ of *B*'s, *C*'s money will be $1\frac{1}{2}$ of this sum ; find it.

14 A person had a legacy left to him, which he thus divided amongst 3 charities. To one he gave $\frac{3}{10}$, to the second $\frac{3}{7}$ of the remainder, and to the third $\frac{3}{4}$ of what now remained ; and he then had *Rs*.1500 left. Find the amount of the legacy, and how much was given to each charity.

15. What number is that of which the fourth, fifth and sixth parts together exceed the half of the number by 112 ?

16. A person making his will, gave to one child $\frac{13}{20}$ of his estate, and the rest to another. When these legacies came to be paid, the one turned out to be £1200 more than the other ; what did the testator die worth ?

17. *A*, *B* and *C* rent a pasture for *Rs*.400. *A* puts in 8 cattle, *B* 9 and *C* 11 ; how much should each pay for his share ?

18. A person dies worth *Rs*.100000, and leaves $\frac{1}{3}$ of his property to his wife, $\frac{1}{2}$ to his son, and the rest to his daughter. The wife at her death leaves $\frac{2}{5}$ of her legacy to the son, and the rest to the daughter ; but the son adds his fortune to his sister's and gives her $\frac{1}{3}$ of the whole. How much will the sister gain by this, and what fraction will her gain be of the whole ?

313. Pipes and Cisterns.

If one or more pipes fill or empty a cistern in **8** min., they fill or empty $\frac{1}{8}$th of it per min.; and *conversely*, if they fill or empty $\frac{1}{8}$th of it per min., they fill or empty the whole in **8** min. Similarly, if they fill or empty a cistern in $5\frac{1}{2}$ hours, they fill or empty $\frac{1}{5\frac{1}{2}}$ or $\frac{2}{11}$ of it in 1 hr.: and *conversely*, if they fill or empty $\frac{2}{11}$ of it per hour, they will fill or empty the cistern in $(1 \div \frac{2}{11})$ or $5\frac{1}{2}$ hrs.

Ex. 1. Two pipes can separately fill a cistern in 10 and 15 minutes. If both the pipes are opened, how soon will the cistern be filled?

The first pipe fills $\frac{1}{10}$ of the cistern in 1 min.

.......second... ......$\frac{1}{15}$................ . ...

$\therefore$ both the pipes fill $(\frac{1}{10}+\frac{1}{15})$ or $\frac{1}{6}$ of the cistern in 1 min.

Hence they fill the cistern in $(1 \div \frac{1}{6})$ min. $= \underline{6}$ min. *Ans.*

Ex. 2. Pipes A and B can fill a cistern in 3 min. and 5 min. respectively, and C can empty it in $7\frac{1}{2}$ min. In what time will the cistern be filled when A, B and C are all turned on?

The first pipe fills in 1 min. $\frac{1}{3}$ of the cistern;

.. second .... $\frac{1}{5}$ .... ;

The third pipe empties in 1 min. $(1 \div 7\frac{1}{2})$ or $\frac{2}{15}$ of the cistern;

$\therefore$ with all open, $(\frac{1}{3}+\frac{1}{5}-\frac{2}{15})$ or $\frac{2}{5}$ of the cistern is filled in 1 min.

Hence the cistern will be filled in $(1 \div \frac{2}{5})$ min. $= \underline{2\frac{1}{2}}$ min. *Ans.*

Ex. 3. Two taps take 4 hours and 6 hours respectively to fill a cistern. When the waste pipe is left open along with the two taps, the cistern is filled in 24 hours. In what time does the waste pipe empty the cistern?

The first tap fills $\frac{1}{4}$ of the cistern in 1 hour.

...second....... $\frac{1}{6}$ ....

The three together fill $\frac{1}{24}$...... ....

$\therefore$ the waste pipe empties $(\frac{1}{4}+\frac{1}{6}-\frac{1}{24})$ or $\frac{3}{8}$ of the cistern in 1 hour. Hence, the waste pipe will empty the cistern in $(1 \div \frac{3}{8}) = 2\frac{2}{3}$ hrs. *Ans.*

Ex. 4. A cistern which would be filled in 8 hours requires 2 hours more to be filled, owing to a leak in the bottom. If the cistern is full, in what time will the leak empty it?

Had there been no leak, $\frac{1}{8}$ of the cistern would have been filled in 1 hr.; but the leak allows only $\frac{1}{10}$ to be filled in 1 hour.

$\therefore (\frac{1}{8}-\frac{1}{10})$ or $\frac{1}{40}$ of the cistern is emptied by the leak in 1 hour.

Therefore the leak requires $(1 \div \frac{1}{40})$ or $\underline{40}$ hours to empty the cistern. *Ans.*

Examples XC.

1. Two taps, A and B, fill a cistern in 10 and 20 hours respectively. In what time will they fill it together?

2. A cistern is filled by two taps in 10 and 15 hours respectively, and is emptied by a tap, C, in 8 hours. If all the three taps are open, in what time will the cistern be filled?

3. A cistern is fed by a spout which can fill it in 3 hrs. How long would it take to fill it, if the cistern has a leak which would empty, when full, in 17 hrs.?

4. Two pipes together can fill a cistern in 8 min., and one of them alone in 24 min. How long would the other alone take?

5 A cistern has three pipes connected with it, two to supply and one to draw off. The first alone can fill $\frac{2}{3}$ of the cistern in 3 hours, and the second $\frac{3}{4}$ in 4 hours; the third can empty $\frac{3}{4}$ of the cistern in 5 hours. If all the pipes be opened together at once, when will the cistern be full?

6. A cistern is filled by two spouts in 20 and 24 minutes respectively, and emptied by a tap in 30 minutes; what portion of it will be filled in 15 minutes when they are all left open together?

7. A cistern has three pipes A, B and C; A and B can fill it in 3 and 4 hours respectively, and C can empty it in 1 hour; if these pipes be opened in order at 1, 2 and 3 o'clock, when will the cistern be empty?

8 A cistern is provided with three spouts A, B and C. A can fill it in 30 minutes, B in 40 and C can empty it in 2 hours. If A, B and C be opened successively for a minute each, in what time will the cistern be filled; and how much of the content of the cistern will have passed out by C?

9. A cistern can be filled by three pipes; by the first in 10 hours, by the second in 9, and by the third in 8 hours. It is supplied by the first pipe till $\frac{1}{4}$ of it is full, then the second is also turned on till it becomes half full, and then all three begin to run. How long would it take to fill the cistern?

10. A tank can be filled by one pipe in 6 min., and by a second in 5 min., there is also a tap by which the tank can be emptied. If the tank be empty at first, and the pipes and tap be all left open, the tank is filled in 3 min. If the pipes are then closed, in what time will the tank be emptied by the tap?

11. A cistern can be filled by two pipes, A, B in 4 and 5 min. respectively and emptied by C in 144 seconds. B is opened 2 min. after A, C is opened 1 min. after B. The cistern contains 361 gallons just before C is opened. In what time will it be filled or emptied after the opening of C and how many gallons will go out by C?

12. Three taps, A, B and C can fill a cistern, A by itself in

24 min., B in 10 min., and C in 27 min. They are all turned on at once, but after $4\frac{1}{2}$ min. B and C are turned off. How much longer will A by itself take then to fill the cistern ?

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314. Time and Work. The following points (if remembered) will greatly help students in solving problems concerning *Time* and *Work.*

(1) If **1** man can do a piece of work in a certain time, then in the same time **2** men will do *twice* as much, 3 men *thrice* as much, and so on. *Conversely*, if **3** men can do a piece of work in a certain time, **1** man will do $\frac{1}{3}$ of the work in the same time.

(2) If one or more men can do a piece of work in **6** days, they can do $\frac{1}{6}$ of the work in 1 day ; so, if a piece of work can be done in $6\frac{1}{2}$ days, $(1 \div 6\frac{1}{2})$ or $\frac{2}{13}$ths of the work can be done in 1 day ; and *conversely*, if $\frac{1}{6}$th of a piece of work is done in 1 day, the whole work can be done in **6** days ; so, also if $\frac{2}{13}$ths of a piece of work can be done in 1 day, the whole work will be done in $(1 \div \frac{2}{13})$ or $6\frac{1}{2}$ days.

(3) If **5** men can do a piece of work in **7** days, then they will do $\frac{1}{7}$th of the work in one day ; therefore 1 man will do $\frac{1}{5} \times \frac{1}{7} = \frac{1}{35}$ of the work in 1 day, or the whole work in 35 or (5×7) days.

(4) If **1** man can do a piece of work in **5** days, then he will do $\frac{1}{5}$th of the work in 1 day ; therefore **3** men will do $\frac{3}{5}$ths of the work in 1 day, and therefore the whole work in $(1 \div \frac{3}{5})$ or $(5 \div 3)$ days.

(5) If **1** man can do $\frac{2}{5}$ of a piece of work in **7** days, he can do $\frac{2}{5} \times \frac{1}{7}$ of it in 1 day, and therefore the whole work in $1 \div (\frac{2}{5} \times \frac{1}{7})$ or $(7 \div \frac{2}{5})$ days.

(6) If A can do $\frac{1}{5}$ of a work in 1 day, and B $\frac{1}{4}$ in 1 day, then A and B together will do $(\frac{1}{5} + \frac{1}{4})$ or $\frac{9}{20}$ of the work in 1 day, and therefore both will finish the work in $(1 \div \frac{9}{20})$ or $2\frac{2}{9}$ days.

Ex. 1. A can do a piece of work in 5 days, B in 6 and C in 7 ; how much of it can they jointly do in 2 days, and how long will they take to do the whole work ?

A can do $\frac{1}{5}$ of the work in 1 day ; B can do $\frac{1}{6}$ of the work in 1 day ; C can do $\frac{1}{7}$ of the work in 1 day.

$\therefore$ A, B and C can jointly do $(\frac{1}{5} + \frac{1}{6} + \frac{1}{7})$ or $\frac{107}{210}$ of the work in 1 day.

Hence, in 2 days they will do $\frac{107}{210} \times 2$ or $\underline{1\frac{2}{105}}$ of the work. *Ans.*

Also, they can jointly do the whole work in $(1 \div \frac{107}{210})$ or $\underline{1\frac{103}{107}}$ days.

Ex. 2. If A and B together can perform a piece of work in 10 days, and A himself can do it in 18 days, what time will it take B to do it alone ?

A and B can do the work in 10 days;

$\therefore$ they can do $\frac{1}{10}$ of the work in 1 day;

A can do the work in 18 days:

$\therefore$ he can do $\frac{1}{18}$ of the work in one day;

$\therefore$ B can do $(\frac{1}{10}-\frac{1}{18})$ or $\frac{2}{45}$ of the work in 1 day;

Hence B can do the whole work in $(1 \div \frac{2}{45})$ or $\underline{22\frac{1}{2}}$ days. *Ans.*

Ex. 3. A does $\frac{7}{10}$ of a piece of work in 14 days; he then calls in B, and they finish the work in 2 days; how long would B take to do the whole work by himself?

A does $\frac{7}{10}$ of the work in 14 days;

$\therefore$ he does $\frac{7}{140}$ or $\frac{1}{20}$ of the work in 1 day;

$\therefore$ in 2 days, A does $\frac{2}{20}$ or $\frac{1}{10}$ of the work.

But $(1-\frac{7}{10})$ or $\frac{3}{10}$ of the work remains to be done;

$\therefore$ B does $(\frac{3}{10}-\frac{1}{10})$ or $\frac{1}{5}$ of the work in 2 days;

$\therefore$ B can do $\frac{1}{10}$ of the work daily, and

$\therefore$ B can do the whole work in $(1 \div \frac{1}{10})$ or 10 days. *Ans.*

Ex. 4. If A and B can do a piece of work in 18 days, A and C in 12 days, and B and C in 9 days, find the time in which A, B and C can together finish it, and also each working singly.

A and B can do $\frac{1}{18}$ of the work in 1 day;

A and C........$\frac{1}{12}$.............. ....;

B and C.........$\frac{1}{9}$....................;

$\therefore$ 2 men like A + 2 men like B + 2 men like C can do $(\frac{1}{18}+\frac{1}{12}+\frac{1}{9})$ or $\frac{1}{4}$ of the work in 1 day;

$\therefore$ A, B and C can do $\frac{1}{8}$ of the work in 1 day;

Hence they can jointly do the whole work in $(1 \div \frac{1}{8})$ or $\underline{8}$ days. *Ans.*

Also A can do $(\frac{1}{8}-\frac{1}{9})$ or $\frac{1}{72}$ of the work in 1 day, and

$\therefore$ the whole work in $\underline{72}$ days.

B ....$(\frac{1}{8}-\frac{1}{12})$ or $\frac{1}{24}$ of the work in 1 day, and

$\therefore$ the whole work in $\underline{24}$ days.

C.......$(\frac{1}{8}-\frac{1}{18})$ or $\frac{5}{72}$ of the work in 1 day, and

$\therefore$ the whole work in $\underline{14\frac{2}{5}}$ days. *Ans.*

Ex. 5. 5 men or 10 women or 15 boys can do a piece of work in 16 days. In how many days will 2 men, 3 women and 4 boys do it?

Since 5 men can do the work in 16 days;

$\therefore$ 1 man will do the work in (16×5) days;

$\therefore$ 1 man in one day will do $\frac{1}{16 \times 5}$ of the work;

$\therefore$ 2 men in one day will do $\dfrac{2}{16 \times 5}$ or $\frac{1}{40}$ of the work ;

Similarly, 3 women in one day will do $\frac{3}{160}$ of the work ;

and 4 boys .... $\frac{1}{60}$ of the work ;

$\therefore$ 2 men + 3 women + 4 boys in one day will do $(\frac{1}{40} + \frac{3}{160} + \frac{1}{60})$ or $\frac{29}{480}$ of the work.

Hence, they will take $(1 \div \frac{29}{480})$ or $\frac{480}{29} = 16\frac{16}{29}$ days. *Ans.*

Ex. 6. A and B can do a piece of work in 10 days, B and C in 15 days, and A and C in 25 days ; they all work at it together for 4 days ; A then leaves, and B and C go on together for 5 days, and then B leaves ; in how many days will C complete the work?

A and B can do $\frac{1}{10}$ of the work daily ; B and C $\frac{1}{15}$ daily, and A and C $\frac{1}{25}$ daily ; $\therefore$ 2 men like A + 2 men like B + 2 men like C can together do $(\frac{1}{10} + \frac{1}{15} + \frac{1}{25})$ or $\frac{31}{150}$ of the work daily, and $\therefore$ $A+B+C$ can do $\frac{31}{300}$ of the work daily. Hence in 4 days, they do $(\frac{31}{300} \times 4) = \frac{31}{75}$ of the work.

$\therefore$ when A leaves, $(1 - \frac{31}{75})$ or $\frac{44}{75}$ of the work remains to be done.

Now, B and C together in 5 days do $(\frac{1}{15} \times 5)$ or $\frac{1}{3}$ of the work.

$\therefore$ when B leaves, $(\frac{44}{75} - \frac{1}{3})$ or $\frac{19}{75}$ of the work remains to be done, and this work C finishes by himself.

Again, C in one day can do $(\frac{31}{300} - \frac{1}{10})$ or $\frac{1}{300}$ of the work.

Hence, C finishes the work in $(\frac{19}{75} \div \frac{1}{300})$ or $\frac{19}{75} \times 300 = 76$ days. *Ans.*

Ex. 7. If 10 excavators can dig 12 loads of earth in 16 hours, whilst 12 others can dig 9 loads in 15 hours ; find the time in which they will jointly dig 100 loads.

The *first* set of men can dig $\frac{12}{16}$ or $\frac{3}{4}$ load in 1 hour ; the *second* set $\frac{9}{15}$ or $\frac{3}{5}$ load in one hour.

$\therefore$ they can jointly dig $(\frac{3}{4} + \frac{3}{5})$ or $\frac{27}{20}$ loads in 1 hour ;

$\therefore$ they can dig 1 load in $(1 \div \frac{27}{20})$ or $\frac{20}{27}$ hour.

Hence they can dig 100 loads in $(\frac{20}{27} \times 100)$ or $74\frac{2}{27}$ hours. *Ans.*

Ex. 8. A can do a piece of work in 10 days, B in 9 days and C in 12 days. All begin together ; but A leaves after 4 days and B 2 days before the work is done. How long did the work last?

A can do $\frac{1}{10}$ of the work in 1 day ; B $\frac{1}{9}$ in 1 day ; and C $\frac{1}{12}$ in 1 day.

A in 4 days does $\frac{4}{10}$ or $\frac{2}{5}$ of the work. Now, C worked 2 days more than B, and during that time did $\frac{2}{12}$ or $\frac{1}{6}$ of the work.

Therefore the work done by B and C together is $(1 - \frac{2}{5} - \frac{1}{6})$ or $\frac{13}{30}$ of the work. Now, B and C in 1 day can do $(\frac{1}{9} + \frac{1}{12})$ or $\frac{7}{36}$ of the work ; therefore they took $(\frac{13}{30} \div \frac{7}{36})$ or $\frac{78}{35}$ days $= 2\frac{8}{35}$ days.

Hence the whole time occupied $= (2\frac{8}{35} + 2)$ or $4\frac{8}{35}$ days. *Ans.*

Ex. 9. If A can do as much work in 5 hours as B can do in

6 hours, or as C can do in 9 hours, how long will it take C to complete a piece of work, one-half of which has been done by A working 12 hours and B 24 hours?

Since 5 hrs. work of $A = 9$ hrs. work of C;

$\therefore$ 1 hr. of $A = \frac{9}{5}$ hrs. of C, or 12 hrs. of $A = \frac{9}{5} \times 12$ or $21\frac{3}{5}$ hrs. of C;

Since 6 hrs. work of $B = 9$ hrs. work of C;

$\therefore$ 24 hrs. of $B = 9 \times 4$ or 36 hrs. of C.

Hence 12 hrs. of $A + 24$ hrs. of $B = (21\frac{3}{5} + 36)$ or $57\frac{3}{5}$ hrs. of C.

But 12 hrs. work of $A + 24$ hrs. work of $B = \frac{1}{2}$ of the work;

$\therefore$ C can finish the remaining half in $\underline{57\frac{3}{5}}$ hrs. *Ans.*

Ex. 10. A is thrice as good a workman as B; and together they finish $\frac{3}{5}$ of a work in 9 days. In how many days will it be done by each separately?

Since 3 days' work of $B = 1$ day's work of A;

$\therefore$ 9 days' work of $B = 3$ days' work of A.

$\therefore$ 9 days' work of $B + 9$ days' work of $A = 12$ days' work of A.

But 9 days' work of $B + 9$ days' work of $A = \frac{3}{5}$ of the work;

$\therefore$ 12 days' work of $A = \frac{3}{5}$ of the work, *i. e.*

A can do $\frac{3}{5}$ of the work in 12 days.

Hence A does the whole work in $(12 \div \frac{3}{5})$ or $\underline{20}$ days, and therefore B does the whole work in 3×20 or $\underline{60}$ days. } *Ans.*

Examples XCI.

1. A alone can do a piece of work in 11 days, and B alone can do it in 17 days: find how long they would take to do it together.

2. A, B and C can complete a piece of work in 10, 12 and 15 days respectively. How long would it take them if they work together?

3. A can finish a piece of work in $2\frac{1}{2}$ days and B in $3\frac{1}{3}$ days: if they work together what part of the work will they finish in $1\frac{2}{7}$ days?

4. A and B can do a piece of work in 12 days: when C joins them they can do it in 9 days; in what time can C do it working alone?

5. A man alone can do a piece of work in 10 days which, if his son helps him, he can do in 6 days; in what time would his son working alone do the work?

6. A can reap $\frac{4}{9}$ of a field in $2\frac{2}{3}$ days, and B can reap $\frac{3}{5}$ of it in $4\frac{1}{2}$ days; in what time could A and B working together reap the field?

7. If A and B can do a piece of work in 24 days, A and C in

16 days, and B and C in 12 days; find the time in which A, B and C can together finish it.

8. A and B can do a piece of work in 6 days which B and C can do in 4 days, and A and C in 3 days. Find the time in which each can separately do it.

9. A and B can do a piece of work in 8 days, A and C in $10\frac{2}{3}$ days, and B and C in $9\frac{1}{5}$ days; in how many days can A alone do it?

10. A, B and C can finish a piece of work in 12 hours, also A and B can do it in 16 hours, and A and C in 18 hours; what part of the whole work can B and C do in $9\frac{1}{7}$ hours?

11. A, B and C can do a piece of work together in 20 days, A alone can do it in 40 days, and B alone in 60 days. In what time can C alone do it?

12. A performs $\frac{2}{7}$ of a piece of work in 13 days, and with the help of B finishes it in 6 days. In what time could each of them do the piece of work separately?

13. A can do $\frac{2}{3}$ of a piece of work in 4 hours, B can do $\frac{3}{4}$ of the remainder in 1 hour, and C can then finish it in 20 minutes; in what time can A, B and C together do it?

14. A certain number of men mow 4 acres of grass in 3 hours, and a certain number of others mow 8 acres in 5 hours; how long will they be in mowing 11 acres, if all work together?

15. A can mow $2\frac{1}{2}$ acres in $6\frac{2}{3}$ days, and B $2\frac{1}{6}$ acres in $5\frac{1}{3}$ days; they mow together a field of 10 acres; how long will it take them to do it, and how many acres will each mow?

16. A and B can do a piece of work in 4 days, working 6 hours a day; B and C can do it in 4 days, working 5 hours a day: and A and C can do it in 4 days, working 4 hours a day. In how many days of 8 hours will each do it separately?

17. A can do a piece of work in 27 days, A and B can do it in 15 days; A works alone for 12 days, and A and C together for 5 days, and B finishes it in 7 days; find in what time B and C together could do it.

18. A can do a piece of work in 27 days and B in 15 days; A works at it alone for 12 days, B then works 5 days and afterwards C finishes it in 4 days; in what time could C have done the whole work?

19. A and B can do a piece of work, each, in 24 days; A and B work together for 6 days, when B goes away and C works with A for 3 days, then B rejoins them, and the work is finished in 2 days more. How long would it have taken A, B and C to do the piece of work, if they had all worked together?

20. A can do a piece of work in 6 days and B in 9 days. They begin together. But 2 days before the completion of the work, A leaves off. In how many days is the work finished?

21. A is twice as good a workman as B; and together they finish a work in 8 days. In how many days can it be done by each separately?

22. 8 men or 12 women or 16 children can do a piece of work in 15 days. In how many days will 3 men, 4 women and 5 children do it?

23. A is thrice as good a workman as B. If the time taken by B to do a piece of work exceed that taken by A by 8 days, find in how many days each can do it.

24. A is twice as good a workman as B and thrice as good as C. Working together for 10 days they can finish a work. They all begin together. But after working for 3 days A leaves off. After 5 days more B also leaves off. In how many days more will C finish the work?

25. A can do a piece of work in 10 days, B in 9 days and C in 12 days. All begin together; A leaves after $3\frac{3}{8}$ days, B leaves $2\frac{3}{5}$ days before the work is done. How long did the work last?

26. A man can do as much work in 3 days as a boy can do in 5. How long will a man take to finish a work, $\frac{1}{4}$ of which has been done by a boy in 8 days?

27. If A in 2 days can do as much work as C in 3 days, and B in 5 days as much as C in 4 days; what time will B require to execute a piece of work which A can accomplish in 6 weeks?

28. If A can do as much work in 5 hours as B can do in 6 hours, or as C can do in 9 hours, how long will it take A to complete a piece of work, one-half of which has been done by B working 12 hours, and C working 24 hours?

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315. Equations. A statement of the equality of two arithmetical expressions is called an **arithmetical equation.**

Thus, $8=5+3$ is an *arithmetical equation*, for it asserts that 8 is equal to the sum of 5 and 3. The numbers 8, 5 and 3 are called **terms** of the equation. If one of the terms be unknown, it can be easily found from the above statement.

316. Although equation is an instrument of great power in all mathematical calculations, yet it is surprising to see that in no text-book of Arithmetic the method of solution by equations has received due favour. The following simple results are very useful in solving equations.

(i) *If equals be added to equals the sums are equal.*

Thus, $15-3=12$, $\therefore 15=12+3$, (adding 3 to each side of the equation).

(ii) *If equals be taken from equals the remainders are equal.*

Thus, $15=12+3$, $\therefore 15-3=12$, (taking 3 from each side of the equation).

(iii) *If equals be multiplied by equals the products are equal.*
Thus, $15=12+3$; $\therefore$ $15\times4=(12+3)\times4$, (multiplying each side by 4).

(iv) *If equals be divided by equals the quotients are equal.*
Thus, $15=12+3$; $\therefore$ $15\div3=(12+3)\div3$, (dividing each side by 3).

Hence from (i) and (ii) we see that *any term of an equation may be transferred from one side of the equation to the other, if its sign be changed, plus becoming minus and minus becoming plus.*

317. In a problem, the number to be found is called the **unknown quantity** or **unknown term,** and the numbers given are called the **known quantities** or **known terms.** To combine them and thus reduce their number, we **transpose** all the terms into which the unknown quantity enters to one side of the equation and the known terms to the other side, changing the sign of each term so transposed.

Ex. 1. If to the sum of $\frac{1}{3}$ and $\frac{1}{4}$ of a number 5 be added, the sum is 19; find the number.

$(\frac{1}{3}+\frac{1}{4})$ of the number $+5=19$; $\therefore$ $\frac{7}{12}$ of the number $+5=19$

Transposing the terms, we have

$\frac{7}{12}$ of the number $=19-5=14$; $\therefore$ number $=14\times\frac{12}{7}=\underline{24}$. *Ans.*

Ex. 2. What is the number from which if you take away 15, the remainder is $\frac{2}{3}$ of the original number?

The number $-15=\frac{2}{3}$ of the number.
Transposing the terms, we have
the number $-\frac{2}{3}$ of the number $=15$;
$\therefore$ $\frac{1}{3}$ of the number $=15$; $\therefore$ the number $=3\times15=\underline{45}$. *Ans.*

Ex. 3. A boy loses $\frac{1}{4}$ of his money, and then gains 6*ps.*; he then loses $\frac{1}{3}$ of what he has, and then gains 4*ps.*; he afterwards loses $\frac{3}{8}$ of what he has, and then finds that he has 6*a.* 1*ps.* left. How much had he at first?

$\frac{1}{4}$ of the money is lost; $\therefore$ $\frac{3}{4}$ of it remains; 6*ps.* is then gained;
$\therefore$ money now remaining $=\frac{3}{4}$ of original money $+$ 6*ps.*; of this $\frac{1}{3}$ is lost;
$\therefore$ $\frac{2}{3}$ of ($\frac{3}{4}$ of original money $+$ 6*ps.*) remains; 4*ps.* is then gained;
$\therefore$ money now remaining $=\frac{2}{3}$ of ($\frac{3}{4}$ of original money $+$ 6*ps.*) $+$ 4*ps.* of this amount $\frac{3}{8}$ is lost;
$\therefore$ $\frac{5}{8}$ of [$\frac{2}{3}$ of ($\frac{3}{4}$ of original money $+$ 6*ps.*) $+$ 4*ps.*] remains,
$=\frac{5}{8}$ of ($\frac{1}{2}$ of original money $+$ 4*ps.* $+$ 4*ps.*),
$=\frac{5}{16}$ of original money $+$ 5*ps.*;
$\therefore$ $\frac{5}{16}$ of original money $+$ 5*ps.* $=$ 6*a.* 1*ps.*;
$\therefore$ $\frac{5}{16}$ of original money $=$ 25*ps.* $-$ 5*ps.* $=$ 20*ps.*
$\therefore$ original money $=$ 20*ps.* $\times\frac{16}{5}=$ 64*ps.* $=$ *Re.* $\underline{1}$. *Ans.*

Ex. 4. From a tank $\frac{4}{5}$ths full of water 12 gals. are drawn, and the tank is then found to be $10\frac{1}{2}$ gals. more than half full; find how many gals. it will hold.

After drawing 12 gals. the quantity of water remaining $= \frac{4}{5}$ of tank $-$ 12 gals.; and it is then found that the tank is $10\frac{1}{2}$ gals. more than half full;

$\therefore \frac{4}{5}$ of tank $-$ 12 gals. $= \frac{1}{2}$ of tank $+ 10\frac{1}{2}$ gals.

$\therefore \frac{4}{5}$ of tank $- \frac{1}{2}$ of tank $= 10\frac{1}{2}$ gals. $+$ 12 gals. $= 22\frac{1}{2}$ gals.;

$\therefore \frac{3}{10}$ of tank $= 22\frac{1}{2}$ gals.; $\therefore$ tank holds $22\frac{1}{2} \times \frac{10}{3}$ or $\underline{75}$ gals. *Ans.*

Examples XCII.

1. If to $\frac{3}{7}$ of a number 18 be added the sum is 42; find the number.

2. If to the sum of $\frac{3}{14}$ and $\frac{5}{8}$ of a number 34 be added the sum is 128; find the number.

3. *If from the sum of* $\frac{5}{6}$ and $\frac{7}{10}$ of a number 41 be taken the remainder is 97; find the number.

4. What is the number to which if you add 60 the sum is 5 times the *original number?*

5. There is a number, to which 3 is added and $\frac{1}{10}$ of the result taken; to this 5 is added and $\frac{1}{15}$ of the result taken; then the result is $1\frac{1}{2}$; what is the number?

6. The sum of two numbers is 5760, and their difference is equal to one-third of the greater. What are the numbers?

7. The sum of four fractions is $2\frac{19}{30}$, and one common result is obtained by adding the fraction $\frac{3}{10}$ to the first, subtracting $\frac{1}{8}$ from the second, multiplying the third by $\frac{7}{8}$ and dividing the fourth by $\frac{10}{11}$. Find the four fractions.

8. A person after paying away one-third of his money together with *Rs.*10, finds that he has remaining *Rs.*15 more than its half; what money had he?

9. *A* spends $\frac{3}{16}$ of his money and then earns *Rs.*5; he afterwards spends $\frac{4}{7}$ of what he then has, and has then *Rs.*10. 8*a.* left; find how much he had at first.

10. *Out of* $\frac{2}{3}$ *of my income I pay to one* person *Rs.*100 and to another *Rs.*150, and then find that I have *Rs.*50 less than $\frac{7}{12}$ of my income left; find my income.

11. Out of a cask two-thirds full of wine 8 gals. are drawn, and it is then found to be 2 gals. less than half-full; how many gals. is the cask able to hold?

12. An army in a defeat loses $\frac{1}{6}$ is to number and 8000 prisoners; after being reinforced by 6000 men it again loses $\frac{1}{4}$ of its number in retreat; and 36000 are then left, what was the original force?

318. Irregular Distributions.

Again means a second time. *As much again* means as much once and as much a second time, (*i. e.*) twice as much. *Half as much again* means as much once and half as much a second time, *i. e.* $1\frac{1}{2}$ times as much.

Proceed as in the following Examples.

Ex. 1. Divide *Rs.*11875 among A, B and C so that as often as A gets *Rs.*4, B shall get *Rs.*3, and as often as B gets *Rs.*6, C shall get *Rs.*5.

As often as A gets *Rs.*4, B gets *Rs.*3; $\therefore$ B's share $= \frac{3}{4}$ of A's.

As often as B gets *Rs.*6, C gets *Rs.*5; $\therefore$ C's share $= \frac{5}{6}$ of B's.

$\therefore$ C's share $= \frac{5}{6}$ of $\frac{3}{4}$ of A's $= \frac{5}{8}$ of A's;

$\therefore$ A's share $+ B$'s $+ C$'s $= (1 + \frac{3}{4} + \frac{5}{8})$ of A's $= 2\frac{3}{8}$ times A's share;

Hence $2\frac{3}{8}$ times A's share $=$ *Rs.*11875;

$\therefore$ A's share $=$ *Rs.*$11875 \div 2\frac{3}{8} =$ *Rs.*5000.

$\therefore$ B's share $= \frac{3}{4}$ of *Rs.*5000 $=$ *Rs* 3750.

and C's share $= \frac{5}{8}$ of *Rs.*5000 $=$ *Rs.*3125.

Otherwise thus : If A gets *Rs.*8, B gets *Rs.*6 and C gets *Rs.*5.

Now, $8+6+5=19$; and $11875 \div 19 = 625$.

$\therefore$ A gets $\frac{8}{19}$ of *Rs* $11875 =$ *Rs.*$8 \times 625 =$ *Rs.*5000; &c.

Ex. 2. Divide *Rs.*640 among A, B and C, so that A may have 3 times as much as B, and C $\frac{1}{3}$ of what A and B together have.

A's share $= 3$ times B's share; C's share $= \frac{1}{3}(A$'s $+ B$'s$)$.

$\therefore$ C's share $= \frac{1}{3}$ $(3\ B$'s $+ B$'s$) = \frac{4}{3}$ B's.

$\therefore$ A's share $+ B$'s $+ C$'s $= (3 + 1 + \frac{4}{3})$ of B's $= 5\frac{1}{3}$ of B's share.

Hence $5\frac{1}{3}$ of B's share $=$ *Rs.* 640; $\therefore$ B's share $=$ *Rs.*$640 \div 5\frac{1}{3} =$ *Rs.*120.

$\therefore$ A's share $=$ *Rs*$120 \times 3 =$ *Rs.*360 and C's $= \frac{4}{3} \times$ *Rs.*120 $=$ *Rs.*160.

Ex. 3. The sum of *Rs.*155 is to be divided amongst 3 men, 5 women and 8 boys, so that for every 3*a.* a man gets, a woman gets 2*a.*, and a boy 1*a.* 6*p.*; find the share of each.

A woman's share $= \frac{2}{3}$ of a man's; a boy's share $= \frac{1}{2}$ of a man's;

$\therefore$ a man's share + a woman's + a boy's $= (1 + \frac{2}{3} + \frac{1}{2})$ of a man's;

$\therefore$ 3 men's shares + 5 women's + 8 boys' $= (3 + \frac{10}{3} + 4)$ of a man's $= 10\frac{1}{3}$ times a man's share;

Hence $10\frac{1}{3}$ times a man's share $=$ *Rs.* 155;

$\therefore$ a man's share $=$ *Rs.*$155 \div 10\frac{1}{3} =$ *Rs.*15; a woman's share $= \frac{2}{3}$ of *Rs.* 15 $=$ *Rs.*10, and a boy's share $= \frac{1}{2}$ of *Rs.*15 $=$ *Rs.*7. 8*a.*

Ex. 4. Divide *Rs.*8424 among A, B and C, so that A shall receive $\frac{4}{9}$ as much as B and C together, and B $\frac{4}{5}$ of what A and C together receive.

A's share $=\frac{4}{9}$ of (B's $+$ C's), and B's share $=\frac{4}{5}$ of (A's $+$ C's).

$\therefore$ A's share $=\frac{4}{9}B$'s $+\frac{4}{9}C$s $=\frac{4}{9}$ of $\frac{4}{5}$ of (A's $+$ C's) $+\frac{4}{9}C$'s $=\frac{16}{45}$ of (A's $+$ C's) $+\frac{4}{9}C$'s $=\frac{16}{45}A$'s $+\frac{16}{45}C$'s $+\frac{4}{9}C$'s $=\frac{16}{45}A$'s $+\frac{4}{5}C$'s ;

$\therefore$ A's $-\frac{16}{45}A$'s $=\frac{4}{5}C$'s, or $\frac{29}{45}A$'s $=\frac{4}{5}C$'s ; $\therefore$ A's $=\frac{4}{5}\times\frac{45}{29}C$'s $=\frac{36}{29}C$s.

$\therefore$ B's $=\frac{4}{5}A$'s $+\frac{4}{5}C$'s $=\frac{4}{5}\times\frac{36}{29}C$'s $+\frac{4}{5}C$'s $=\frac{260}{145}C$'s $=\frac{52}{29}C$'s.

$\therefore$ A's share $+$ B's $+$ C's $=(\frac{36}{29}+\frac{52}{29}+1)$ of C's $=\frac{117}{29}$ of Cs share ;
hence $\frac{117}{29}$ of C's share $=Rs.8424$.

$\therefore$ C's share $=Rs.8424\div\frac{117}{29}=Rs.2088$,
$\therefore$ A's share $=\frac{36}{29}$ of $Rs.2088=Rs.2592$,
and B's share $=\frac{52}{29}$ of $Rs.2088=Rs.3744$. *Ans.*

Examples XCIII.

1. Divide *Rs.*6488. 7*a.* 10*p.* amongst three persons A, B and C, so that $\frac{1}{17}$ of A's share $=\frac{1}{22}$ of B's $=\frac{1}{26}$ of Cs.

2. Divide *Rs.*75. 8*a.* between A, B and C giving B half as much again as A less *Re.*1, and C as much as A and B together.

3. Divide *Rs.*1400 among A, B and C in such a manner that as often as A gets *Rs.*5, B shall get *Rs.*4, and as often as B gets *Rs.*3, C shall get *Rs.*2.

4. Divide *Rs.*352. 9*a.* among A, B and C, so that B may get twice, and C 3 times as much as A.

5. Divide *Rs.*1800 among A, B and C, so that A may receive 3 times as much as B, and B and C together $\frac{1}{2}$ as much as A.

6. Divide *Rs.*12540 among A, B and C, so that A shall receive $\frac{3}{7}$ as much as B and C together, and B $\frac{2}{9}$ of what A and C together receive.

7. Divide *Rs.*2000 among A, B and C, so that B's share may be $\frac{7}{8}$ of A's share, and C's share $\frac{5}{8}$ of B's.

8. Divide *Rs.*95. 10*a.* 8*p.* among 10 men, 6 women and 4 children, giving a woman 3 times as much as a child and a man twice as much as a woman.

9. Divide £1650 among A, B, C and D, so that A may have half as much as B, B a third as much as C and C a fourth as much as D.

10. If $\frac{3}{4}$ of A's money $=\frac{2}{3}$ of B's $=\frac{6}{11}$ of C's and A, B and C's money together amount to *Rs.* 8260 ; how much has each?

11. If $\frac{5}{6}$ of A's money $=\frac{7}{8}$ of B's $=\frac{8}{9}$ of C's $=\frac{9}{10}$ of D's and A, B, C and D together have *Rs.*23078 ; determine how much money each has.

12. If $\frac{7}{10}$ of A's money $=\frac{9}{11}$ of B's, and C's money $=\frac{2}{3}$ ($\frac{6}{7}$of

318. Irregular Distributions.

Again means a second time. *As much again* means as much once and as much a second time, (*i. e*) twice as much. *Half as much again* means as much once and half as much a second time, *i. e.* $1\frac{1}{2}$ times as much.

Proceed as in the following Examples.

Ex. 1. Divide *Rs.*11875 among A, B and C so that as often as A gets *Rs.*4, B shall get *Rs.*3, and as often as B gets *Rs.*6, C shall get *Rs.*5.

As often as A gets *Rs.*4, B gets *Rs.*3 ; $\therefore$ B's share $=\frac{3}{4}$ of A's.

As often as B gets *Rs.*6, C gets *Rs.*5 ; $\therefore$ C's share $=\frac{5}{6}$ of B's.

$\therefore$ C's share $=\frac{5}{6}$ of $\frac{3}{4}$ of A's $=\frac{5}{8}$ of A's ;

$\therefore$ A's share $+B$'s$+C$'s$=(1+\frac{3}{4}+\frac{5}{8})$ of A's $=2\frac{3}{8}$ times A's share ;

Hence $2\frac{3}{8}$ times A's share $=$ *Rs.*11875 ;

$\therefore$ A's share $=$ *Rs.*$11875\div2\frac{3}{8}=$ *Rs.*5000.

$\therefore$ B's share $=\frac{3}{4}$ of *Rs.*5000 $=$ *Rs* 3750.

and C's share $=\frac{5}{8}$ of *Rs.*5000 $=$ *Rs.*3125.

Otherwise thus : If A gets *Rs.*8, B gets *Rs.*6 and C gets *Rs.*5.

Now, $8+6+5=19$; and $11875\div19=625$.

$\therefore$ A gets $\frac{8}{19}$ of *Rs* 11875 $=$ *Rs.*$8\times625=$ *Rs.*5000 ; &c.

Ex. 2. Divide *Rs.*640 among A, B and C, so that A may have 3 times as much as B, and C $\frac{1}{3}$ of what A and B together have.

A's share $=3$ times B's share ; C's share $=\frac{1}{3}(A$'s$+B$'s$)$.

$\therefore$ C's share $=\frac{1}{3}(3\ B$'s$+B$'s$)=\frac{4}{3}\ B$'s.

$\therefore$ A's share $+B$'s$+C$'s$=(3+1+\frac{4}{3})$ of B's $=5\frac{1}{3}$ of B's share.

Hence $5\frac{1}{3}$ of B's share $=$ *Rs.* 640 ; $\therefore$ B's share $=$ *Rs.*$640\div5\frac{1}{3}=$ *Rs.*120.

$\therefore$ A's share $=$ *Rs*$120\times3=$ *Rs.*360 and C's $=\frac{4}{3}\times$ *Rs.*120 $=$ *Rs.*160.

Ex. 3 The sum of *Rs.*155 is to be divided amongst 3 men, 5 women and 8 boys, so that for every 3*a.* a man gets, a woman gets 2*a.*, and a boy 1*a.* 6*p.* ; find the share of each.

A woman's share $=\frac{2}{3}$ of a man's ; a boy's share $=\frac{1}{2}$ of a man's ;

$\therefore$ a man's share $+$ a woman's $+$ a boy's $=(1+\frac{2}{3}+\frac{1}{2})$ of a man's ;

$\therefore$ 3 men's shares $+5$ women's $+8$ boys' $=(3+\frac{10}{3}+4)$ of a man's $=10\frac{1}{3}$ times a man's share ;

Hence $10\frac{1}{3}$ times a man's share $=$ *Rs.* 155 ;

$\therefore$ a man's share $=$ *Rs.*$155\div10\frac{1}{3}=$ *Rs.*15 ; a woman's share $=\frac{2}{3}$ of *Rs.* 15 $=$ *Rs.*10, and a boy's share $=\frac{1}{2}$ of *Rs.*15 $=$ *Rs.*7. 8*a.*

Ex. 4. Divide *Rs.*8424 among A, B and C, so that A shall receive $\frac{4}{11}$ as much as B and C together, and B $\frac{4}{5}$ of what A and C together receive.

A's share $= \frac{4}{9}$ of (B's $+ C$'s), and B's share $= \frac{4}{5}$ of (A's $+ C$'s).

$\therefore$ A's share $= \frac{4}{9}B$'s $+ \frac{4}{9}C$s $= \frac{4}{9}$ of $\frac{4}{5}$ of (A's $+ C$'s) $+ \frac{4}{9}C$'s $= \frac{16}{45}$ of (A's $+ C$'s) $+ \frac{4}{9}C$'s $= \frac{16}{45}A$'s $+ \frac{16}{45}C$'s $+ \frac{4}{9}C$'s $= \frac{16}{45}A$'s $+ \frac{4}{5}C$'s ;

$\therefore$ A's $- \frac{16}{45}A$'s $= \frac{4}{5}C$'s, or $\frac{29}{45}A$'s $= \frac{4}{5}C$'s ; $\therefore$ A's $= \frac{4}{5} \times \frac{45}{29}C$'s $= \frac{36}{29}C$'s.

$\therefore$ B's $= \frac{4}{5}A$'s $+ \frac{4}{5}C$'s $= \frac{4}{5} \times \frac{36}{29}C$'s $+ \frac{4}{5}C$'s $= \frac{260}{145}C$'s $= \frac{52}{29}C$'s.

$\therefore$ A's share $+ B$'s $+ C$'s $= (\frac{36}{29} + \frac{52}{29} + 1)$ of C's $= \frac{117}{29}$ of C's share ;

hence $\frac{117}{29}$ of C's share $= Rs.8424$.

$\therefore$ C's share $= Rs.8424 \div \frac{117}{29} = Rs.2088$,
$\therefore$ A's share $= \frac{36}{29}$ of $Rs.2088 = Rs.2592$, *Ans.*
and B's share $= \frac{52}{29}$ of $Rs.2088 = Rs.3744$.

Examples XCIII.

1. Divide *Rs.*6488. 7*a.* 10*p.* amongst three persons A, B and C, so that $\frac{1}{17}$ of A's share $= \frac{1}{22}$ of B's $= \frac{1}{26}$ of Cs.

2. Divide *Rs.*75. 8*a.* between A, B and C giving B half as much again as A less *Re.*1, and C as much as A and B together.

3. Divide *Rs.*1400 among A, B and C in such a manner that as often as A gets *Rs.*5, B shall get *Rs.*4, and as often as B gets *Rs.*3, C shall get *Rs.*2.

4. Divide *Rs.*352. 9*a.* among A, B and C, so that B may get twice, and C 3 times as much as A.

5. Divide *Rs.*1800 among A, B and C, so that A may receive 3 times as much as B, and B and C together $\frac{1}{2}$ as much as A.

6. Divide *Rs.*12540 among A, B and C, so that A shall receive $\frac{3}{7}$ as much as B and C together, and B $\frac{3}{8}$ of what A and C together receive.

7. Divide *Rs.*2000 among A, B and C, so that B's share may be $\frac{7}{8}$ of A's share, and C's share $\frac{5}{6}$ of B's.

8. Divide *Rs.*95. 10*a.* 8*p.* among 10 men, 6 women and 4 children, giving a woman 3 times as much as a child and a man twice as much as a woman.

9. Divide £1650 among A, B, C and D, so that A may have half as much as B, B a third as much as C and C a fourth as much as D.

10. If $\frac{3}{4}$ of A's money $= \frac{2}{3}$ of B's $= \frac{8}{9}$ of C's and A, B and C's money together amount to *Rs.* 8260 ; how much has each?

11. If $\frac{5}{6}$ of A's money $= \frac{7}{8}$ of B's $= \frac{8}{9}$ of C's $= \frac{9}{10}$ of D's and A, B, C and D together have *Rs.*23078 ; determine how much money each has.

12. If $\frac{7}{10}$ of A's money $= \frac{9}{11}$ of B's, and C's money = [illegible]

A's $+\frac{3}{5}$ of B's), and C's money $-$ A's money $=$ *Rs.*667 ; find how much A, B and C each has.

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319. Travelling round a Circle.

When two or more persons start simultaneously from the same place to travel round a circular course either in the same direction or in opposite directions, (i) they should first be together again at an interval of time which is the L. C. M. of the times during which one of the walkers gains one complete round over each of the others, for each pair will be together after this time ; (ii) they should first be together at the starting post again at an interval of time which is the L. C. M. of the times during which each makes one complete round, for in that interval each shall make an integral number of rounds.

Ex. 1. A can go round a circular course in 18 min., B can go round it in 24 min., and C in 32 min. If they start simultaneously from the same point and travel in the same direction, in what time will they come together again ?

Take 1 for the length of the course ;

then A travels $\frac{1}{18}$, B $\frac{1}{24}$ and C $\frac{1}{32}$ of the course in 1 min. ;

$\therefore$ A gains on B $(\frac{1}{18}-\frac{1}{24})$ or $\frac{1}{72}$ of the course in 1 min.

$\therefore$ A gains on B one complete round in $(1\div\frac{1}{72})$ or 72 min.

Hence A and B will be together after 72 min.

Again, A gains on C $(\frac{1}{18}-\frac{1}{32})$ or $\frac{7}{288}$ of the course in 1 min.

$\therefore$ A gains on C one complete round in $(1\div\frac{7}{288})$ or $\frac{288}{7}$ min.

Hence A and C will be together after $\frac{288}{7}$ min.

Therefore A, B and C will be together after a time which is the L. C. M. of 72 and $\frac{288}{7}$; but the L. C. M. of 72 and $\frac{288}{7}$ is 288.

$\therefore$ A, B and C are first together after 288 min. *Ans.*

Ex. 2. In the above question if A and B travel in the same direction but C in the opposite direction, when will they meet again ?

As in the above question,

A and B will be together at the end of 72 min. :

Again, A and C together pass over $(\frac{1}{18}+\frac{1}{32})$ or $\frac{25}{288}$ of the course in 1 min.

$\therefore$ they come together at the end of $(1\div\frac{25}{288})$ or $\frac{288}{25}$ min.

Now, the L. C. M. of 72 and $\frac{288}{25}$ is 288 ;

$\therefore$ A, B and C will be together at the end of 288 min. *Ans.*

Ex. 3. A, B and C start from the same point and travel in the same direction round an island 6 miles in circumference, A at the rate of 3, B at the rate of $2\frac{1}{2}$ and C at the rate of $1\frac{1}{4}$ miles an hour. In how many hours will they come together again ?

A gains on B $(3-2\frac{1}{2})$ or $\frac{1}{2}$ mile in 1 hour, $\therefore$ he gains 6 miles or a complete round in $(6\div\frac{1}{2})$ or 12 hours.

Hence A and B are together at the end of every 12 hours.

Again, A gains on C $(3-1\frac{1}{4})$ or $1\frac{3}{4}$ miles in 1 hour, $\therefore$ he gains 6 miles or a complete round in $(6 \div 1\frac{3}{4})$ or $\frac{24}{7}$ hours.

Hence, A and C are together at the end of every $\frac{24}{7}$ hours. Therefore A, B and C are together at the end of any number of hours which is a common multiple of 12 and $\frac{24}{7}$;

but the L. C. M. of 12 and $\frac{24}{7}$ is 24;

hence A, B and C are first together at the end of 24 hrs. *Ans.*

Ex. 4. In the above question, when will they be together again at the starting point?

Here, A takes $\frac{6}{3}$ or 2 hrs., B $(6 \div 2\frac{1}{2})$ or $\frac{12}{5}$ hrs. and C $(6 \div 1\frac{1}{4})$ or $\frac{24}{5}$ hrs. to make one round.

Now, the L. C. M. of 2, $\frac{12}{5}$ and $\frac{24}{5}$ is 24;

$\therefore$ they will be together again at the starting point 24 hrs. after.

Examples XCIV.

1. Two persons A and B start from the same point to walk round a circular course in the same direction. A takes 9 min. and B takes 24 min. to complete one round; in what time will they be together again?

2. Three persons, A, B and C can respectively go round a circular path in 8, 18 and 30 min. If they start simultaneously from the same point and travel in the same direction, when will they meet again?

3. A, B and C start from the same point and travel in the same direction round an island 73 miles in circumference. A at the rate of 10, B at the rate of 14 and C at the rate of 16 miles a day; in how many days will they come together again?

4. There is a park $1\frac{7}{8}$ miles in circumference. Five persons start from the same point to travel round it in the same direction at the respective rates of 3, $3\frac{3}{4}$, 4, $4\frac{1}{2}$ and 5 miles per hour. When will they be together again at the starting point?

5. A, B and C start from the same point and travel in the same direction round an island 36 miles in circumference, A at the rate of 3 miles, B at the rate of $3\frac{1}{2}$ miles and C at the rate of 4 miles an hour; when will they be together again?

6. An island is 43 miles in circumference. Three men A, B and C start from the same place to walk round it, at the rates of 4, $4\frac{1}{3}$ and $5\frac{1}{2}$ miles per hour respectively. In how many hours will they come together again, supposing them to travel in the same direction?

7. In the above question, if A and B travel in the same direction and C in the opposite direction, when will they come together again for the first time?

8. An island is 120 miles in circumference. Three persons A, B and C start from the same place to walk round at the respective

rates of 10, 12 and 15 miles per hour. When will they next meet, supposing (i) that they walk in the same direction, (ii) that A walks in one direction and B and C in opposite directions?

320. Chain Rule. If we wish to express one quantity A in terms of another quantity R, and have data from which we can form the following series of relations, *viz.*,

$$a\,A = m\,M \dots\dots (1)$$
$$b\,M = n\,N \dots\dots (2)$$
$$c\,N = p\,P \dots\dots (3)$$
$$d\,P = q\,Q \dots\dots (4)$$
$$e\,Q = r\,R \dots\dots (5)$$

which may be as numerous as we choose, then will

$$A = \frac{mnpqr}{abcde}R.$$

Hence we see that the quantity required is found by dividing the product of the numbers on the right-hand side of these equations by the numbers on the left-hand side.

Ex. 1. If 3 lbs. of tea be worth 4 lbs. of coffee, and 6 lbs. of coffee be worth 20 lbs. of sugar, and 15 lbs of sugar be worth 24 lbs. of rice; how many lbs. of rice are equal to 18 lbs. of tea?

lbs. reqd. rice = 18 lbs. tea,
3 lbs. tea = 4 lbs. coffee,
6 lbs. coffee = 20 lbs. sugar,
15 lbs. sugar = 24 lbs. rice;

$$\therefore \text{ lbs. reqd. rice} = \frac{18 \times 4 \times 20 \times 24}{3 \times 6 \times 15} = \underline{128}.$$

321. In the preceding equations the quantity on the right-hand side of one equation is of the *same kind* as that on the left-hand side of the next equation, and thus the Chain of quantities from one kind to another is unbroken. And not only must they be of the *same kind* but also of the *same denomination*; for if not, the one or more missing links must be supplied.

Ex. 2. If 3 lbs. of rice be worth 5 oz. of tea, and 4 lbs. of tea be worth 9 lbs. of coffee, how many lbs. of coffee are worth 48 lbs. of rice?

Here, we must either supply the missing link 16 oz. tea = 1 lb. tea, or we must express 5 oz. tea as $\frac{5}{16}$ lb. tea; so that we have

lbs. reqd. coffee = 48 lbs. rice,	lbs. reqd. coffee = 48 lbs. rice,
3 lbs. rice = 5 oz. tea,	3 lbs. rice = $\frac{5}{16}$ lbs. tea.
16 oz. tea = 1 lb. tea,	4 lbs. tea = 9 lbs. coffee;
4 lbs. tea = 9 lbs. coffee;	$\therefore$ lbs. reqd. coffee $= \dfrac{48 \times \frac{5}{16} \times 9}{3 \times 4}$
$\therefore$ lbs. reqd. coffee $= \dfrac{48 \times 5 \times 1 \times 9}{3 \times 16 \times 4}$ $= \underline{11\frac{1}{4}}$.	$= \dfrac{48 \times 5 \times 9}{16 \times 3 \times 4} = \underline{11\frac{1}{4}}$.

322. It is unnecessary to name the quantity on the *left-hand* side of any equation ; for it must be the same as the quantity on the right-hand side of the preceding equation.

Ex. 3. If $\frac{1}{5}$ of a sheep be worth £$\frac{2}{3}$, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of an ox, what must be given for 100 oxen ?

$$\text{£}s.\ \text{reqd.} = 100 \text{ oxen},$$
$$\tfrac{1}{14} = \tfrac{3}{7} \text{ sheep},$$
$$\tfrac{1}{5} = \text{£}\tfrac{2}{3},$$
$$\therefore \text{£}s.\ \text{reqd.} = \frac{100 \times \frac{3}{7} \times \frac{2}{3}}{\frac{1}{14} \times \frac{1}{5}} = \frac{100 \times 3 \times 2 \times 14 \times 5}{7 \times 3} = \underline{2000.}$$

Ex. 4. If 1 lb. of standard gold, of which 11 parts out of 12 are fine gold, be worth £46. 14s. 6d., find the value of 595 gold rupees of Bombay, each weighing 7 dwts. $10\frac{1}{2}$ grs. of which 187 parts are fine gold and 13 alloy.

£46. 14*s.* 6*d.* = £$46\frac{29}{40}$ = £$\frac{1869}{40}$; $\therefore$ 40 lbs. standard = £1869 ;
7 dwts. $10\frac{1}{2}$ grs. = $7\frac{7}{16}$ dwts. = $\frac{119}{16}$ dwts. ; 16 Bombay rupees = 119 dwts. ;
187 + 13 = 200 ; $\therefore$ 187 parts out of 200 are fine ; hence

$$\text{£}s.\ \text{reqd.} = 595 \text{ Bombay rupees},$$
$$16 = 119 \text{ dwts. Bombay standard},$$
$$20 \times 12 = 1 \text{ lb.}$$
$$200 = 187 \text{ lbs. fine},$$
$$11 = 12 \text{ lbs. English standard},$$
$$40 = \text{£}1869\ ;$$
$$\therefore \text{£}s.\ \text{reqd.} = \frac{595 \times 119 \times 187 \times 12 \times 1869}{16 \times 20 \times 12 \times 200 \times 11 \times 40} = \text{£}878\tfrac{401453}{512000}$$
$$= \underline{\text{£}878.\ 15s.\ 8\tfrac{1159}{6400}d.}$$

Examples XCV.

1. When 25 yards of muslin are equal to 16 yds. of calico, 21 yds. of calico to 13 yds. of flannel, 40 yds. of flannel to 27 yds. of linen, $58\frac{1}{2}$ yds. of linen to 28 yds. of silk, 47 yds. of silk to 35 yds. of velvet ; find how many yards of velvet are equal in value to 60 yds. of muslin

2. If 16 mangoes be equal in price to 25 apples, and 18 oranges equal to 12 mangoes, and 20 lemons equal to 27 oranges, and lemons cost 9*d.* a dozen, what is the cost of 15 apples ?

3. If 12 of A count for 13 of B, 6 of B for 18 of C, and 13 of C for 2 of D ; how many of A count for 100 of D ?

4. If £3 = 20 thalers ; 25 thalers = 93 francs ; 27 francs = 5 scudi ; and 62 scudi = 135 gulden ; how many gulden = £1 ?

5. If 16 darics make 17 guineas, 19 guineas make 24 pistoles, 31 pistoles make 38 sequins ; how many sequins are there in 1581 darics ?

6. If 72 carlini be worth 25 shillings, 4 shillings worth 5 francs and 8 scudi worth 45 francs, how many carlini are equal to 100 scudi ?

7. If 35 metres = 39 yards, and 17 metres = 9 toises, and 5 plethera = 124 toises, how many yards are there in 1575 plethera?

8. If 6 horses cost as much as 24 cows, 10 cows as much as 8 buffaloes, 4 buffaloes as much as 15 asses, 8 asses as much as 32 sheep, and if the price of 9 sheep be *Rs.* 25, find the cost of 8 horses.

9. If $\frac{1}{7}$ of a sheep be worth £$\frac{3}{8}$, and $\frac{3}{5}$ of a sheep worth $\frac{1}{14}$ of an ox; how much must be given for 300 oxen?

10. If 40 lbs. of standard gold, of which 11 parts out of 12 are fine, be coined into 1869 sovereigns; how many grains of pure gold are there in 1 sovereign?

11. If 1 lb. of standard gold, of which 11 parts out of 12 are fine, be worth £46. 14*s.* 6*d.*, find the value of 550 Madras gold rupees, each weighing 7 dwts. 12 grs., of which 916 parts out of 1000 are fine.

12. If 1 lb. of standard silver, of which 37 parts out of 40 are fine, be worth 66*s.*, find the value of an Arcot Rupee, weighing 7 dwts. 9 grs., of which 941 parts out of 1000 are fine.

Examples worked out.

Ex. 1. What least number must be added to $8\frac{1}{9}$, that the result being divided by $1\frac{2}{3}$, the quotient shall be an integer?

$$8\tfrac{1}{9} \div 1\tfrac{2}{3} = \tfrac{73}{9} \div \tfrac{5}{3} = \tfrac{73}{9} \times \tfrac{3}{5} = \tfrac{73}{15} = 4\tfrac{13}{15}.$$

Now, the least number that should be added to $4\frac{13}{15}$ to make it an integer is $\frac{2}{15}$, for $\frac{13}{15} + \frac{2}{15} = 1$.

Then the question reduces to "What number divided by $1\frac{2}{3}$ will give $\frac{2}{15}$ as quotient?"

Hence the required number $= \frac{2}{15} \times 1\frac{2}{3} = \frac{2}{15} \times \frac{5}{3} = \frac{2}{9}$. *Ans.*

Ex. 2. Find two least integers such that $\frac{5}{6}$ of the first shall be equal to $\frac{7}{8}$ of the second.

If $\frac{5}{6}$ of 1st number be $= 1$, then also $\frac{7}{8}$ of 2nd number $= 1$.

$\therefore$ 1st number $= (1 \div \frac{5}{6}) = \frac{6}{5}$, and 2nd number $= (1 \div \frac{7}{8}) = \frac{8}{7}$.

Now to transform these fractions to least integers, multiply each of them by the L. C. M. of their denominators, and divide the numbers thus found by their G. C. M.

The L. C. M. of 5 and 7 is 35; $\therefore$ from 1st we have $\frac{6}{5} \times 35 = 42$, and from 2nd $\frac{8}{7} \times 35 = 40$. Now the G. C. M. of 42 and 40 is 2.

Hence the numbers are $\frac{42}{2}$ and $\frac{40}{2}$, or 21 and 20. *Ans.*

Ex. 3. By selling an article for £12. 7*s.* 6*d.*, I cleared $\frac{4}{5}$ of what it cost me; what was the original cost?

Taking 1 for the original cost, the gain is $\frac{4}{5}$, and the selling price $(1 + \frac{4}{5})$ or $\frac{9}{5}$.

$\therefore \frac{9}{5}$ of the original cost $=$ £12. 7*s.* 6*d.*;

$\therefore$ the original cost $=$ £12. 7*s.* 6*d.* $\div \frac{9}{5} =$ £12. 7*s.* 6*d.* $\times \frac{5}{9}$
$=$ £1. 7*s.* 6*d.* $\times 5 =$ £6. 17*s.* 6*d.* *Ans.*

Ex. 4. By selling 15 seers of tea at *Rs.*5. 4*a.* per seer, a grocer clears $\frac{1}{8}$ of his outlay. He then raises the price to *Rs.*6 per seer and sells 50 seers more. What does he gain on the whole outlay for 65 seers?

Taking 1 for the original cost, the selling price is $(1+\frac{1}{8})$ or $\frac{9}{8}$.

∴ the original cost = *Rs.*5. 4*a.* $\div \frac{9}{8}$ = *Rs.*4. 10*a.* 8*p.*

∴ in the 1st case gain per seer = *Rs.*5. 4*a.* − *Rs.*4. 10*a.* 8*p.* = 9*a.* 4*p.*

in the 2nd case ... = *Rs.*6. − *Rs.*4. 10*a.* 8*p.* = *Re.*1. 5*a.* 4*p.*

Now, gain on 15 seers = 9*a.* 4*p.* × 15 = *Rs.* 8. 12*a.*

and gain on 50 seers = *Re.*1. 5*a.* 4*p.* × 50 = *Rs.*66. 10*a.* 8*p.*

∴ his whole gain = *Rs.*75. 6*a.* 8*p.* *Ans.*

Ex. 5. Find the least number of sovereigns that contains an exact number of 20-franc pieces of 15*s.* $11\frac{1}{4}$*d.* each.

Here, 15*s.* $11\frac{1}{4}$*d.* = $191\frac{1}{4}$*d.* = $\frac{765}{4}$*d.* and a sovereign = 240*d.*

∴ $\frac{765}{4}$*d.* × no. of 20-franc pieces = 240*d.* × no. of sovereigns;

∴ no. of 20-franc pieces = $240 \times \frac{4}{765}$ × no. of sovereigns

= $\frac{64}{51}$ × no. of sovereigns.

Hence the least no. of sovereigns that will make an exact number of 20-franc pieces is 51. *Ans.*

Ex. 6. A man bought 4 sorts of rice at an average price of *Rs.*6 a maund. If the prices increase by a common difference of 5*a.* per maund, find the cost of each sort per maund.

The price of 4 sorts at *Rs.*6 per maund = *Rs.*6 × 4 = *Rs.*24.

Each maund of second sort cost 5*a.* more than a md. of 1st sort,

.... third .... 10*a.* ....

.... fourth .... 15*a.* ....

∴ these 3 maunds cost *Re.*1. 14*a.* more ....

Now, leaving out this sum, the cost of 4 maunds is *Rs.*24 − *Re.*1 14*a.* or *Rs.*22. 2*a.*; ∴ the cost of 1 md. = *Rs.*22. 2*a.* ÷ 4 = *Rs.*5. 8*a.* 6*p.*

Hence the cost of 1 md. of 1st sort = *Rs.*5. 8*a.* 6*p.*
............................2nd sort = *Rs.*5. 8*a.* 6*p.* + 5*a.* = *Rs.*5. 13*a.* 6*p.*
............................3rd sort = *Rs.*5. 13*a.* 6*p.* + 5*a.* = *Rs.*6. 2*a.* 6*p.*
............................4th sort = *Rs.*6. 2*a.* 6*p.* + 5*a.* = *Rs.*6. 7*a.* 6*p.*
Ans.

Ex. 7. *A* and *B* undertake to do a piece of work for *Rs.*12. 8*a.*, *A* can do the work alone in 20 days and *B* in 15 days. They work together for 3 days, and then with the assistance of *C* finish it in 5 days more. How should the sum be divided?

Here, *A* and *B* each worked for (5+3) or 8 days, and *C* for 5 days.

As *A* can do $\frac{1}{20}$ of the work in 1 day, he did $\frac{8}{20}$ or $\frac{2}{5}$ of the work in 8 days.

.... *B* .... .. $\frac{1}{15}$ .... he did $\frac{8}{15}$ ....

∴ *A* and *B* did in 8 days $(\frac{2}{5}+\frac{8}{15})$ or $\frac{14}{15}$ of the work.

Hence the work done by C in 5 days $=(1-\frac{14}{15})$ or $\frac{1}{15}$ of the work.

$$\left.\begin{array}{l} \therefore A \text{ received } \frac{2}{5} \text{ of } Rs.12.\ 8a. = \underline{Rs.5.} \\ B \ldots\ldots \frac{8}{15} \text{ of } Rs.12.\ 8a. = \underline{Rs.6.\ 10a.\ 8p.} \\ \text{and } C \ldots\ldots \frac{1}{15} \text{ of } Rs.12.\ 8a. = \underline{13a.\ 4p.} \end{array}\right\} \textit{Ans.}$$

Ex. 8. A man's income from Government Securities is $\frac{3}{4}$ of what he receives from his landed property. An income-tax of $5p.$ in the rupee is charged on the first and of $4p.$ in the rupee on the second, and he has to pay altogether $Rs.31$ as income-tax. Find his total income.

Suppose his income from landed property to be $Rs.4$,

then .... Government Securities is $Rs.3$;

income-tax on 1st $=(4 \times 4)$ or $16p.$ and on second $=(3 \times 5)=15p.$, and $16p. + 15p. = 31p. = Re.\frac{31}{192}$.

$\therefore$ he has to pay $Re.\frac{31}{192}$ as tax on every $Rs.7$ of income.

$\therefore$ .... $Re.1$ as tax .... $Rs.7 \times \frac{192}{31}$ ....

$\therefore$ .... $Rs.31$ as tax .... $Rs.7 \times \frac{192}{31} \times 31$ ....

Hence, required income $= Rs.7 \times \frac{192}{31} \times 31 = \underline{Rs.1344.}$ *Ans.*

Ex. 9. A can do as much work in one day as B can do in 2 days, or as C can do in 3 days or as D can do in 4 days. They together finish a piece of work in 8 days. How many days would each take to do it singly?

Suppose A's one day's work to be 1, then B's one day's work is $\frac{1}{2}$, C's $\frac{1}{3}$ and D's $\frac{1}{4}$.

$\therefore A, B, C$ and D's one day's work

$=(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4})$ or $\frac{25}{12}$ times A's work per day;

$\therefore A, B, C$ and D's 8 days' work $= 8 \times \frac{25}{12}$ or $\frac{50}{3}$ times A's work per day.

But A, B, C and D's 8 day's work = whole work;

$\therefore \frac{50}{3}$ times A's work per day = whole work;

or A's work per day $= \frac{3}{50}$ of the whole work.

Hence A can do the whole work in $(1 \div \frac{3}{50})$ or $16\frac{2}{3}$ days.

Therefore B's time $=(2 \times 16\frac{2}{3})$ or $33\frac{1}{3}$ days; C's time $=(3 \times 16\frac{2}{3})$ or 50 days, and D's time $=(4 \times 16\frac{2}{3})$ or $66\frac{2}{3}$ days.

Examples XCVI.

1. Determine the least number which must be added to $3\frac{1}{5}$ that when the result is divided by $\frac{1}{4}$, the quotient shall be an integer.

2. What least number must be subtracted from $8\frac{2}{3}$, that when the difference is divided by $\frac{4}{7}$, the quotient shall be an integer?

3. If a pound weight of standard gold is worth £61. 18*s.* 9*d.*, find the least integral number of pounds of gold that can be coined into an integral number of sovereigns.

4. If the rupee is worth 1*s.* 9*d.* and the mohur 30*s.*, find the least number of pounds which can be paid exactly in rupees or mohurs.

5. By selling an article for *Rs.*460, I cleared $\frac{3}{20}$ of the prime cost. Find the cost price.

6. By selling a horse for *Rs.*2520, a man lost $\frac{4}{25}$ of what it cost him. What did it cost him?

7. Find the least number of sovereigns that contains an exact number of thalers and of dollars; 48 thalers being worth £7. 3*s.* and 8 dollars £1. 13*s.*

8. *A* has twice as much money as *B*. They play together, and at the end of the first game *B* wins from *A* one-third of *A*'s money; what fraction of the sum which *B* now has must *A* win back in the second game that they may have exactly equal sums?

9. How many maunds of rice at *Rs.*4 per maund must a corn-merchant mix with 1 maund of rice at *Rs.*5 per maund, that by selling the mixed rice at *Rs.*4. 8*a.* per maund, he may gain $\frac{1}{16}$ of his outlay?

10. Find the least number that must be added to $75\frac{1}{4}$, that the sum being severally divided by $\frac{3}{4}$, $\frac{5}{12}$, $\frac{7}{8}$ or $\frac{9}{16}$, the quotient in each case shall be an integer.

11. *A* and *B* undertake to do a piece of work for *Rs.*7. 8*a.* *A* can do it alone in 8 days and *B* in 6 days. With the assistance of *C* it is finished in 3 days. How should the money be divided?

12. *A* and *B* engage to do a piece of work for *Rs.* 40. *A* can do it alone in 16 days and *B* can do it in 12 days. After working together for 4 days, *A* leaves off, when *C*, who can alone finish the work in 8 days, joins. How should the sum be distributed after the work is completed?

13. A man derives his income from three sources. His income from Government Securities is $\frac{1}{2}$ of his income from trade, and his landed property yields an income equal to $\frac{2}{3}$ of the sum of both. The rate of tax on income from trade is 6*p.* per rupee, on Securities 5*p.* per rupee, and on landed property 4*p.* per rupee. If his total income-tax amounts to *Rs.*260, find his gross income.

14. By selling tea at *Rs.*2. 10*a.* 8*p.* per lb., a grocer clears $\frac{1}{8}$ of his outlay; he then raises the price to *Rs.*3. What does he clear on every *Rs.* 200 of his outlay by this price?

15. A tradesman buys 5 mds. 24 sr. of goods for *Rs.*150 intending to gain $\frac{3}{25}$ of his outlay by the sale; but *Rs.*10. 8*a.* worth at this calculation being damaged, at what price shall he sell the remainder per maund, to gain as much upon the whole outlay as he intended?

16. *A* can do as much work in 1 day as *B* in 3 days, *C* in

5 and D in 7 days. They together complete a piece of work in 8 days. In how many days will each do it singly?

17. A cloth-merchant bought a bale of cloth containing 150 pieces each, of cloths $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4 and $4\frac{1}{2}$ yds. in length for *Rs.*626. 9*a.* If the prices increase by a common difference of 3*a.*, at what price per piece must he sell them that he may gain *Rs.* 100 by the transaction?

18. By selling a horse for *Rs.*345, I lost $\frac{2}{25}$ of the prime cost. What would have been my gain had I sold it for *Rs.*380?

19. A and B can finish a piece of work in $1\frac{1}{2}$ days, A and C in 2 days and B and C in 3 days. If *Rs.*6 be paid for the piece of work, what are a day's wages of each workman?

20. An elastic ball after striking the ground rises to $\frac{1}{4}$ of the height from which it fell. After striking the ground the third time it rises $3\frac{7}{8}$ inches: from what height did it fall at first?

Miscellaneous Examples III.

1. Divide the sum of 10 and $\frac{1}{10}$ by their difference, and also the difference by their sum; and find the sum and difference of the two quotients.

2. Add together $1\frac{3}{4}$, $2\frac{2}{3}$, and $3\frac{1}{2}$; multiply this sum by the product of these fractions; subtract from the result the difference of $2\frac{2}{3}$ and $1\frac{1}{2}$; and divide the remainder by the sum of $5\frac{1}{3}$ and $1\frac{1}{3}$ of $3\frac{3}{4}$.

3. Simplify $\frac{3}{4}$ of $\dfrac{£3.\ 10s.}{£4.\ \ 4s.} \div \frac{4}{5}(\frac{7}{12}+\frac{29}{36})$ of $\dfrac{2 \text{ tons } 4 \text{ cwt.}}{3 \text{ tons } 6 \text{ cwt.}}$

4. Divide *Rs.* 19000 among A, B, C and D, so that B may receive $\frac{3}{4}$ of A, C $\frac{4}{5}$ of B and D $\frac{5}{6}$ of C.

5. If $\frac{4}{5}$ of $\frac{9}{10}$ of A's money $=\frac{7}{8}$ of $\frac{12}{13}$ of B's and the difference of their moneys be *Rs.*291, find A's and B's money.

6. If 3 men and 2 boys can do a piece of work in 15 days, and 2 men and 3 boys can do the same in 18 days, in what time will a man and a boy jointly do the work?

7. In an orchard, $\frac{1}{3}$ of the trees are apple trees, $\frac{1}{4}$ pear trees, $\frac{1}{5}$ cherry trees, $\frac{1}{6}$ filbert trees, and there are 12 walnut trees; what is the number of each sort?

8. If A can do half a piece of work in 3 hours, which is twice as much as B can do, and A, B and C can together do the whole in $2\frac{1}{2}$ hours; shew that C can do in 5 hours as much as B can do in 9 hours.

9. A 38-gallon cask of wine cost a wine-merchant *Rs.*250; but he lost 8 gallons of it by leakage; how must he sell the remainder per gallon to gain $\frac{1}{10}$th of the prime cost?

10. A man owns $\frac{5}{16}$ of an estate. He sells $\frac{1}{5}$ of his share and then finds that his remaining share is worth *Rs.*525. 10*a.* What is the value of the whole property?

11. A can do a piece of work in 8 days, B in 12 days and C in 15 days. They all work together for 3 days at the same piece of work. In what time will B finish the remaining work?

12. A grocer mixes 5 maunds of rice at *Rs.*4 per md. with $3\frac{1}{3}$ mds. at *Rs.*4. 8*a.* per md. At how much per maund must he sell the mixed rice that he may gain $\frac{3}{32}$ of his outlay?

13. If A takes 8 days to complete a piece of work, B takes 9 days to do $\frac{3}{4}$ of the same, and if B takes 10 days to complete a piece of work, C takes 8 days to do $\frac{2}{3}$ of the same. In what time will B and C together finish a work which A alone can do in 25 days?

14. Reduce $\frac{3}{8}$ of *Rs.*3. 12*a.* + $\frac{1}{5}$ of *Rs.*7. 6*a.* − $\frac{1}{10}$ of *Rs.*8. 4*a.* 6*p.* to the fraction of *Rs.*20. 10*a.*

15. Reduce $\frac{11}{12}$ of £7. 6*s.* 8*d.* + $\frac{5}{24}$ of £9. 13*s.* 4*d.* − $\frac{1}{16}$ of £10. 3*s.* 4*d.* to the fraction of £25. 10*s.*

16. If 2 men and 1 boy do a piece of work in 5 days, 1 man and 2 women do it in 6 days and 1 woman and 2 boys do the same in 8 days, in what time will a man, a woman and a boy do it jointly?

17. Of the population of a certain town $\frac{1}{3}$ can read, $\frac{1}{4}$ can write, $\frac{1}{5}$ can read and write and the remaining 130 can neither read, nor write. Find the total population of the town.

18. Simplify—

(*a*) $\dfrac{\frac{5}{12}+\frac{3}{16}}{\frac{5}{12}-\frac{3}{16}}$ of $\dfrac{8}{9\frac{1}{3}} \div \dfrac{\frac{7}{8}+\frac{1}{9}}{\frac{7}{8}-\frac{1}{9}}$ of $\dfrac{13}{14} \times \dfrac{\frac{15}{16}+\frac{3}{4}}{\frac{15}{16}-\frac{3}{4}}$ of $\dfrac{2}{3}\left(\dfrac{3}{4}+\dfrac{1}{5}\right)$.

(*b*) $\dfrac{9\frac{1}{3}}{11\frac{1}{5}}$ of $\dfrac{Rs\ 3.\ 12a.}{Rs.4.\ \ 8a.} \div \dfrac{7\frac{1}{4}}{11\frac{3}{8}}$ of $\dfrac{£2.\ 6s.}{£3.\ 9s.}$.

19. The adult population of a country is 22815210; the adult females are $\frac{5}{16}$ of the whole population, and the adult males are $\frac{16}{17}$ of the adult females; find the whole population.

20. The wages of A and B together for $22\frac{1}{2}$ days amount to the same sum as the wages of A alone for $38\frac{4}{7}$ days. For how many days will this sum pay the wages of B alone?

21. A farmer paid a corn-rent of 5 qrs. of wheat and 3 qrs. of barley, Winchester measure. What was the value of his rent when wheat was at 60*s.* and barley 54*s.* per quarter, Imperial measure, it being assumed that 32 Imperial gallons are equivalent to 33 Winchester gallons?

22. A man's debts amount to $\frac{1}{14}$ of his property, but before paying them he loses $\frac{7}{8}$ of his property: afterwards he recovers a portion equal to $\frac{1}{5}$ of what he has left, and then loses $\frac{1}{2}$ of what he has got. Can he pay his debts? What part of his property remains over?

23. A man can do 4 times a certain work in 9 hours, a woman 3 times the work in 10 hours, and a child twice the work in 11 hours;

if a man, a woman and a child work together, in what time can they do 7 times the work ?

24. Five brothers join in paying a sum of money ; the eldest pays a third of it, and the others pay the remainder in equal shares, and thereby each of them pays *Rs.*840 less than the eldest brother. What is the sum of money ?

25. If 9 men or 16 women can do a piece of work in 144 days, in what time would 7 men and 9 women do it, working together ?

26. Out of a cistern, which is $\frac{2}{3}$rds full, 20 gallons are drawn, the cistern is then found to be $\frac{3}{5}$ths full How much will the cistern hold ?

27. The product of three numbers is 340 ; the first is $7\frac{3}{5}$, the second is less than the first by $1\frac{7}{20}$. Find the third number.

28. A owned $\frac{5}{6}$ of a mine and sold $\frac{3}{4}$ of his share to B, who sold $\frac{1}{2}$ of his share to C, who sold $\frac{2}{3}$ of his share to D ; D's share was worth *Rs* 20565. What was the worth of B's remaining share, and what the worth of the whole mine ?

29. There are two fractions whose sum is $\frac{113}{120}$, and whose difference is $\frac{7}{18}$; find the fractions, and the quotient of the greater by the less.

30. If a turkey cost £$\frac{4}{7}$ and a goose £$\frac{3}{10}$, how many turkeys and geese, an equal number of each, can be bought for £14. 4*s.* ?

31. A boy, in flying his kite, lost $\frac{2}{5}$ of the string ; he then added 65 ft., and then found that it was $\frac{5}{6}$ of the original length. What was the length at first ?

32. If $2\frac{1}{3}$ of $(A+\frac{3}{7}$ of $A)=\frac{1}{3}$ of $(B-\frac{1}{3}B)$, find the value of A in terms of B.

33. A man bequeathed $\frac{5}{12}$ of his estate to one son, $\frac{7}{12}$ of the remainder to another son, and the balance to his widow. The sons' shares differ by *Rs.*1320 ; find the widow's share.

34. A man gives away in charity $\frac{1}{8}$ of his income, and pays $\frac{1}{10}$ of it in rates and taxes ; with these deductions he has *Rs.*4736. 8*a.* 8*p.* left. What is his gross income ?

35. Find the whole annual cost of a house, of which the rent is *Rs.* 360 ; the poor-rate being 2*a.* 8*p.* in the rupee, the gas-rate $\frac{2}{3}$ of the poor-rate, and the paving-rate $\frac{3}{5}$ of the gas-rate.

36. What sum must be added to or subtracted from £12. 7*s.* 6*d.*, so that £5. 3*s.* 4*d.* shall be the same fraction of the sum or difference that £3. 6*s.* 8*d.* is of £8. 6*s.* 8*d.* ?

37. Divide *Rs.*4200 among A, B, C and D, so that A may get twice as much as B, A and C may get thrice as much as B and A, and D may get four times as much as B and C.

38. The sum of $\frac{2}{5}$ and $\frac{1}{3}$ of a man's debts amounts to *Rs.*198.

$7a.\ 4p.$ and his assets are $Rs.45.\ 1a.\ 8p.$; how much in the rupee will his creditors lose ?

39. One-third of A's money is equal to $\frac{1}{2}$ of C's and $\frac{1}{3}$ of C's is equal to $\frac{2}{3}$ of B's ; B gives to A $\frac{1}{2}$ of his money and to C $\frac{1}{4}$ of the remainder, and has $2s.\ 6d.$ left. What amount had each at first ?

40. Express $\frac{2}{3}$ of $\frac{6}{15}$ of £$1.\ 10s. + \frac{3}{4}$ of $\frac{5}{6}$ of $5s.\ 4d - 8\frac{1}{2}$ of $\dfrac{1}{4\frac{1}{4}}$ of $5s.\ 3\frac{3}{4}d.$ as the fraction of $2s.\ 1\frac{1}{2}d.$

41. Find the value of

$$\frac{1}{4} \text{ of} \left(\frac{4\frac{7}{9} \text{ of } 6\frac{2}{3}}{7\frac{26}{27}} \right) \times \frac{3\frac{2}{5} - 3\frac{1}{6}}{3\frac{3}{5} + 2\frac{1}{6}} \text{ of } Rs.\ 184.\ 11a.\ 5p.$$

42. What sum must be added to or subtracted from $Rs.\ 8.\ 12a.\ 6p.$ so that the sum or difference shall be the same fraction of $Rs.\ 20\ 10a.$ that $Rs.7.\ 6a.\ 6p.$ is of $Rs.18.\ 8a.\ 3p.$?

43. A can do in 6 days as much work as C can do in 4 days, and B in 10 days as much as C in 8 days : what time would B require to finish a piece of work which A can do in 12 weeks ?

44 When rice is at 12 sr. per rupee, the expenses of a family amount to $Rs.140$; but they amount to $Rs.134$ only, when the price falls to 16 sr. per rupee. What will the expenses be, when rice is at 18 sr. per rupee ?

45. A can do in 2 days as much work as B in 3 days, and B in 5 days as much work as C in 4 days ; what time will C require to finish a piece of work which A can do in 9 days ?

46. A can by himself perform a certain quantity of work in 5 days, B twice as much in 7 days, and C four times as much in 11 days ; in what time can A, B and C together perform three times the original work ?

47. A was owner of $\frac{5}{17}$ of a privateer, and sold $\frac{3}{11}$ of $\frac{2}{9}$ of his share for £$12\frac{4}{33}$; what was the value of $\dfrac{1\frac{1}{7}}{5\frac{2}{3}}$ of $\frac{7}{12}$ of the vessel at the same rate ?

48. How much ore must be raised, that on losing $\frac{17}{40}$ in roasting, and $\frac{18}{19}$ of the residue in smelting, there may result 506 tons of pure metal ?

49. Simplify—

$$\frac{\frac{1}{2}+\frac{1}{4}-\frac{1}{12}}{\frac{1}{2}-\frac{1}{4}+\frac{1}{12}} \text{ of } \frac{\frac{1}{2}-(\frac{1}{4}+\frac{1}{12})}{\frac{1}{2}-(\frac{1}{4}-\frac{1}{12})} \div \frac{\frac{1}{2}\times\frac{1}{4}-\frac{1}{12}}{\frac{1}{2}\times(\frac{1}{4}-\frac{1}{12})} \text{ of } \frac{\frac{1}{2}-\frac{1}{4}\times\frac{1}{12}}{(\frac{1}{2}-\frac{1}{4})\times\frac{1}{12}}.$$

50. A is $1\frac{1}{2}$ times as good a working person as B, and twice as good as C. They all three can do a piece of work together in 6 days. They begin together, but after working for 2 days A goes away. After 2 days more B goes away, and C then completes the work alone. In how many days from the commencement is the whole work finished ?

if a man, a woman and a child work together, in what time can they do 7 times the work ?

24. Five brothers join in paying a sum of money ; the eldest pays a third of it, and the others pay the remainder in equal shares, and thereby each of them pays *Rs.*840 less than the eldest brother. What is the sum of money ?

25. If 9 men or 16 women can do a piece of work in 144 days, in what time would 7 men and 9 women do it, working together ?

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27. The product of three numbers is 340 ; the first is $7\frac{3}{5}$, the second is less than the first by $1\frac{7}{20}$. Find the third number.

28. A owned $\frac{5}{6}$ of a mine and sold $\frac{3}{4}$ of his share to B, who sold $\frac{1}{2}$ of his share to C, who sold $\frac{2}{3}$ of his share to D ; D's share was worth *Rs* 20565. What was the worth of B's remaining share, and what the worth of the whole mine ?

29. There are two fractions whose sum is $\frac{113}{126}$, and whose difference is $\frac{7}{18}$; find the fractions, and the quotient of the greater by the less.

30. If a turkey cost £$\frac{5}{7}$ and a goose £$\frac{3}{10}$, how many turkeys and geese, an equal number of each, can be bought for £14. 4*s.* ?

31. A boy, in flying his kite, lost $\frac{3}{5}$ of the string ; he then added 65 ft., and then found that it was $\frac{5}{6}$ of the original length. What was the length at first ?

32. If $2\frac{1}{3}$ of $(A+\frac{2}{7}$ of $A)=\frac{1}{3}$ of $(B-\frac{1}{3}B)$, find the value of A in terms of B.

33. A man bequeathed $\frac{5}{12}$ of his estate to one son, $\frac{7}{12}$ of the remainder to another son, and the balance to his widow. The sons' shares differ by *Rs.*1320 ; find the widow's share.

34. A man gives away in charity $\frac{1}{8}$ of his income, and pays $\frac{1}{10}$ of it in rates and taxes ; with these deductions he has *Rs.*4736. 8*a.* 8*p.* left. What is his gross income ?

35. Find the whole annual cost of a house, of which the rent is *Rs.* 360 ; the poor-rate being 2*a.* 8*p.* in the rupee, the gas-rate $\frac{2}{3}$ of the poor-rate, and the paving-rate $\frac{3}{5}$ of the gas-rate.

36. What sum must be added to or subtracted from £12. 7*s.* 6*d.*, so that £5. 3*s.* 4*d.* shall be the same fraction of the sum or difference that £3. 6*s.* 8*d.* is of £8. 6*s.* 8*d.* ?

37. Divide *Rs.*4200 among A, B, C and D, so that A may get twice as much as B, A and C may get thrice as much as B and A, and D may get four times as much as B and C.

38. The sum of $\frac{2}{5}$ and $\frac{1}{3}$ of a man's debts amounts to *Rs.*198.

7*a*. 4*p*. and his assets are *Rs*.45. 1*a*. 8*p*. ; how much in the rupee will his creditors lose ?

39. One-third of *A's* money is equal to $\frac{1}{2}$ of *C's* and $\frac{1}{3}$ of *C's* is equal to $\frac{3}{5}$ of *B's* ; *B* gives to *A* $\frac{1}{2}$ of his money and to *C* $\frac{1}{4}$ of the remainder, and has 2*s*. 6*d*. left. What amount had each at first ?

40. Express $\frac{2}{3}$ of $\frac{6}{15}$ of £1. 10*s*. + $\frac{3}{4}$ of $\frac{5}{6}$ of 5*s*. 4*d* − $8\frac{1}{2}$ of $\frac{1}{4\frac{1}{4}}$ of 5*s*. $3\frac{3}{4}$*d*. as the fraction of 2*s*. $1\frac{1}{2}$*d*.

41. Find the value of

$$\frac{1}{4}\text{ of}\left(\frac{4\frac{7}{6}\text{ of }6\frac{2}{3}}{7\frac{26}{27}}\right)\times\frac{3\frac{2}{3}-3\frac{1}{6}}{3\frac{2}{3}+2\frac{1}{6}}\text{ of } Rs.\ 184.\ 11a.\ 5p.$$

42. What sum must be added to or subtracted from *Rs*. 8. 12*a*. 6*p*. so that the sum or difference shall be the same fraction of *Rs*. 20 10*a*. that *Rs*.7. 6*a*. 6*p*. is of *Rs*.18. 8*a*. 3*p*. ?

43. *A* can do in 6 days as much work as *C* can do in 4 days, and *B* in 10 days as much as *C* in 8 days : what time would *B* require to finish a piece of work which *A* can do in 12 weeks ?

44 When rice is at 12 sr. per rupee, the expenses of a family amount to *Rs*.140 ; but they amount to *Rs*.134 only, when the price falls to 16 sr. per rupee. What will the expenses be, when rice is at 18 sr. per rupee ?

45. *A* can do in 2 days as much work as *B* in 3 days, and *B* in 5 days as much work as *C* in 4 days ; what time will *C* require to finish a piece of work which *A* can do in 9 days ?

46. *A* can by himself perform a certain quantity of work in 5 days, *B* twice as much in 7 days, and *C* four times as much in 11 days ; in what time can *A*, *B* and *C* together perform three times the original work ?

47. *A* was owner of $\frac{5}{17}$ of a privateer, and sold $\frac{3}{11}$ of $\frac{2}{9}$ of his share for £12 $\frac{4}{33}$; what was the value of $\frac{1\frac{1}{2}}{5\frac{2}{3}}$ of $\frac{7}{12}$ of the vessel at the same rate ?

48. How much ore must be raised, that on losing $\frac{17}{40}$ in roasting, and $\frac{18}{19}$ of the residue in smelting, there may result 506 tons of pure metal ?

49. Simplify—

$$\frac{\frac{1}{2}+\frac{1}{4}-\frac{1}{12}}{\frac{1}{2}-\frac{1}{4}+\frac{1}{12}}\text{ of }\frac{\frac{1}{2}-(\frac{1}{4}+\frac{1}{12})}{\frac{1}{2}-(\frac{1}{4}-\frac{1}{12})}\div\frac{\frac{1}{2}\times\frac{1}{4}-\frac{1}{12}}{\frac{1}{2}\times(\frac{1}{4}-\frac{1}{12})}\text{ of }\frac{\frac{1}{2}-\frac{1}{4}\times\frac{1}{12}}{(\frac{1}{2}-\frac{1}{4})\times\frac{1}{12}}.$$

50. *A* is $1\frac{1}{2}$ times as good a working person as *B*, and twice as good as *C*. They all three can do a piece of work together in 6 days. They begin together, but after working for 2 days *A* goes away. After 2 days more *B* goes away, and *C* then completes the work alone. In how many days from the commencement is the whole work finished ?

51. *A* and *B* are engaged to do a piece of work, which can be done by each in 15 and 20 days respectively. If *A* leaves off 3 days before the completion of the work, how should a sum of *Rs*.12. 8*a*. be distributed among them?

52. *A* and *B* undertake to do a piece of work in 15 days for *Rs*.22. 8*a*. After working for 12 days they call *C* to their help, and finish it in time. A could have done the work alone in 25 days. If they give *C Rs*.2. 4*a*., how many days would *B* take to finish the work?

53. A man is thrice as good a workman as a boy. If the time taken by a boy to do a piece of work exceed that taken by a man by $4\frac{1}{2}$ days, find the time in which a man can do it.

54. *A* and *B* can do a piece of work in 6 days, *B* and *C* in 7 days, and *A*, *B* and *C* can do it in 4 days. How long will *A* and *C* take to do it?

55. There is a leak in the bottom of a cistern. When the cistern was in thorough repair it could be filled in $\frac{3}{4}$ of an hour. It now takes 10 min. longer. If the cistern were full, how long would it be in leaking itself to become empty?

56. 10 men can do a piece of work in 30 days. After working for 10 days, a certain number of men are allowed to leave off, and then the work is finished in $43\frac{1}{3}$ days from the commencement. How many men are allowed to leave off?

57. The work which can be done by a certain number of men in 60 days, can be done by 15 men more in 40 days. Find the number of men required to do the work in 60 days.

58. There are two numbers of which the difference is 91. A third number is contained in them 13 and 20 times respectively. Find the numbers.

59. The number 483 divided by another gives 4 for the quotient and 7 for the remainder; find what number, when multiplied by the remainder, will give that divisor.

60. A boy was told to divide one-half of a certain number by 7, and the other half by 9, and then to add the two quotients. To save trouble he divided the number by 8, and his result was 6 wrong. What was the number?

61. At 3 o'clock I had completed $\frac{2}{5}$ of my journey, and at 5 o'clock $\frac{5}{8}$ of the same; when did I start and arrive?

62. 40 men can do a piece of work in a certain number of days; if only 30 men be employed it requires 6 days more. Find the time in which 60 men can do it.

63. 20 men do a piece of work in 24 days. After working for 6 days, an additional number of men is taken for assistance, and the work is finished in 21 days from the beginning. Required the additional number of men.

64. There are 4 casks of different sizes. The 1st is filled with water, the rest are empty. The 2nd cask is filled from the 1st and $\frac{4}{5}$ths of the original water in the 1st remains. The third is then filled from the 2nd, and $\frac{1}{4}$th of the water in the 2nd remains. The water in the 3rd is then poured into the 4th, and fills $\frac{9}{16}$ ths of it. Had the 3rd and 4th casks been filled from the contents of the 1st, 15 seers would still have remained in the 1st. Find the size of each cask.

CHAPTER VI.

The Theory of Decimals.

323. In the Notation of Integers, it has been seen that the figures in the units' place alone retain their *absolute* values, whilst the *local* values of figures in other situations increase **tenfold** for every figure we advance towards the left hand from that place. Therefore, in beginning at the *left* hand figure of any number and proceeding towards the *right* hand, it follows that the *local* value of every figure will be a **tenth** part of that which immediately precedes it ; and if we suppose figures to be situated to the right of the units' place, and this kind of tenfold *sub-division* to be extended to them, it is manifest that the local values of such figures in order from the place of units, will be a *tenth*, a *hundredth*, a *thousandth*, *&c.*, parts of their absolute values.

Hence we are enabled to represent integers and fractions by one uniform system of notation, by merely marking the **place of units** ; and whilst *Integers* are expressed by figures in the units' place and in places to the *left* of it, *Fractions* will be represented by figures situated in places, on the *right* of the units, called the places of *tenths*, *hundredths*, *thousandths*, *&c.*

324. In this manner originates the system of **Decimals**, being merely an extension of the Notation of Integers ; and though there are decimals of all denominations as *Decimal Integers*, yet from the circumstance of the system representing only *tenth*, *hundredth*, *thousandth*, *&c.*, parts of the unit, all *fractions* belonging to it are termed **Decimal Fractions**, in contradistinction to *Vulgar Fractions*, whereof the denominations may be any parts whatever.

Whence, **Decimals** may be *defined* to be *Fractions* whose denominators are 10, 100, 1000, &c., these denominators not being *written* as in Vulgar Fractions, but *expressed* by the position of *dot* or *point*, called the **decimal point**.

I. NOTATION AND NUMERATION OF DECIMALS.

325. If we suppose the digit 1 to occupy the units' place, the following scheme will point out the denominations of the figures to

the left and right of it, and it may be extended so as to include both integers and fractions of all local values whatever.

&c.	&c.	Ten-thousands	Thousands	Hundreds	Tens	**Units**	Tenths	Hundredths	Thousandths	Ten-thousandths	&c.	&c.
...	...	5	4	3	2	**1**	2	3	4	5	...	...

A mixed quantity, formed of integers and fractions is separated into its *integral* and *fractional* portions by means of the *Decimal Point* placed on the right of the units' place towards the *top*, (to distinguish it from the sign of Multiplication), which dispenses with the description of the local denominations, given above.

Thus, in 54321·2345, the figures 54321 on the *left* of the point denote so many integers, and the figures 2345 on the *right* of it, so many fractions, namely, 2 *tenths*, 3 *hundredths*, 4 *thousandths*, 5 *ten-thousandths*, and so on.

326. A number thus expressed, composed of units and *decimal* parts of unity, or of *decimal* parts of unity only, is called a **decimal number**, or simply a **decimal**. The part to the left of the point is called the **integral**, and to the right the **decimal** part of the given number.

Thus, 452·3678 is a *decimal* ; 452 is the *integral* part and 3678 the *decimal* part.

327. From what has been said above, it appears that the *expressing* and *reading* of *Decimals* will evidently be conducted upon the respective principles of the *Notation* and *Numeration* of integers : also, inasmuch as *Integers* denote assemblages of two or more *units*, *Decimals* will represent assemblages of two or more tenth, hundredth, &c., *parts* of a unit. Thus, to express,

(1) 45 units 3 tenths 2 hundredths 6 thousandths 8 ten-thousandths we write 45·3268.

(2) 45 units 2 hundredths 8 ten-thousandths we write 45·0208.

(3) 2 hundredths 8 ten-thousandths we write 0·0208 or simply ·0208.

328. In reading a decimal, we read off the decimal part as an integer annexing the denomination of its *last* figure on the right hand. Thus,

(1) 45·3268 is read 45 and 3268 *ten thousandths*.

(2) $3{\cdot}141596$ is read 3 and 141596 *millionths*.
(3) 00047 is read 47 *hundred-thousandths*.

329. In practice, however, we do not annex the decimal denomination, but saying (*decimal*) *point* read off the figures of the decimal *separately* in order. Thus,

(1) $45{\cdot}3268$ is read 45, point 3, 2, 6, 8.
(2) $3{\cdot}141596$ is read 3, point 1, 4, 1, 5, 9, 6.
(3) $\cdot 00047$ is read point 0, 0, 0, 4, 7.

II. RELATION OF DECIMALS TO VULGAR FRACTIONS.

330. From the statements made in the preceding Articles, it is obvious that every magnitude made of one or more decimals is equivalent to, and may be expressed by, one or more vulgar fractions having 10, 100, 1000, &c., for their denominators; and that all *mixed* quantities expressed decimally may be represented by means of *whole* numbers and *vulgar fractions* of similar denominations.

Thus, $24{\cdot}387 = 24 + \frac{3}{10} + \frac{8}{100} + \frac{7}{1000}$; $\cdot 045 = \frac{0}{10} + \frac{4}{100} + \frac{5}{1000}$.

331. *To convert a decimal into an equivalent vulgar fraction.*

RULE. Write down the given number for the numerator (omitting the decimal point), and for the denominator write 1 followed by as *many* ciphers as there are figures in the decimal part.

Ex. 1. $\cdot 327 = \frac{327}{1000}$; for $\cdot 327 = \frac{3}{10} + \frac{2}{100} + \frac{7}{1000} = \frac{327}{1000}$.

Ex. 2. $\cdot 0459 = \frac{459}{10000}$; for $\cdot 0459 = \frac{0}{10} + \frac{4}{100} + \frac{5}{1000} + \frac{9}{10000} = \frac{459}{10000}$.

Ex. 3. $13{\cdot}816 = \frac{13816}{1000}$; for $13{\cdot}816 = 13 + \frac{8}{10} + \frac{1}{100} + \frac{6}{1000}$
$= 13\frac{816}{1000} = \frac{13816}{1000}$.

In these instances, we see that the reduction to a common denominator, so tedious in vulgar fractions, is entirely dispensed with, and the *immediate* comparison of fractional quantities is one of the great advantages of the system.

332. *Conversely*, every vulgar fraction having 10, 100, 1000, &c., for its denominator, may be immediately represented by an equivalent decimal.

RULE. Write down the numerator and by beginning at the figure on the *right* hand, mark off by the decimal point as many figures as there are ciphers in the denominator. If the number of figures in the numerator be less than the number of ciphers in the denominator, prefix in the numerator the necessary number of ciphers.

Ex. 1. $\frac{15243}{1000} = 15{\cdot}243$; for there are 3 o's in the denominator.

Ex. 2. $\frac{5243}{10000} = \cdot 5243$; for there are 4 o's in the denominator.

Ex. 3. $\frac{243}{100000} = \cdot 00243$; for there are 5 o's in the denominator.

Examples XCVII.

1. Express as decimals :—

(1) Four-tenths ; eighteen hundredths ; six-thousandths.

(2) Three and seven-tenths ; one and fifty-four hundredths.

(3) Twenty-four and seventy-nine thousandths ; one millionth.

(4) Five-tenths, seven hundredths and nine thousandths.

(5) Eight ten-thousandths ; seventy millionths ; five ten-millionths.

(6) Three hundred and fifteen, eight thousandths and fifty millionths.

2. Express the following decimals in words :—

(1) ·5 ; ·35 ; ·326 ; ·1 ; ·01 ; ·0001 ; 5·37 ; ·0025.

(2) ·43268 ; ·003405 ; ·0000456 ; 98·7654321 ; ·100001.

3. Transform the following decimals into vulgar fractions :—

(1) ·7 ; ·71 ; ·751 ; ·1461 ; ·03 ; ·0037 ; ·00057 ; ·0000457.

(2) 3·7 ; 52·19 ; 475·913 ; 276·00483 ; 1475·2045709.

4. Convert the following decimals into equivalent vulgar fractions in their lowest terms :—

(1) ·5 ; ·25 ; ·75 ; ·625 ; ·1875 ; 2·56 ; ·432 ; ·00625 ; ·222464.

(2) 4·375 ; ·8125 ; 47·256 ; 4·3125 ; 13·00085 ; ·0006875.

(3) ·1015625 ; 1·075 ; 3·01875 ; 7·0046875 ; 13·0005859375.

5. Express as mixed numbers with the fractional parts in their lowest terms :—

(1) 5·2 ; 6·024 ; 41·015 ; 72·75 ; 9·4158 ; 307·251 ; 8·9125.

(2) 376·275 ; 970·00004 ; 7321·0505 ; 23·067138671875.

6. Express the following as decimals :—

(1) $\frac{8}{10}$; $\frac{7}{10}$; $\frac{13}{10}$; $\frac{47}{10}$; $\frac{13}{100}$; $\frac{407}{1000}$; $\frac{801}{100}$; $\frac{559}{10}$; $\frac{57}{1000}$.

(2) $\frac{17}{10000}$; $\frac{965}{1000000}$; $\frac{303}{10000}$; $\frac{9}{1000000}$; $\frac{57}{100000000}$; $\frac{4878}{100}$.

(3) $\frac{213}{100}$; $\frac{213}{10000}$; $\frac{213}{1000000}$; $\frac{300507}{10000000}$; $\frac{76358}{100000000}$; $\frac{20408005}{10000}$.

(4) 16 tenths ; 75 hundredths ; 31 thousandths ; 4 ten-thousandths ; 275 ten-thousandths ; 79 millionths ; 1001 ten-millionths ; 79 hundred-millionths ; two hundred and sixty-one hundred-thousandths ; 328 billionths.

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333. *Ciphers annexed to the right hand of a decimal fraction have no effect upon its value.*

Thus, $·37 = \frac{37}{100}$; $·370 = \frac{370}{1000} = \frac{37}{100}$; $·3700 = \frac{3700}{10000} = \frac{37}{100}$, and so on ; as appears also from the consideration, that there are *no* thousandths, &c., in addition to the tenths and hundredths expressed by ·37.

334. *Hence an integer or a whole number can be expressed as a decimal by writing ciphers in the decimal part.*

Thus, $317\cdot000 = 317 + \frac{0}{10} + \frac{0}{100} + \frac{0}{1000} = 317 + 0 + 0 + 0 = 317$;

$\therefore\ 317 = 317\cdot000$.

Again, $31\cdot72 = 31 + \frac{7}{10} + \frac{2}{100} = 31 + \frac{72}{100}$; and

$31\cdot720 = 31 + \frac{7}{10} + \frac{2}{100} + \frac{0}{1000} = 31 + \frac{72}{100}$; $\therefore\ 31\cdot72 = 31\cdot720$.

335. *Every cipher affixed to the left hand of a decimal fraction after the point diminishes its value tenfold.*

Thus, $\cdot43 = \frac{43}{100}$; $\cdot043 = \frac{43}{1000}$; $\cdot0043 = \frac{43}{10000}$; &c.;

where each fraction is a tenth part of that which immediately precedes it; and indeed this is evident from the circumstance of every figure being reduced *one* denomination lower by means of *each* cipher.

336. Hence, *Multiplication* and *Division* of a decimal by 10, 100, 1000, &c., are immediately effected, by shifting the decimal point *one*, *two*, *three*, &c., places towards the *right* and *left* respectively, adding ciphers, if necessary.

Ex. 1. $23\cdot45 \times 10 = 234\cdot5$; for $23\cdot45 \times 10 = \frac{2345}{100} \times 10 = \frac{2345}{10} = 234\cdot5$.

Ex. 2. $23\cdot45 \times 10000 = 234500$; for $23\cdot45 \times 10000 = \frac{2345}{100} \times 10000$

$= 2345 \times 100 = 234500$.

Ex. 3. $23\cdot45 \div 10 = 2\cdot345$; for $23\cdot45 \div 10 = \frac{2345}{100} \times \frac{1}{10} = \frac{2345}{1000} = 2\cdot345$.

Ex. 4. $23\cdot45 \div 10000 = \cdot002345$; for $23\cdot45 \div 10000 = \frac{2345}{100} \times \frac{1}{10000}$

$= \frac{2345}{1000000} = \cdot002345$.

Examples XCVIII.

1. Multiply :—

(1) $\cdot8$ separately by 10, 100, 1000, 100000, 10000000.
(2) $\cdot0053$ separately by 100, 10000, 1000000, 100000000.
(3) $13\cdot0014$ separately by 10, 100, 1000, 10000, 1000000.
(4) $8\cdot003056$ separately by 100, 10000, 10000000.

2. Divide :—

(1) $\cdot71$ separately by 10, 100, 10000, 1000000.
(2) $73\cdot58$ separately by 1000, 10000, 1000000, 10000000.
(3) $\cdot007$ separately by 100, 1000, 100000, a million.
(4) $\cdot1$ by 100; $\cdot001$ by 10000; $5742\cdot6$ by 10000000.

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337. The operations of Addition, Subtraction, Multiplication, and Division of decimals are performed in the same way as in the case of whole numbers. Hence it is an advantage to use decimals in preference to vulgar fractions.

III. ADDITION OF DECIMALS.

338. RULE. Place the numbers so that all the decimal points may be in the same vertical line, to insure the combination of those

of the *same* denominations ; and add them together as in integers, taking care to place the decimal point in the sum, immediately under those of the numbers proposed.

Ex. Add together 25·61, 4·805, ·0096, 653·27, 23.

25·61	For 25·61 + 4·805 + ·0096 + 653·27 + 23
4·805	$= \frac{2561}{100} + \frac{4805}{1000} + \frac{96}{10000} + \frac{65327}{100} + 23$
·0096	
653·27	$= \frac{256100 + 48050 + 96 + 6532700 + 230000}{10000}$
23·	
706·6946	$= \frac{7066946}{10000} = 706{\cdot}6946.$

339. Hence, decimals are said to be reduced to a *common denominator*, when ciphers are supplied so that there is the *same* number of decimal places in each.

Examples XCIX.

1. Add together :—

(1) ·295, 3·086, 12·87, ·0051, 729·54, 7·419, 3·0256.
(2) 3608·26, 360·826, 36·0826, 3·60826, ·360826, ·22314.
(3) 36·053, ·0079, ·000952, 417, 85·5803, ·0000501.
(4) 16, 12·2, 371·057, ·8241, 9·1, 1·235, 23·000358.
(5) 17·215, 3·0567, ·009, 2·07195, 365, 54·75.
(6) 231·8, 45·001, 2·7169, 4567·21, ·00087, 6·05.
(7) 20·02, 576·89174, 1·0008159, ·423564, 29, 7·21685.
(8) 61, 3·16004, ·0478, 21·805, 1·00006, 12·9871.
(9) ·00625, 30·698, 2·7535, 19·84, ·1875, 8·096.
(10) 100, ·1, ·01, ·001, ·0001, ·00001, ·000001, ·0000001.

2. Find the values of :—

(1) 69·563 + 1307·2345 + 16·27 + 18·03 + 59·327 + 116·2491 + 3·0002.
(2) 15·063 + ·002857 + 308·62 + 769·3276 + 58·739127 + ·69325.
(3) 77·3 + 160·6734 + 26·345 + 46 + 31·1 + 117·154 + ·0002 + 2343·008
 + 1·0000123 + 213·7 + 2·913 + 14·769 + ·007871.
(4) ₨1159·217 + ₨387·61 + ₨71·316 + ₨91·204 + ₨74·031.
(5) £573·162 + £83·017 + £92·159 + £30·031 + £99·999.
(6) 1596·131 cwt. + 702·021 cwt. + ·170038 cwt. + 319 7 cwt. + 5·93 cwt.
(7) 97·316 yds. + 1597·308 yds. + 316·2917 yds. + ·03 yd. + 159·1 yds.
(8) 27 tenths + 345 hundredths + 17 thousandths + 4256 millionths.

IV. SUBTRACTION OF DECIMALS.

340. RULE. Place the less number under the greater as in Addition ; suppose ciphers to be supplied if necessary, in the upper line ;

and the difference, found as in integers, will have as many decimal places as are contained in each, either expressed or understood.

Ex. 1. Subtract 34·917 from 41·62.

$$\begin{array}{r} 41{\cdot}62 \\ \underline{34{\cdot}917} \\ \underline{\underline{6{\cdot}703}} \end{array} \qquad \text{For } 41{\cdot}62 - 34{\cdot}917 = \frac{4162}{100} - \frac{34917}{1000} = \frac{41620 - 34917}{1000}$$
$$= \tfrac{6703}{1000} = 6{\cdot}703.$$

Ex. 2. Is ·90437532 more nearly represented by ·90438 or by ·90437 ?

$${\cdot}90438 - {\cdot}90437532 = {\cdot}00000468 \;;\; {\cdot}90437532 - {\cdot}90437 = {\cdot}00000532.$$

∴ ·90438 is nearer to ·90437532 than ·90437.

Examples C.

1. Subtract :—

(1) ·3806 from ·57031 ; 7·998 from 19·201 ; 3·4796 from 56·036.
(2) ·013096 from ·13096 ; ·21968 from 1·026103 ; 6·90086 from 7.
(3) ·99999 from 9 ; ·00071961 from ·03107 ; ·5968 from 1·11315.
(4) ·01 from ·1 ; ·0009 from ·001 ; ·672163 from 1·29613.

2. Find the difference between :—

(1) 27·903 and ·054 ; 7295·06 and 254·738 ; 35·08989 and 3·508989.
(2) 2·057 and 1·0097 ; 3·025 and ·003025 ; ·7053 and ·6729.
(3) 5·0009 and ·089898 ; 136·159 and 136·0159 ; 13 and 5·90516.

3. Find the values of :—

(1) 1500·5 − 714·286 ; 15·903 − 4·696843 ; ·001 − ·00001.
(2) ₨45·21 − ₨38·793 ; ₨8·264 − ₨6·03176 ; ₨5·71021 − ₨2·369684.
(3) £83·6 − £83·47916 ; £70·151 − £15·8261 ; £70·107 − £69·89706.
(4) 6·4 mds. − ·000064 md. ; 23·5 tons − ·9876 ton ; 1·44 ft. − ·00144 ft.

4. What number subtracted from 13·007 leaves 3·594 ?

5. What number added to 13·265 makes up 100·0008 ?

6. Simplify :—

(1) 5 − 3·22 + 2·333 − 1·4444 ; 2·194 + 15·367 − 10·009 − 11·25 + 5·8.
(2) 227·9 − (420·315 + 27·291) + 865·21 − 1·057.
(3) 17·073 + 1·3591 − 10·84 − (11·03796 − 15·8 + 6·9).
(4) 105·09 − 211·748 − 21·1748 − 15·73241 + 670·6 − ·0053.

7. Find the complement of ·7781513 ; ·000456 ; 98·654321 ; 9542·425 ; 998·899 ; and ·00001. (See Art. 58.)

8. Whether is 3·1415926535 more accurately represented by 3·1415926 or by 3·1415927 ?

9. Express in the decimal notation, the value of 8·0625 − 6·04 − ·00375 + 1·09236 − $\tfrac{25679}{10000}$.

V. MULTIPLICATION OF DECIMALS.

341. RULE. Multiply together the numbers proposed as if they were integers; and the product will contain as many places of decimals, as there are decimal places in the multiplicand and multiplier together. If there are not figures enough, prefix the necessary number of ciphers.

Ex. 1. Multiply ·627 by 1·59.

$$\begin{array}{r} {\cdot}627 \\ \underline{1{\cdot}59} \\ 5643 \\ 3135 \\ \underline{627} \\ {\cdot}99693 \end{array}$$

The number of decimal places in the multiplicand and multiplier is 3 and 2 respectively; therefore the number in the product is $3+2=5$.

$\therefore$ the required product $= {\cdot}99693$.

For $\cdot 627 \times 1{\cdot}59 = \dfrac{627}{1000} \times \dfrac{159}{100} = \dfrac{99693}{100000} = {\cdot}99693$.

Ex. 2. Multiply 7·5 by ·000084.

$$\begin{array}{r} 7{\cdot}5 \\ \underline{{\cdot}000084} \\ 300 \\ \underline{600} \\ 6300 \end{array}$$

The number of decimal places in the multiplicand and multiplier is 1 and 6 respectively; therefore the number in the product is $1+6=7$. But there are only 4 figures in the product; therefore prefix 3 ciphers.

$\therefore$ the product $= {\cdot}0006300 = {\cdot}00063$.

Examples CI.

1. Multiply :—

(1) ·718 by ·57; 16·8 by ·0024; 144 by ·0625; 12·5 by ·062216.
(2) 270·56 by ·37025; ·00579 by 3796·8; 36·2185 by ·229.
(3) 421·619 by ·547; 34·6875 by 119·808; ·007853 by ·00476.
(4) 384·759375 by ·00032; ·00082175 by 2·38645; ·002 by ·0004.
(5) ·0000051472 by ·0625; 948·7096 by ·007089; 170·71 by ·0325.
(6) ·00015625 by 8·192; ·00025 by ·0000625; ·00711858 by ·00024.

2. Find the values of :—

(1) 3·51 × ·075; ·0167 × ·008448; ·354178 × ·005; 3·12 × 2·0001.
(2) 3·005 × 40·23; 1·279 × ·0008787; 35·04 × ·0008 × 5·25.
(3) ·275 × 2·75 × 27·5; 3·24 × ·0028 × 2·9375; 11·01 × 110 × ·1102.
(4) 1·02 × 102 × 10·2 × ·102; 5·107 × ·05107 × ·05 × 700.
(5) ·4 × ·05 × ·006 × ·0007 × 800000; ·004 × ·04 × ·4 × ·0004 × 40000.
(6) ·01 × ·001 × ·0001 × ·00001 × 100000; ·845 × ·0017 × 7·4 × ·09 × 10000

3. Find the values of :—

(1) 7·94 × 2·638 + 32·56 × ·00457 − ·007853 × ·00476 − ·000076 × 18·9.
(2) 592·9 × 61·6 × ·0064 + 1562·5 × ·0625 × 2·5 − 45·08 × 64·4 × ·092.
(3) (37·1 − 19·08) × ·703; 37·1 − 19·08 × ·703; $({\cdot}05)^3+({\cdot}025)^2+{\cdot}00025$.
(4) $(36{\cdot}73)^2-(25{\cdot}894)^2$; $({\cdot}888)^2-({\cdot}8008)^2$; $(3{\cdot}025)^2-3{\cdot}025\times{\cdot}003025$.

4. Multiply 325 tenths by 547 millionths; 128 thousandths by 78125 ten millionths.

VI. DIVISION OF DECIMALS.

342. *When the divisor is an integer.*

RULE. Divide, as if dividend and divisor were whole numbers; and when, in the process of division, the decimal point of the dividend is arrived at, place a decimal point in the quotient. If the division do not terminate with the last digit of the dividend, annex ciphers to the dividend and continue the operation until it terminates or the required number of decimal places in the quotient is obtained.

Ex. Divide 187·5 by 25; 1770·89 by 4735 and 3217 by 625.

(1)
```
25)187·5(7·5
   175
    125
    125
```
∴ the quotient = 7·5.

(2)
```
4735)1770·890(·374
     14205
      35039
      33145
       18940
       18940
```
∴ the quotient = ·374.

(3)
```
625)3217·0000(5·1472
    3125
      920
      625
      2950
      2500
       4500
       4375
        1250
        1250
```
∴ the quotient = 5·1472.

(1) For $187·5 \div 25$
$= \frac{1875}{10} \times \frac{1}{25} = \frac{1875}{25} \times \frac{1}{10}$
$= 75 \times \frac{1}{10} = \frac{75}{10} = 7·5.$

343. When the divisor does not exceed 20, or when it can easily be separated into factors none of which exceeds 20, the division should be performed by the method of **short division**.

Ex. Divide 56·787 by 12, and 1·21968 by 693.

(1)
```
12)56·78700
    4·73225  Ans.
```

(2)
```
     ⎧  7)1·21968
693  ⎨  9) ·17424
     ⎩ 11) ·01936
            ·00176  Ans.
```

344. *When the divisor is a decimal.*

RULE. Make the divisor a whole number by removing its decimal point altogether, and shift the decimal point of the dividend as many places to the right as there were decimal figures in the divisor, annexing for this purpose ciphers, if necessary, to the dividend. Then, divide as if the terms were integers; in the quotient, count off as many decimal places from the right as there are in the altered dividend, prefixing ciphers, if necessary.

Ex. Divide 10·836 by 5·16 ; ·1875 by 2·5 and 62·5 by ·025.

(1)
```
5·16)10·836
516)1083·6(2·1
    1032
      516
      516
```
∴ the quotient = 2·1.

(2)
```
2·5)·1875
25)1·875(·075
    175
     125
     125
```
∴ the quotient = ·075.

(3)
```
·025)62·5
25)62500(2500
   50
    125
    125
```
∴ the quotient = 2500.

345. In the course of the division, if there be any remainder after the last figure from the altered dividend has been brought down, add ciphers to the right of the dividend, and proceed as in Art. 342.

Ex. Divide ·01029 by 1·68.

```
1·68)·01029
168)1·029000(6125
    1008
      210
      168
       420
       336
        840
        840
```

Here the altered dividend has 3 decimal figures, and we have added to it 3 ciphers; therefore in the quotient, we *must* count off 6 decimal places. . . . . .

∴ the quotient = ·006125.

For $\cdot 01029 \div 1\cdot 68 = \frac{1029}{100000} \div \frac{168}{100} = \frac{1029}{168} \times \frac{1}{1000}$
$= 6\frac{1}{8} \times \frac{1}{1000} = (6 + \frac{125}{1000}) \times \frac{1}{1000}$
$= \frac{6\cdot 125}{1000} = \cdot 006125.$

346. In this case also, the method of **short division** can advantageously be employed when the divisor has been made an integer, as in Art. 343.

Ex. Divide 90·65 by ·049, and 171·99 by 27·3.

(1)
```
   ·049)90·65
49 { 7)90650
   { 7)12950
        1850 Ans.
```

(2)
```
     27·3)171·99
    {  3)1719·9
273 {  7) 573·3
    { 13)  81·9
            6·3 Ans.
```

347. If the division do not terminate, the quotient may be required to a given number of decimal places, as in the following examples.

Ex. Divide ·02 by 1·7 ; 1 by ·013 and 1 by ·007, each to 5 places of decimals.

(1)
```
1·7)·02
17)·20000
   ·01176...Ans.
```

(2)
```
·013)1·000
13)1000·00000
   76·92307 . Ans.
```

(3)
```
·007)1·000
7)1000·00000
  142·85714..Ans.
```

348. An integral divisor ending with ciphers may be deprived of the ciphers, if we remove the decimal point of the dividend one place *to the left* for every cipher withdrawn.

Thus, $\cdot 78 \div 60 = \cdot 078 \div 6$; $\cdot 78 \div 600 = \cdot 0078 \div 6$, and so on.

Ex. Divide 1·5625 by 25000, and 7 by 796·3 to 5 places of decimals.

(1) 25000)1·5625

25 { 5)·0015625
     5)·0003125
       ·0000625 *Ans.*

(2) 796·3)7·0

7963)70·00000(·00879...*Ans.*
63704
 62960
 55741
  72190
  71667

349. In the above divisions, it should be very carefully noticed that *for each digit in the decimal part of the dividend there is a digit in the decimal part of the quotient.*

Examples CII.

1. Divide :—

(1) 783·5 separately by 5, 25, 125, 625 and 6250.
(2) 773·682 separately by 6, 13, 78, 169, 507 and 1014.
(3) ·00750116 separately by 677, 1354, 2708 and 10832.
(4) 35·9424 by 7·02 ; ·278831 by ·653 ; 11·444495 by 4·735.
(5) 1·68 by ·024 ; 971·7 by ·123 ; 142·025 by ·0437 ; 84·375 by ·00375.
(6) ·020872522 by ·08635 ; ·0020925 by ·000864 ; ·39538 by 5300.
(7) ·1 by ·01 : ·01001 by ·001 ; 92·7 by ·06 ; 99 by ·0009 ; ·001 by ·0001.
(8) 9864·1698175 by 35·0645 ; 124·59993 by 3194·87.
(9) 1·365 separately by 1·25, 12·5, ·00125 and 12500.
(10) 7·835 separately by ·5, ·25, 12·5, 6·25, ·625, ·0625 and 625000.
(11) ·0003738028 by ·0476 ; ·0064096 by 2·003 ; 614·50824 by ·0010201.
(12) 2 and 22 hundredths by 74 ten-thousandths.

2. Find the values of (to 5 places of decimals) :—

(1) 3 ÷ ·876 ; ·0257 ÷ ·0041 ; 325·46 ÷ ·0187 ; ·0719 ÷ 27·53.
(2) ·5 ÷ 76·91342 ; 11·121 ÷ 3·4571 ; 16·1 ÷ 63572·45 ; 25 ÷ 19.
(3) ·046 ÷ ·00762089 ; ·32165 ÷ ·0035216 ; 314159·26 ÷ ·008597.

3. Find the quotient, by *short division*, of :—

(1) 3·6288 separately by ·3, ·7, ·9, 6·3, 12·6, ·189 and ·024.
(2) ·0255 separately by ·03, ·005, 3·4, 60, ·0102 and 2·55.

4. Divide, by *short division*, to 5 places of decimals :—

(1) ·009384 separately by 7, ·07, ·007, 1·8, ·0018 and ·00063.
(2) 57982·6966 by ·00000076 ; 346·72361 by ·00016.

5. Find the values of :—

(1) ·01385 × 61·37 ÷ 2·77 ; ·399 × ·007 ÷ ·000019 ; 24·01 × ·0039 ÷ 133·77.

(2) $(3{\cdot}124+1{\cdot}25)\times(2{\cdot}237-{\cdot}026)\div{\cdot}0003$; $(2{\cdot}05)^2\times2{\cdot}24\div{\cdot}0041$.

(3) $3833336\div(8{\cdot}99\times20{\cdot}8)$; ${\cdot}000785\div{\cdot}0005-{\cdot}000075\div15$.

(4) $206{\cdot}59\div1872\times{\cdot}001$ to 5 places ; $15{\cdot}8402+3{\cdot}689\div672{\cdot}4$ to 6 places.

VII. REDUCTION OF FRACTIONS TO DECIMALS.

350. *To reduce a vulgar fraction to a decimal.*

RULE. Reduce the fraction to its lowest terms ; then divide the numerator with as many ciphers annexed to the right of it, as may be deemed necessary, by the denominator ; and the quotient comprising as *many* decimal places (counting from the right) as there are *ciphers* annexed, will be the decimal required.

Ex. 1. Reduce $\frac{3}{8}$, $\frac{1}{6400}$ and $4\frac{7}{125}$ to decimals.

(1) $8)3{\cdot}000$
${\cdot}375$
$\therefore$ decimal $= {\cdot}375$. *Ans.*

(2) $100)1{\cdot}0$
$64\left\{\begin{array}{l} 8)\ {\cdot}01000 \\ 8)\ {\cdot}00125000 \end{array}\right.$
$\ {\cdot}00015625$
$\therefore$ decimal $= {\cdot}00015625$. *Ans.*

(3) $125\left\{\begin{array}{l} 5)7{\cdot}0 \\ 5)1{\cdot}4 \\ 5)\ {\cdot}28 \end{array}\right.$
$\ {\cdot}056$
$\therefore$ decimal $= 4{\cdot}056$. *Ans.*

For $\frac{3}{8}=\frac{30}{80}=\dfrac{\frac{30}{8}}{10}=\dfrac{3\frac{6}{8}}{10}=\frac{3}{10}+\frac{6}{80}=\frac{3}{10}+\frac{3}{40}=\frac{3}{10}+\frac{30}{400}=\frac{3}{10}+\dfrac{\frac{30}{4}}{100}$

$=\frac{3}{10}+\dfrac{7\frac{2}{4}}{100}=\frac{3}{10}+\frac{7}{100}+\frac{2}{400}=\frac{3}{10}+\frac{7}{100}+\frac{1}{200}=\frac{3}{10}+\frac{7}{100}+\frac{10}{2000}$

$=\frac{3}{10}+\frac{7}{100}+\dfrac{\frac{10}{2}}{1000}=\frac{3}{10}+\frac{7}{100}+\frac{5}{1000}=\frac{375}{1000}={\cdot}375.$

Similarly the other two can be explained.

Ex. 2. Convert $5\frac{5}{640}+{\cdot}75$ of $1\frac{1}{5}$ of $7\frac{1}{2}$ into a decimal.

$640\left\{\begin{array}{l} 10)5{\cdot}0 \\ 8)\ {\cdot}5000 \\ 8)\ {\cdot}0625000 \end{array}\right.$
$\ {\cdot}0078125$

${\cdot}75$ of $1\frac{1}{5}$ of $7\frac{1}{2}={\cdot}75$ of $\frac{6}{5}$ of $\frac{15}{2}$
$={\cdot}75\times9=6{\cdot}75$.
$\therefore$ the reqd. decimal $=5{\cdot}0078125+6{\cdot}75$
$=11{\cdot}7578125$.

Ex. 3. Find the value of $\dfrac{{\cdot}01491\times2{\cdot}204}{{\cdot}0071\times2{\cdot}1\times{\cdot}029}$.

since $\dfrac{1491\times2204}{71\times21\times29}=76$,
$\therefore \dfrac{{\cdot}01491\times2{\cdot}204}{{\cdot}0071\times2{\cdot}1\times{\cdot}029}=76$.

There are 8 decimal places in the numerator and 8 in the denominator.
$\therefore$ the quotient is an integer.

351. In some cases if the division do not terminate, neither is the corresponding decimal *finite*, and the vulgar fraction is expressed only *approximately* by the decimal fraction thus found ; five or six figures are generally sufficient for all practical purposes, but the approximation will be nearer, the further the division is continued, inasmuch as by *every succeeding* step of the operation a

decimal fraction of an *inferior* denomination is added to the value already obtained.

Ex. Express $\frac{15}{22}$ as a decimal, as far as the sixth place.

$$22\left\{\begin{array}{l} 2)\underline{15\cdot0} \\ 11)\underline{7\cdot5000000} \\ \quad\ \cdot6818181\ldots\ldots \end{array}\right. \qquad \therefore \text{ the decimal reqd.} = \underline{\cdot681818}.$$

Note. The following results are useful, and should be verified and remembered :—

$\frac{1}{4}=\cdot25$; $\frac{1}{2}=\cdot5$; $\frac{3}{4}=\cdot75$; $\frac{1}{8}=\cdot125$; $\frac{3}{8}=\cdot375$;
$\frac{5}{8}=\cdot625$; $\frac{7}{8}=\cdot875$; $\frac{1}{16}=\cdot0625$; $\frac{1}{5}=\cdot2$; $\frac{1}{32}=\cdot03125$.

Examples CIII.

1. Reduce to decimals :—

(1) $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{5}{8}$; $\frac{3}{16}$; $\frac{4}{25}$; $\frac{399}{128}$; $\frac{13}{1600}$; $\frac{9}{125}$; $\frac{97}{128}$.

(2) $\frac{17}{2500}$; $\frac{238}{6250}$; $\frac{2504}{625}$; $3\frac{4}{25}$; $4\frac{5}{16}$; $6\frac{3}{5120}$; $7\frac{51}{120}$.

(3) $\frac{7}{5}$; $\frac{17}{320}$; $\frac{34567}{20480}$; $1\frac{71}{1024}$; $7\frac{3}{6250}$; $3\frac{7}{3125}$; $4\frac{83}{78125}$.

2. Convert into decimals :—

(1) $1\frac{2}{15}$ of $11\frac{1}{4}$; $3\frac{1}{2}$ of $4\frac{1}{4}$ of $5\frac{1}{8}$; $3\frac{1}{8}$ of $2\frac{4}{5}$; $\frac{3}{5}+\frac{7}{8}+\frac{9}{10}+\frac{7}{32}$.

(2) $\dfrac{12\frac{3}{4}}{10\frac{1}{5}}$; $\dfrac{1\frac{1}{10}}{62\frac{1}{2}}$; $\dfrac{5\frac{2}{5}}{7\frac{1}{2}}$ of $\dfrac{21\cdot25}{\cdot046875}$; $\dfrac{14\frac{1}{10}}{125}$; $\dfrac{6\frac{3}{4}}{11\frac{1}{4}}$; $\dfrac{17\frac{1}{8}}{12\frac{1}{2}}$.

(3) $17\frac{3}{10}+25\frac{5}{8}+6\frac{25}{32}+13\frac{8}{25}+20\frac{73}{10000}$; $3\frac{5}{9}$ of $\frac{1}{512}$.

(4) $1\cdot26$ of $66\frac{2}{3}+5\frac{1}{5}$ of $1\cdot0375$; $3\frac{5}{8}$ of $\cdot003-\cdot0011$ of $7\frac{1}{4}$.

3. Express as decimals, as far as the sixth decimal place :—

$\frac{1}{3}$; $\frac{2}{7}$; $\frac{3}{11}$; $\frac{5}{13}$; $\frac{7}{26}$; $\frac{13}{42}$; $2\frac{11}{12}$; $8\frac{9}{14}$; $24\frac{6}{17}$; $7\frac{15}{46}$.

4. Arrange in order of magnitude, by reducing to decimals :—

(1) $\frac{2}{3}, \frac{5}{6}, \frac{3}{8}$. (2) $\frac{5}{12}, \frac{7}{16}, \frac{3}{4}$. (3) $\frac{5}{24}, \frac{7}{32}, \frac{4}{9}$. (4) $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}$.

5. Simplify the following :—

(1) $\dfrac{\cdot003125\times\cdot48}{\cdot0000125}$. (2) $\dfrac{\cdot0075\times2\cdot1}{\cdot0175}$. (3) $\dfrac{4\cdot255\times\cdot0064}{\cdot00032}$.

(4) $\dfrac{2\cdot004\times3\cdot375}{4\times\cdot167}$. (5) $\dfrac{\cdot0295\times\cdot00152}{3\cdot04\times1\cdot18}$. (6) $\dfrac{1\cdot18\times3\cdot04}{\cdot152\times2\cdot95\times\cdot00125}$.

(7) $3\cdot25-2\cdot765+3\cdot125\times8-\cdot607095\div\cdot027$.

(8) $\dfrac{22\cdot4}{\cdot25}+\dfrac{250}{\cdot8}+\dfrac{1\cdot2}{\cdot0075}$; $5\cdot634\times\cdot0057+\dfrac{4\cdot2325}{8465}\div\dfrac{\cdot39424}{5632}$.

VIII. G. C. M. AND L. C. M. OF DECIMALS.

352. *To find the G. C. M. and the L. C. M. of Decimals.*

RULE. Make the same number of decimal places in each of the numbers (Art. 339) ; find their G. C. M. or L. C. M. as if they

were integers, and then mark off the said number of decimal places in the result, prefixing ciphers, if necessary.

Ex. Find the G. C. M. and the L. C. M. of 1·6, ·24 and 14.

Here, the numbers are equivalent to 1·60, ·24 and 14·00.

The G. C. M. of 160, 24 and 1400 = 8 ; their L. C. M. = 16800.

∴ the G. C M. reqd. = ·08 ; and the L. C. M. reqd. = 168·00 = 168.

Examples CIV.

1. Find the G. C. M. of :—

(1) 1353·6 and 231·48. (2) 4·2237 and 755·82. (3) 36·795 and 57·98.
(4) 376·1034 and 1081. (5) ·14, 1·8 and ·024. (6) ·009, 1·8 and ·24.
(7) 2·4, ·48, ·64 and 1·92. (8) ·016, ·0024, 4·8 and 74.

2. Find the L. C. M. of :—

(1) 1·5, 35, ·063 and 7·2. (2) 6·3, ·12, ·084 and ·0014.
(3) 2·4, ·39 and 3·76. (4) ·312, ·0124, 3·41 and 37·2.
(5) 4·2237 and 755·82. (6) 1·36652 and 246·8642.

IX. RECURRING DECIMALS.

353. In the conversion of a vulgar fraction into a decimal, we find that the division performed according to the Rule laid down in Art. 350 terminates in some cases and does not terminate in others. Thus, $\frac{5}{8}$ = ·625, and here the division terminates ; but $\frac{3}{11}$ = ·272727......, and in this case the division does not terminate and can be extended to an unlimited length. The former is called a **terminating** or **finite** decimal, and the latter a **non-terminating** decimal.

354. It has already been shewn in Art. 331 that to reduce a vulgar fraction in its lowest terms to a decimal is the same as reducing it to an equivalent one having 10 or some power of 10 for its denominator. Thus, it follows that no vulgar fraction can be reduced to a terminating decimal, unless it can be expressed as one having 10 or some power of 10 for its denominator. Now, no number can, by multiplication, be made a power of 10, unless it be composed of prime factors, each of which is 2 or 5. Hence, to find whether a vulgar fraction can be expressed as a terminating decimal or not, we have the following Rule.

RULE. *Reduce the given vulgar fraction to its lowest terms, and resolve its dinominator into its prime factors ; if these prime factors be only 2 and 5, it can be expressed as an exact or terminating decimal ; otherwise, it cannot.*

Ex. 1. Can $\frac{3}{50}$ and $\frac{81}{1250}$ be expressed as a terminating decimal ?

(1) Yes ; for 50 = 2 × 5 × 5, and involves factors of 2 and 5 only.

(2) Yes ; for 1250 = 2 × 5^4, and involves factors of 2 and 5 only.

Ex. 2. Can $\frac{137}{576}$ be expressed as a finite decimal?

No; for $576 = 2^6 \times 3^2$, and involves other factors than 2 and 5.

Examples CV.

1. Which of the following fractions can be expressed as finite or terminating decimals:—

$\frac{5}{12}$; $\frac{13}{16}$; $\frac{9}{48}$; $\frac{11}{25}$; $\frac{31}{80}$; $\frac{19}{64}$; $\frac{85}{192}$; $\frac{79}{405}$; $\frac{91}{560}$; $\frac{512}{875}$; $\frac{18041}{65536}$.

2. Write down those numbers between 1 and 25, of which if any one be the denominator of a fraction in its lowest terms, that fraction can be reduced to a terminating decimal.

355. *In non-terminating decimals the figures of the quotient must recur over and over again.*

Take the fraction $\frac{5}{7}$. To reduce it to a decimal, we annex ciphers to 5 and divide by 7. Since the division does not terminate, we cannot have the remainder 0, and the only *possible* remainders that can arise are 1, 2, 3, 4, 5, and 6 and consequently after *six steps at most* (after as many divisions at least as there are units in the denominator) we must come to the given numerator or to one of the remainders that has occurred before, and therefore from that point we must have a recurrence of the remainders, and therefore of the quotient figures in the same order over and over again. Thus,

$$
\begin{array}{l}
7)5\cdot0(\cdot714285 \\
\underline{49} \\
\begin{array}{lll}
10 & 20 & 40 \\
\underline{7} & \underline{14} & \underline{35} \\
30 & 60 & 5 \\
\underline{28} & \underline{56} &
\end{array}
\end{array}
$$

Here, after 6 figures, we get 5 for remainder, and therefore the whole process will recur again from the beginning.

$\therefore \frac{5}{7} = \cdot714285714285714142\ldots\ \ldots$

356. When, beginning from a certain point in the decimal part of a number, the figures repeat themselves indefinitely and in the same order, the number is called a **recurring, circulating, repeating** or **periodic** decimal; and the whole set of figures which recurs constantly in the same order is called the **period** or **repetend.**

357. The *period* is termed *simple* or a *compound repetend* according as it consists of *one* or *more* figures; and the *extent* of the period is denoted by means of *dots* ($\cdot$) placed over the *first* and *last* of the figures which compose it.

Thus, $\frac{8}{3} = 2\cdot6666\ldots\ldots = 2\cdot\dot{6}$; and $\frac{5}{37} = \cdot135135\ldots\ldots = \cdot\dot{1}3\dot{5}$.

So, $\cdot\dot{5}\dot{7} = \cdot575757\ldots$; $\cdot\dot{0}2\dot{4} = \cdot024024024\ldots$; $\cdot3\dot{2}\dot{6} = \cdot3262626\ldots\ldots$

The several *periods* in the above are 6, 135, 57, 024 and 26.

358. Recurring Decimals are either **Pure** or **Mixed.**

(i) A **pure circulating decimal** is one which recurs from the first figure of the decimal part; as, $\cdot\dot{3}$, $\cdot\dot{0}7\dot{8}$.

(ii) A **mixed circulating decimal** is one which recurs after some figures and thus consists of a *non-recurring* and a *recurring* part; as, $\cdot 1\dot{7}$, $\cdot 24\dot{6}\dot{8}$.

359. A vulgar fraction in its lowest terms, whose denominator contains neither of the prime factors 2 and 5, produces a *pure circulating decimal*, whereas one, whose denominator contains 2 or 5 and one or more other prime factors, produces a *mixed circulating decimal*.

Thus, $\frac{2}{3} = \cdot\dot{6}$; $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$; $\frac{7}{22} = \frac{7}{2 \times 11} = \cdot 3\dot{1}\dot{8}$.

Ex. 1. Convert $\frac{7}{3}$ and $\frac{4}{27}$ into decimals.

(1)

$$\begin{array}{l} 3)7(2\cdot 3\ldots \\ \underline{6} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

Here, a repetition of 1 in the remainder gives a repetition of the figure 3 in the quotient.

$\therefore \frac{7}{3} = 2\cdot\dot{3}$.

(2)

$$\begin{array}{l} 27)4\cdot 0(\cdot 148\ldots \\ \underline{27} \\ 130 \\ \underline{108} \\ 220 \\ \underline{216} \\ \quad 4 \end{array}$$

The figure 4 occurs again in the remainder after 3 steps, therefore the digits 1, 4 and 8 must recur in the quotient.

$\therefore \frac{4}{27} = \cdot\dot{1}4\dot{8}$.

For $\frac{7}{3} = 2\frac{1}{3} = 2 + \frac{10}{30} = 2 + \frac{\frac{10}{3}}{10} = 2 + \frac{3\frac{1}{3}}{10} = 2 + \frac{3}{10} + \frac{1}{30} = 2 + \frac{3}{10} + \frac{10}{300}$

$$= 2 + \frac{3}{10} + \frac{\frac{10}{3}}{100} = 2 + \frac{3}{10} + \frac{3\frac{1}{3}}{100} = 2 + \frac{3}{10} + \frac{3}{100} + \frac{1}{300} = 2\cdot 33\ldots\ldots$$

Similarly, the second can be explained.

Ex. 2. Reduce $\frac{5}{36}$ to a decimal.

$$\begin{array}{l} 36)5\cdot 0(\cdot 138\ldots \\ \underline{36} \\ 140 \\ \underline{108} \\ \;320 \\ \;\underline{288} \\ \quad 32 \end{array}$$

Here, the remainder 32 which occurred after the second step occurs again in the third, and therefore the figure 8 will recur in the quotient.

$\therefore \frac{5}{36} = \cdot 13\dot{8}$.

Examples CVI.

Reduce to recurring decimals :—

1. $\frac{2}{9}$; $\frac{3}{11}$; $\frac{13}{99}$; $\frac{5}{37}$; $\frac{129}{55}$; $\frac{5}{3}$; $\frac{44}{9}$; $\frac{21}{11}$; $\frac{5}{7}$; $\frac{97}{13}$; $\frac{8}{63}$; $9\frac{17}{55}$.

2. $\frac{4}{13}$; $\frac{5}{41}$; $\frac{8}{33}$; $3\frac{59}{98}$; $4\frac{45}{56}$; $7\frac{25}{132}$; $5\frac{29}{101}$; $3\frac{883}{2475}$.

3. $\frac{5}{396}$; $\frac{214}{37}$; $\frac{809}{296}$; $\frac{625}{576}$; $\frac{425}{328}$; $\frac{313}{419}$; $\frac{3542}{1025}$; $\frac{506}{505}$.

4. $\frac{1000}{407}$; $\frac{20000}{5291}$; $\frac{7624}{4320}$; $\frac{28172}{49995}$; $\frac{997}{1375}$; $\frac{541}{1084}$; $\frac{661}{728}$.

---

360. (i) In a given recurring decimal, the period may be supposed to begin at any point we please after the first repeating figure.

Thus, $15\cdot45387387\ldots = 15\cdot45\dot{3}8\dot{7} = 15\cdot453\dot{8}7\dot{3} = 15\cdot4538\dot{7}3\dot{8} =$ &c.

(ii) Sometimes the period is made to commence in the *integral* part.

Thus, $6\dot{4}\cdot2\dot{5} = 64\cdot\dot{2}5\dot{4} = 64\cdot2\dot{5}4\dot{2} =$ &c.

(iii) The number of digits in the period may be repeated as often as we please without altering the value of the decimal.

Thus, $8\cdot5\dot{4}\dot{6} = 8\cdot5\dot{4}64\dot{6} = 8\cdot5\dot{4}6464\dot{6} =$ &c.

(iv) In the conversion of a fraction to a recurring decimal, we may often shorten the work by expressing the remainder at some step as a fraction. Thus,

$\frac{1}{7} = \cdot142\frac{6}{7}$; $\therefore \frac{6}{7} = \cdot142\frac{6}{7} \times 6 = 857\frac{1}{7}$; and $\therefore \frac{1}{7} = \cdot142857\frac{1}{7} = \cdot\dot{1}4285\dot{7}$.

361. When recurring decimals have the same number of non-recurring figures and also the same number of recurring figures, they are said to be **similar**.

Thus, $\cdot34\dot{2}5\dot{8}$ and $6\cdot17\dot{8}6\dot{3}$ are *similar* recurring decimals.

362. *All recurring decimals can be made similar.*

RULE. Extend each decimal as far as the farthest non-recurring figure in any of them ; then find the L. C. M. of the numbers of figures in each period, and extend each period so many places further.

Ex. Make $4\cdot23\dot{8}$, $\cdot1\dot{2}3\dot{4}$ and $54\cdot0\dot{2}\dot{3}$ similar.

$4\cdot23\dot{8} =$	$4\cdot23 \mid \dot{8}8888\dot{8}$	Here, we see that the first term has the largest number of non-recurring figures ; *i. e.* 2 figures. So extend each decimal 2 places.
$\cdot1\dot{2}3\dot{4} =$	$\cdot12 \mid \dot{3}4234\dot{2}$	
$54\cdot0\dot{2}\dot{3} =$	$54\cdot02 \mid \dot{3}2323\dot{2}$	

The periods which consist of 1, 3, 2 figures respectively, are then extended 6 places, for 6 is the L. C. M. of 1, 2 and 3.

Examples CVII.

1. In the following recurring decimals begin the period at the fifth decimal place :—

$3\cdot\dot{2}\dot{5}$; $4\cdot\dot{7}$; $290\cdot0\dot{2}$; $\cdot3\dot{6}$; $21\cdot\dot{1}\dot{4}$; $\cdot0\dot{3}5\dot{2}$; $\cdot706\dot{5}$; $\cdot0\dot{4}6\dot{3}$; $\dot{3}\cdot4\dot{5}$.

2. Extend $\cdot5\dot{7}$, $2\cdot\dot{3}\dot{4}$ and $\cdot0\dot{6}4\dot{5}$ so that they may have the same number of figures in the period.

3. Extend $\cdot1\dot{2}\dot{3}$, $\cdot1\dot{2}3\dot{4}$ and $\cdot\dot{1}23\dot{4}$ so that they may have the same number of recurring figures.

4. Convert the following vulgar fractions into recurring decimals by the method of Art. 360 (iv) :—

$\frac{11}{13}$; $\frac{1}{14}$; $\frac{5}{17}$; $\frac{2}{19}$; $\frac{8}{28}$; $\frac{1}{29}$; $\frac{6}{31}$; $\frac{19}{53}$.

5. Make the following recurring decimals similar :—

(1) $3\cdot07\dot{6}$, $9\cdot\dot{2}4\dot{5}$, $\cdot2\dot{0}\dot{3}$. (2) $\cdot\dot{8}$, $\cdot\dot{8}\dot{7}$, $\cdot\dot{8}7\dot{6}$.

(3) $\cdot4\dot{1}\dot{4}$, $\cdot0\dot{3}5\dot{2}$, $6\cdot\dot{1}01\dot{3}$. (4) $\cdot550\dot{7}$, $\cdot04\dot{6}\dot{3}$, $1\cdot\dot{4}1\dot{3}$, $\cdot\dot{7}06\dot{5}$.

(5) $\cdot7\dot{8}5\dot{4}$, $\cdot\dot{5}\dot{9}$, $14\cdot5\dot{7}$, $\cdot00\dot{4}\dot{5}$. (6) $9\cdot70\dot{1}\dot{2}$, $4\cdot40\dot{3}$, $10\cdot8492\dot{1}3\dot{7}$, $\cdot2\dot{1}86\dot{5}$.

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363. *To find the vulgar fraction which shall be equivalent to a pure recurring decimal.*

RULE. Make the period the *numerator* of a fraction whose *denominator* shall consist of as many *nines* as there are figures in the said period ; and this reduced to its simplest terms will be the vulgar fraction required.

Ex. Convert $\cdot\dot{6}$ and $\cdot\dot{9}\dot{6}$ into equivalent vulgar fractions in their lowest terms.

(1) $\cdot\dot{6}=\frac{6}{9}=\frac{2}{3}$. (2) $\cdot\dot{9}\dot{6}=\frac{96}{99}=\frac{32}{33}$.

Proof. For the sake of conciseness, let x and y represent their values respectively ; then, we shall have

$x= \cdot6666...$ | $y= \cdot9696...$
$\therefore$ 10 times $x=6\cdot6666...$ | $\therefore$ 100 times $y=96\cdot9696...$

whence, subtracting in each case, the former from the latter, we obtain

9 times $x=6$, | 99 times $y=96$,
and $\therefore$ $x=\frac{6}{9}=\frac{2}{3}$. | and $\therefore$ $y=\frac{96}{99}=\frac{32}{33}$.

364. *To find the vulgar fraction which shall represent the value of a mixed recurring decimal.*

RULE. Make the non-recurring and the recurring parts taken *together*, diminished by the non-recurring part *alone*, the numerator of a fraction whose denominator shall consist of as many *nines* as there are recurring figures, followed by as many *ciphers* as there are non-recurring figures ; and this reduced to its lowest terms will be the vulgar fraction required.

Ex. Convert $\cdot2\dot{7}$, $\cdot24\dot{5}\dot{7}$ and $\cdot011\dot{3}\dot{6}$ into equivalent vulgar fractions in their lowest terms.

(1) $\cdot2\dot{7}=\dfrac{27-2}{90}=\dfrac{25}{90}=\dfrac{5}{18}$. (2) $\cdot24\dot{5}\dot{7}=\dfrac{2457-24}{9900}=\dfrac{2433}{9900}=\dfrac{811}{3300}$.

(3) $\cdot011\dot{3}\dot{6}=\dfrac{1136-11}{99000}=\dfrac{1125}{99000}=\dfrac{1}{88}$.

Proof. For the sake of conciseness, suppose x and y to represent the values of (1) and (2) respectively ; then, we shall have

$x = \cdot 27777\ldots\ldots$	$y = \cdot 2457575757\ldots\ldots$
$10x = 2\cdot 7777\ \ldots\ldots$	$100y = 24\cdot 57575757\ldots\ldots$
$100x = 27\cdot 7777\ \ldots\ldots$	$10000y = 2457\cdot 575757\ldots\ldots\ldots\ldots$

whence, subtracting the second line from the third in each case, we find

$90x = 27 - 2 = 25,$	$9900y = 2457 - 24 = 2433,$
$\therefore\ x = \frac{27-2}{90} = \frac{25}{90} = \frac{5}{18}.$	$\therefore\ y = \frac{2457-24}{9900} = \frac{2433}{9900} = \frac{811}{3300}.$

365. The above method is also applicable if there should be some integral figures in the decimal, but the equivalent vulgar fraction is improper. If it is required as a mixed number, we may either reduce this to mixed number or apply the method given below and thus obtain it at once in that form.

Ex. Express $2\cdot\dot{2}\dot{7}$ and $4\cdot 5\dot{8}\dot{3}$ as vulgar fractions.

(1) $2\cdot\dot{2}\dot{7} = \frac{227-2}{99} = \frac{225}{99} = \frac{25}{11} = 2\frac{3}{11}$; or $2\cdot\dot{2}\dot{7} = 2 + \cdot\dot{2}\dot{7} = 2 + \frac{27}{99} = 2\frac{3}{11}$.

(2) $4\cdot 5\dot{8}\dot{3} = \frac{4583-45}{990} = \frac{4538}{990} = \frac{2269}{495} = 4\frac{289}{495}$;

or $4\cdot 5\dot{8}\dot{3} = 4 + \cdot 5\dot{8}\dot{3} = 4 + \frac{583-5}{990} = 4 + \frac{578}{990} = 4\frac{289}{495}$.

366. It follows from the Rule that $\cdot\dot{9} = \frac{9}{9} = 1$; $\cdot 0\dot{9} = \frac{9}{90} = \frac{1}{10} = \cdot 1$. Similarly, $\cdot 06\dot{9} = \cdot 07$; $\cdot 025\dot{9} = \cdot 026$. Hence, whenever $\cdot\dot{9}$ occurs at the end of a decimal, it should be omitted, and the preceding figure increased by 1.

367. The following equivalent forms with their converses should be verified and committed to memory :—

$\frac{1}{3} = \cdot\dot{3}$; $\frac{2}{3} = \cdot\dot{6}$; $\frac{1}{6} = \cdot 1\dot{6}$; $\frac{5}{6} = \cdot 8\dot{3}$; $\frac{1}{9} = \cdot\dot{1}$; $\frac{1}{11} = \cdot\dot{0}\dot{9}$.

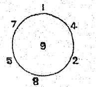
$\frac{1}{7} = \cdot\dot{1}4285\dot{7}$; $\frac{2}{7} = \cdot\dot{2}8571\dot{4}$; $\frac{3}{7} = \cdot\dot{4}2857\dot{1}$; $\frac{4}{7} = \cdot\dot{5}7142\dot{8}$;

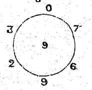
$\frac{5}{7} = \cdot\dot{7}1428\dot{5}$; $\frac{6}{7} = \cdot\dot{8}5714\dot{2}$.

Also $\frac{1}{13} = \cdot\dot{0}7692\dot{3}$; $\frac{3}{13} = \cdot\dot{2}3076\dot{9}$;

$\frac{4}{13} = \cdot\dot{3}0769\dot{2}$; $\frac{9}{13} = \cdot\dot{6}9230\dot{7}$;

$\frac{10}{13} = \cdot\dot{7}6923\dot{0}$; $\frac{12}{13} = \cdot\dot{9}2307\dot{6}$.





Students should carefully notice the decimals equivalent to vulgar fractions with denominators 7 and 13. All are pure circulating decimals, and the same digits 142857, 076923 and 153846 occur in all respectively. Now, if these digits be placed round a circle, and read off, beginning with 1, 2, 4, 5, 7, 8 ; 0, 2, 3, 6, 7, 9 and 1, 3, 4, 5, 6, 8, in turn, with the other digits in order as they stand round the circle, decimals equivalent respectively to $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$; $\frac{1}{13}$, $\frac{3}{13}$, $\frac{4}{13}$, $\frac{9}{13}$, $\frac{10}{13}$, $\frac{12}{13}$ and $\frac{2}{13}$, $\frac{5}{13}$, $\frac{6}{13}$, $\frac{7}{13}$, $\frac{8}{13}$, $\frac{11}{13}$ will be obtained.

4. Convert the following vulgar fractions into recurring decimals by the method of Art. 360 (iv) :—

$$\tfrac{11}{13}\ ;\ \tfrac{1}{14}\ ;\ \tfrac{5}{17}\ ;\ \tfrac{2}{19}\ ;\ \tfrac{8}{23}\ ;\ \tfrac{1}{29}\ ;\ \tfrac{6}{31}\ ;\ \tfrac{10}{53}.$$

5. Make the following recurring decimals similar :—

(1) $3\cdot07\dot{6}$, $9\cdot\dot{2}4\dot{5}$, $\cdot2\dot{0}\dot{3}$. (2) $\cdot\dot{8}$, $\cdot\dot{8}\dot{7}$, $\cdot\dot{8}7\dot{6}$.

(3) $\cdot4\dot{1}\dot{4}$, $\cdot0\dot{3}5\dot{2}$, $6\cdot\dot{1}01\dot{3}$. (4) $\cdot550\dot{7}$, $\cdot04\dot{6}\dot{3}$, $1\cdot\dot{4}1\dot{3}$, $\cdot\dot{7}06\dot{5}$.

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Proof. For the sake of conciseness, let x and y represent their values respectively ; then, we shall have

$$x=\cdot6666\ldots \qquad\qquad y=\cdot9696\ldots$$

$$\therefore\ 10 \text{ times } x=6\cdot6666\ldots \qquad \therefore\ 100 \text{ times } y=96\cdot9696\ldots$$

whence, subtracting in each case, the former from the latter, we obtain

$$9 \text{ times } x=6, \qquad\qquad 99 \text{ times } y=96,$$

$$\text{and } \therefore\ x=\tfrac{6}{9}=\tfrac{2}{3}. \qquad \text{and } \therefore\ y=\tfrac{96}{99}=\tfrac{32}{33}.$$

364. *To find the vulgar fraction which shall represent the value of a mixed recurring decimal.*

RULE. Make the non-recurring and the recurring parts taken *together*, diminished by the non-recurring part *alone*, the numerator of a fraction whose denominator shall consist of as many *nines* as there are recurring figures, followed by as many *ciphers* as there are non-recurring figures ; and this reduced to its lowest terms will be the vulgar fraction required.

Ex. Convert $\cdot2\dot{7}$, $\cdot24\dot{5}\dot{7}$ and $\cdot011\dot{3}\dot{6}$ into equivalent vulgar fractions in their lowest terms.

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Proof. For the sake of conciseness, suppose x and y to represent the values of (1) and (2) respectively ; then, we shall have

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whence, subtracting the second line from the third in each case, we find

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$\frac{1}{3} = \cdot\dot{3}$; $\frac{2}{3} = \cdot\dot{6}$; $\frac{1}{6} = \cdot 1\dot{6}$; $\frac{5}{6} = \cdot 8\dot{3}$; $\frac{1}{9} = \cdot\dot{1}$; $\frac{1}{11} = \cdot\dot{0}\dot{9}$.

$\frac{1}{7} = \cdot\dot{1}4285\dot{7}$; $\frac{2}{7} = \cdot\dot{2}8571\dot{4}$; $\frac{3}{7} = \cdot\dot{4}2857\dot{1}$; $\frac{4}{7} = \cdot\dot{5}7142\dot{8}$;

$\frac{5}{7} = \cdot\dot{7}1428\dot{5}$; $\frac{6}{7} = \cdot\dot{8}5714\dot{2}$.

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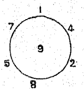

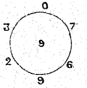

Students should carefully notice the decimals equivalent to vulgar fractions with denominators 7 and 13. All are pure circulating decimals, and the same digits 142857, 076923 and 153846 occur in all respectively. Now, if these digits be placed round a circle, and read off, beginning with 1, 2, 4, 5, 7, 8 ; 0, 2, 3, 6, 7, 9 and 1, 3, 4, 5, 6, 8, in turn, with the other digits in order as they stand round the circle, decimals equivalent respectively to $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$; $\frac{1}{13}$, $\frac{3}{13}$, $\frac{4}{13}$, $\frac{9}{13}$, $\frac{10}{13}$, $\frac{12}{13}$ and $\frac{2}{13}$, $\frac{5}{13}$, $\frac{6}{13}$, $\frac{7}{13}$, $\frac{8}{13}$, $\frac{11}{13}$ will be obtained.

Ex. Express $\cdot 382\dot{1}4285\dot{7}$ as a vulgar fraction.

$$\cdot 382\dot{1}4285\dot{7} = \frac{382\frac{1}{7}}{1000} = \frac{2675}{7000} = \frac{107}{280}.$$

Examples CVIII.

1. Convert the following recurring decimals into vulgar fractions in their lowest terms :—

(1) $\cdot\dot{5}$; $\cdot\dot{0}2\dot{7}$; $\cdot\dot{5}3\dot{4}$; $\cdot\dot{4}26\dot{3}$; $\cdot\dot{5}\dot{6}$; $\cdot\dot{2}5\dot{9}$; $\cdot\dot{7}22\dot{7}$; $\cdot\dot{6}2026\dot{8}$.

(2) $\cdot 3\dot{6}2\dot{1}$; $\cdot 47\dot{5}4\dot{3}$; $\cdot 0\dot{5}$; $\cdot 00\dot{4}9\dot{5}$; $3\cdot 5\dot{4}\dot{5}$; $\cdot 1\dot{9}\dot{6}$; $\cdot 16\dot{5}2\dot{7}$; $\cdot 541\dot{6}$.

(3) $\cdot 0\dot{4}3\dot{2}$; $21\cdot 9\dot{0}4\dot{5}$; $\cdot 6\dot{7}6190\dot{4}$; $\cdot 008\dot{4}9713\dot{3}$; $\cdot 811\dot{3}\dot{6}$; $\cdot 444\dot{1}0\dot{8}$.

(4) $\cdot 24\dot{1}25\dot{4}$; $1\cdot 0\dot{4}2857\dot{1}$; $2\cdot 6\dot{4}2857\dot{1}$; $3\cdot 8643\dot{0}1\dot{8}$; $13\cdot 94\dot{2}3076\dot{9}$.

(5) $6\cdot\dot{7}6923\dot{0}$; $50\cdot\dot{2}3076\dot{9}$; $4\cdot 15\dot{0}7692\dot{3}$; $\cdot\dot{0}1234567\dot{9}$; $\cdot 27\dot{8}4615\dot{3}$.

2. Express the following as finite decimals :—

$\cdot 0\dot{9}$; $4\cdot 36\dot{9}$; $\cdot 457\dot{9}$; $25\cdot\dot{9}9\dot{9}$; $15\cdot 8\dot{9}$; $3\cdot 78\dot{9}\dot{9}$; $59\cdot\dot{9}9\dot{9}$; $\cdot 00\dot{9}$.

3. Required the least numbers of which $\cdot 476190$ is the recurring quotient ; and find the error in the corresponding fraction when $\cdot 47619$ is taken to represent it.

4. Prove that $\frac{1}{9} = \frac{\cdot\dot{1}}{1} = \frac{\cdot\dot{2}}{2} = \frac{\cdot\dot{3}}{3} = \frac{\cdot\dot{4}}{4} = \frac{\cdot\dot{5}}{5} = \frac{\cdot\dot{6}}{6} = \frac{\cdot\dot{7}}{7} = \frac{\cdot\dot{8}}{8} = \frac{\cdot\dot{9}}{9}$.

5. Prove that $\frac{1}{11} = \frac{\cdot\dot{4}\dot{5}}{5} = \frac{\cdot\dot{5}\dot{4}}{6} = \frac{\cdot\dot{6}\dot{3}}{7} = \frac{\cdot\dot{7}\dot{2}}{8} = \frac{\cdot\dot{8}\dot{1}}{9} = \frac{\cdot\dot{9}\dot{0}}{10}$.

X. ADDITION OF RECURRING DECIMALS.

368. *To find the accurate sum of several recurring decimals.*

RULE. Write down the decimals under one another making them all similar (Art. 362), and afterwards extend two places more to make sure that we are carrying the correct figure to the last place of the second extension. Add in the usual way. Then in the sum the first extension will give *the non-recurring part*, and the second *the recurring part.*

Ex. Add together $32\cdot 01\dot{0}1\dot{1}$, $76\cdot 0\dot{9}1\dot{4}$, $5\cdot 1\dot{3}7\dot{5}$, $98\cdot 86\dot{3}$.

32·01	011101110111	01
76·09	149149149149	14
5·13	753753753753	75
98·86	333333333333	33
212·10	$\dot{2}4733734634\dot{7}$ *Ans.*	

Here, the greatest number of non-recurring figures is 2 ; so extend each decimal 2 places. The periods consist of 4, 3, 1 figures, of which the L. C. M. is 12 ; so extend each to 12 places, and two places more to ensure accuracy of the last figure retained. In the sum, 10 is the non-recurring part and 247337346347 is the recurring part.

369. *To find the sum of several recurring decimals approximately correct to a given number of decimal places.*

RULE. Set down the decimals under one another repeating the period of each 2 or 3 places more than what is required in the sum. Then add in the usual way, taking care that the last figure retained be increased by 1, if the succeeding figure be 5, or greater than 5.

Ex. Find the sum of $13\cdot\dot{5}$, $2\cdot0\dot{2}\dot{5}$, $111\cdot000\dot{4}$, $3\cdot14\dot{1}5\dot{9}$, and $2\cdot\dot{0}2\dot{4}$ *approximately correct* to 6 decimal places.

13·555555	55
2·025252	52
111·000444	44
3·141591	59
2·024024	02
131·746868	12

Here, by carrying out the decimals to 8 places, we ensure the accuracy of the first 6 places. Also in the sum, as we stop at 8 and the succeeding figure is 1, we need not increase 8 by 1.

Examples CIX.

1. Add together accurately :—

(1) $4\cdot\dot{6}+\cdot2\dot{5}\dot{1}+\cdot02\dot{5}1\dot{4}$; $\cdot4187+\cdot\dot{3}0\dot{6}+\cdot1\dot{2}\dot{5}$; $2\cdot001+\cdot1\dot{8}1\dot{8}+\cdot\dot{5}$.

(2) $\cdot1\dot{0}\dot{1}+\cdot24318\dot{3}+\cdot12\dot{3}\dot{6}+45\cdot2\dot{9}$; $3\cdot\dot{0}9\dot{0}+\cdot4\dot{0}7\dot{1}+35\cdot13+76\cdot\dot{5}$.

(3) $27\cdot642\dot{3}\dot{5}+9\cdot2642\dot{0}3\dot{7}+5\cdot49\dot{2}\dot{5}+1\cdot49\dot{8}+\cdot\dot{6}0330\dot{6}$.

(4) $4\cdot003\dot{5}\dot{8}+\cdot08\dot{3}8899\dot{4}+36\cdot1612+1\cdot00\dot{6}$.

(5) $3\cdot1416+8\cdot25\dot{1}4285\dot{7}+\cdot03\dot{4}+23\cdot257\dot{6}3\dot{5}+5\cdot456\dot{2}\dot{7}$.

2. Find the values (app. correct to 7 places of decimals) of :—

(1) $7\cdot\dot{9}\dot{0}+\cdot3416+3\cdot\dot{2}4\dot{5}+1\cdot\dot{8}$; $6\cdot\dot{1}2\dot{7}+3\cdot\dot{8}0\dot{1}+1\cdot0\dot{3}1\dot{3}+6$.

(2) $45\cdot67\dot{0}\dot{1}+41\cdot209+513\cdot3\dot{1}\dot{7}+6\cdot74\dot{9}\dot{3}+4\cdot45\dot{6}\dot{7}$.

(3) $\cdot7395+71\cdot\dot{3}+16\cdot2\dot{8}\dot{4}+162\cdot7\dot{3}5\dot{4}+18\cdot\dot{2}\dot{9}+1\cdot\dot{6}+3\cdot9\dot{7}$.

(4) $\cdot13\dot{8}+\cdot\dot{1}4285\dot{7}+2\cdot4\dot{1}\dot{8}+2\cdot0\dot{6}+42\cdot\dot{6}\dot{3}+\cdot008\dot{4}9713\dot{3}$.

XI. SUBTRACTION OF RECURRING DECIMALS.

370. The RULES given for Addition are also applicable in the Subtraction of recurring decimals.

Ex. 1. Subtract $5\cdot98\dot{7}6\dot{5}$ from $28\cdot0354\dot{7}\dot{1}$.

28·0354	71717171
5·9876	57657657
22·0478	$\dot{1}4059\dot{3}$ *Ans.*

Here, the periods have 2 and 3 figures; their L. C. M. is 6; therefore the recurring part in the difference contains 6 figures.

Ex. 2. Find (1) the difference of $2\cdot02\dot{3}4\dot{1}$ and $\cdot62\dot{8}$ *approximately correct* to 6 decimal places; (2) the complement of $\cdot6\dot{1}4285\dot{7}$.

(1)

2·023413	413
·628888	888
1·394524	525

∴ difference = 1·394525.

(2)

1·0000000	00
·6142857	14
·3857142	86

∴ complement = ·3857143.

Examples CX.

1. Find the accurate difference of :—

(1) $17.21\dot{6}\dot{3} - 12.\dot{4}\dot{6}$. (2) $.306\dot{8}\dot{4} - .23\dot{4}\dot{6}$. (3) $.36\dot{8}0\dot{1} - .24\dot{9}\dot{2}$.

(4) $15.\dot{6}2\dot{3} - 11.\dot{2}\dot{7}$. (5) $365.27\dot{3}2\dot{1} - 148.9\dot{7}$. (6) $25.4\dot{7} - 16.85\dot{7}\dot{8}$.

(7) $6.734\dot{5}\dot{9} - 3.0\dot{7}2\dot{6}$. (8) $.\dot{7}1428\dot{5} - .0011\dot{3}\dot{6}$. (9) $7.321\dot{4} - 1.\dot{2}0\dot{7}$.

2. Find the values (app. correct to 6 places of decimals) of :—

(1) $.\dot{0}\dot{4} - .00\dot{7}6923\dot{8}$. (2) $78.\dot{3}\dot{1} - 19.6\dot{8}\dot{4}$. (3) $142.3\dot{4}\dot{5} - 109.\dot{3}\dot{2}$.

(4) $314.\dot{2}90\dot{5} - 180.41\dot{6}\dot{2}$. (5) $52.\dot{8}\dot{6} - 8.3\dot{7}23\dot{5}$. (6) $3.85\dot{6}\dot{4} - 2.038\dot{7}$.

3. Find the complements of $.045\dot{6}\dot{3}$; $.0\dot{7}8\dot{9}$; $25.64\dot{2}037\dot{6}$.

4. Find the values of :—

(1) $5.\dot{7}89\dot{2} - 2.3\dot{6}\dot{8} + 17.5\dot{4} + .2105 - 12.97\dot{6}\dot{1} - 3.21\dot{5}$.

(2) $14.89\dot{7}\dot{6} - 27.\dot{3}15\dot{0} - 49.\dot{8}\dot{1} + 15.763 + 183 + 21.\dot{0}\dot{5}$.

(3) $18.\dot{7}13\dot{0} - 5.\dot{8}\dot{7} + 161.02\dot{3}\dot{5} + 21 - 8.004$.

(4) $7.\dot{5} + 12.\dot{3}\dot{0} - 59.736\dot{5} + 90.02\dot{8} - 6.12\dot{5}\dot{7} - 20.\dot{7}\dot{1}$.

XII. MULTIPLICATION OF RECURRING DECIMALS.

371. *To multiply a recurring decimal by an integer or by a terminating decimal.*

RULE. Proceed in the usual way, extending the decimal 2 or 3 places beyond the end of the period, in order to ensure the correctness of the last digit retained, and in the product point off as many decimal places as there are decimal places in both the multiplicand and multiplier. The product will also be a recurring decimal of the same kind as the multiplicand *i. e.* with a period containing the same number of digits.

Ex. 1. Multiply $37.83\dot{4}5\dot{9}$ by 7, and $37.82\dot{3}\dot{6}$ by 11.

$$\text{(1)}\quad \begin{array}{r|l} 37.83459 & 459 \\ 7 & \\ \hline \underline{264.84\dot{2}1\dot{6}} & \end{array} \qquad\qquad \text{(2)}\quad \begin{array}{r|l} 37.8236 & 36 \\ 11 & \\ \hline 416.05\dot{9}\dot{9} & \end{array} = \underline{416.06}.\ \text{(Art. 366.)}$$

Ex. 2. Multiply $6.\dot{3}91782\dot{5}$ by 6·924.

$$\begin{array}{r|l} 6.\dot{3}91782\dot{5} & 917 \\ 6.924 & \\ \hline 25\dot{5}671303\dot{3} & \\ 127\dot{8}35651\dot{1} & \\ 575\dot{2}60433\dot{3} & \\ 383\dot{5}06955\dot{5} & \end{array} \qquad\qquad \begin{array}{r|l} 2556713033 & 671 \\ 12783565181 & 356 \\ 575260433326 & 043 \\ 3835069555506 & 955 \\ \hline 44.25670\dot{2}6655\dot{5} & 025 \end{array}\ \textit{Ans.}$$

372. *To multiply one recurring decimal by another.*

RULE. Convert the given decimals into equivalent vulgar

fractions, and multiply as in Art. 270. Then reduce the resulting fraction to a decimal.

Ex. Multiply $\cdot 08\dot{9}$ by $\cdot 02\dot{8}$.

$$\cdot 08\dot{9} = \frac{89-8}{900} = \frac{81}{900} = \frac{9}{100}\ ;\ \cdot 02\dot{8} = \frac{28-2}{900} = \frac{26}{900} = \frac{13}{450}.$$

$$\therefore \text{ the product reqd.} = \frac{9}{100} \times \frac{13}{450} = \frac{13}{5000} = \frac{26}{10000} = \underline{\cdot 0026}. \quad \textit{Ans.}$$

Examples CXI.

1. Multiply :—

(1) $\cdot 3764\dot{2}$ by 9 ; $\cdot 376\dot{4}\dot{2}$ by 11 ; $\cdot 37\dot{6}4\dot{2}$ by 37 ; $\cdot 008\dot{3}7\dot{6}$ by 762.

(2) $\cdot 43224431\dot{1}\dot{8}$ by 88 ; $\cdot 785\dot{3}98\dot{1}$ by 3457 ; $6\cdot 34\dot{2}8\dot{7}$ by 5·01723.

(3) $3\cdot\dot{5}$ by ·8 ; $3\cdot\dot{9}\dot{1}$ by ·022 ; $3\cdot 54\dot{2}6\dot{8}$ by ·144 ; $15\cdot\dot{0}7\dot{3}$ by 2·4.

(4) $2\cdot 3\dot{8}5714\dot{2}$ by 5·6 ; $27\cdot 38\dot{4}4\dot{3}$ by 26·7 ; $9\cdot 385\dot{0}78\dot{7}$ by 7·659.

2. Find the values of :—

(1) $4\cdot\dot{8} \times \cdot\dot{2}\dot{4}$; $7\cdot\dot{6}\dot{3} \times 8\cdot 8\dot{3}$; $19\cdot 7\dot{2} \times 29\cdot\dot{4}\dot{5}$; $7\cdot\dot{5} \times \cdot 015\dot{9}\dot{6}$.

(2) $6\cdot\dot{3}\dot{6} \times \cdot\dot{5}7142\dot{8}$; $1\cdot\dot{1}\dot{8} \times \cdot\dot{5}3846\dot{1}$; $5598\cdot 9\dot{2}4\dot{3} \times 8\cdot 24\dot{7}$.

(3) $2\cdot 2\dot{7} \times \cdot 24\dot{9}$; $\cdot 07\dot{3} \times 2\cdot\dot{7}\dot{2}$; $49\cdot\dot{3} \times \cdot 2\dot{9}95\dot{4}$; $\cdot 12\dot{8}3\dot{7} \times 2\cdot 52\dot{2}\dot{7}$.

(4) $\cdot 0\dot{0}2\dot{1} \times 48\cdot 92\dot{6}$; $\cdot\dot{4}2857\dot{1} \times \cdot\dot{3}$ of $3\cdot\dot{8}$; $44\cdot 20\dot{6}4\dot{5} \times 1\cdot 582\dot{3}70\dot{7}$.

XIII. DIVISION OF RECURRING DECIMALS.

373 *To divide a recurring decimal by a whole number or by a terminating decimal.*

RULE. Proceed as in ordinary division, bringing down the digits of the period in succession. The quotient will also be a recurring decimal.

Ex. Divide $8\cdot 985\dot{4}$ by 12 and $\cdot 655\dot{9}90\dot{3}$ by 48·76.

(1)
```
12)8 98544444...(·748787̇03̇
   84
    58
    48        84
    105       84
     96        44
      94       36
      84        8
      104
       96
```
$\therefore$ the quotient $= \underline{\cdot 74878\dot{7}0\dot{3}}$

(2)
```
4876)65·5990399...(134534...
     4876
     16839
     14628
      22110        16839
      19504        14628
       26063        22119
       24380        19504
        16839        2615
```
$\therefore$ the quotient $= \underline{\cdot 0134534\ldots}$

374. *To divide one recurring decimal by another.*

RULE. Convert the given decimals into vulgar fractions, and divide as in Art. 274. Then reduce the resulting fraction to a decimal.

Ex. Divide $1\cdot1\dot{3}$ by $\cdot00\dot{0}13\dot{2}$.

$$1\cdot1\dot{3}=\frac{113-11}{90}=\frac{102}{90}=\frac{17}{15}\ ;\ \cdot00\dot{0}13\dot{2}=\frac{132}{999900}=\frac{1}{7575}.$$

$$\therefore \text{ the quotient reqd.}=\frac{17}{15}\div\frac{1}{7575}=\frac{17}{15}\times\frac{7575}{1}=\underline{8585}.\quad \textit{Ans.}$$

Examples CXII.

1. Divide :—

(1) $\cdot\dot{3}$ by 5, by 7 ; $37\cdot\dot{0}8\dot{7}$ by 5, by 45 ; $\cdot332\dot{5}$ by 125 ; $\cdot\dot{4}6153\dot{8}$ by 30.

(2) $3\cdot4579\dot{5}\dot{4}$ by 8 ; $37\cdot635\dot{8}4\dot{2}$ by 7 ; $539\cdot634\dot{3}\dot{6}$ by 112.

(3) $235\cdot4\dot{7}$ by 24×20 ; $\cdot74\dot{7}\dot{6}$ by $\cdot07$; $9\cdot\dot{4}\dot{0}$ by $1\cdot5$; $3\cdot\dot{6}$ by $2\cdot4$.

(4) $\cdot02834\dot{2}01\dot{2}$ by $14\cdot156$; $20\cdot13\dot{9}7\dot{2}$ by $42\cdot1$; $\cdot101\dot{0}\dot{1}$ by $\cdot00036$.

2. Find the values of :—

(1) $3\cdot\dot{8}\div2\cdot\dot{7}\dot{3}$; $1\cdot\dot{9}\dot{0}\div\cdot58\dot{3}$; $60\cdot\dot{4}\dot{5}\div7\cdot3\dot{8}$; $11\cdot8\dot{3}\div\cdot24\dot{9}$; $\cdot\dot{3}\dot{7}\div\cdot\dot{1}4\dot{8}$.

(2) $4\cdot0\dot{3}\div\cdot\dot{1}40\dot{7}$; $\cdot012\dot{3}\dot{6}\div\cdot0\dot{5}\dot{1}$; $9\cdot5\dot{3}\div3\cdot208\dot{3}$; $6\cdot\dot{8}9\dot{1}\div15\cdot\dot{4}\dot{5}$.

(3) $\cdot89\dot{1}\div1\cdot\dot{2}\dot{9}$; $\cdot0\dot{0}5\dot{7}\div\cdot21\dot{3}$; $\cdot12\dot{5}\div\cdot25\dot{1}$; $7\cdot3\dot{9}\div\cdot07\dot{9}$.

(4) $411\cdot3\dot{5}1\dot{9}\div19\cdot5\dot{8}8\dot{1}$; $14\cdot\dot{4}7619\dot{0}\div2\cdot15\dot{9}\dot{0}$; $77\cdot670\dot{2}\dot{7}\div9\cdot48\dot{6}$.

XIV. SIMPLIFICATION OF DECIMAL FRACTIONS.

Ex. 1. Simplify $\dfrac{\cdot13\times\cdot14\times\cdot01-\cdot12\times\cdot14\times\cdot02+\cdot12\times\cdot13\times\cdot01}{\cdot01\times\cdot2\times\cdot01}$

$$\text{The given fraction}=\frac{\cdot000182-\cdot000336+\cdot000156}{\cdot00002}=\frac{\cdot000002}{\cdot00002}=\underline{\cdot1}.\ \textit{Ans.}$$

Ex. 2. Find the value of $\dfrac{2\cdot8 \text{ of } 2\cdot\dot{2}\dot{7}}{1\cdot1\dot{3}\dot{6}}+\dfrac{4\cdot\dot{4}-2\cdot8\dot{3}}{1\cdot\dot{6}+2\cdot\dot{6}2\dot{9}}$ of $\dfrac{6\cdot8 \text{ of } 3}{2\cdot25}$.

$$\text{The value}=\frac{2\frac{4}{5}\times2\frac{3}{11}}{1\frac{135}{990}}+\frac{4\cdot4444\ldots-2\cdot83333}{1\cdot6666\ldots+2\cdot629629\ldots}\text{ of }\frac{20\cdot4}{2\cdot25}$$

$$=\frac{14}{5}\times\frac{25}{11}\times\frac{990}{1125}+\frac{1\cdot6\dot{1}}{4\cdot\dot{2}9\dot{6}}\times\frac{2040}{225}=\frac{28}{5}+\frac{\frac{145}{90}}{\frac{4292}{999}}\times\frac{136}{15}$$

$$=\frac{28}{5}+\frac{145}{90}\times\frac{999}{4292}\times\frac{136}{15}=\frac{28}{5}+\frac{17}{5}=\frac{45}{5}=\underline{9}.\quad \textit{Ans.}$$

Examples CXIII.

Simplify :—

1. $1\cdot7\dot{2}$ of $\cdot\dot{2}7\dot{6}$ of 15.

2. $1\cdot8\dot{3}$ of $\cdot9\dot{5}\dot{4}$ of $\cdot\dot{4}2857\dot{1}$ of $2\cdot25$.

3. $\cdot\dot{6}\dot{5}$ of $4\cdot\dot{1}\dot{1}$ of $\frac{3\frac{3}{5}}{13}$ of $2\cdot\dot{4}3\dot{2}$.

4. $\frac{2\frac{3}{5}}{3\frac{1}{4}}$ of $\cdot000\dot{6}$ of $\frac{\cdot\dot{4}\dot{5}}{\cdot0024}$.

5. $\frac{\cdot064+12\cdot25}{\cdot9375}$.

6. $\frac{13\cdot5+\cdot078-\cdot003}{\cdot005}$.

7. $\frac{\cdot011\times133\cdot1-\cdot723\times\cdot00723}{1\cdot1377}$.

8. $\frac{5\cdot1183}{\cdot0070\dot{5}}$ of $11\cdot\dot{1}$ of $\cdot2\dot{9}$ of $\cdot1\dot{1}\dot{7}$.

9. $\frac{\cdot12(\cdot02\times\cdot03-\cdot04\times\cdot01)+\cdot16\times\cdot21}{\cdot1\times\cdot023\times\cdot01}$.

10. $\frac{2\cdot5+1\cdot25-2\cdot125}{3\cdot75\div2\cdot\dot{3}-4\cdot25}$.

11. $\frac{\cdot0\dot{3}-\cdot\dot{0}\dot{3}}{\cdot12\dot{3}}$.

12. $\frac{1+5\cdot4\times6\cdot4}{1+2\cdot\dot{3}\times3\cdot\dot{3}}$.

13. $\frac{\cdot005}{\frac{2}{3}\text{ of }13\frac{1}{2}}$ of $\frac{26\cdot25}{\frac{3}{4}\text{ of }2\cdot75}$.

14. $\left(37+\frac{3\cdot7\dot{0}3\dot{7}}{100}\right)\times\cdot54$.

15. $\frac{\cdot\dot{4}2857\dot{1}}{\cdot01\dot{7}1428\dot{5}}$.

16. $\frac{7\frac{1}{2}\times3\frac{3}{5}}{\cdot75\times36\cdot6}+\frac{2\cdot\dot{3}\text{ of }15}{2\frac{1}{3}\text{ of }3\frac{3}{4}}+\frac{7\cdot25}{11\frac{3}{5}}$.

17. $\frac{\cdot04275}{3\cdot0\dot{5}}\times\frac{4\cdot21\dot{6}}{\cdot\dot{3}4\dot{2}}\times\frac{2\cdot\dot{7}}{1\cdot5318}$.

18. $\frac{\cdot125}{100}-\frac{\cdot0625}{25}-2\cdot25-\frac{\cdot005\times1\cdot25}{2\cdot5}+3\cdot1-\frac{\cdot5}{1000}$.

19. $\frac{\cdot\dot{8}5714\dot{2}+\cdot\dot{1}4285\dot{7}}{\cdot\dot{5}7142\dot{8}-\cdot\dot{4}2857\dot{1}}$.

20. $\frac{8\cdot\dot{5}7142\dot{8}\times1\cdot7}{2\frac{3}{5}\text{ of }1\cdot\dot{2}8571\dot{4}}\times\frac{\cdot21\dot{6}\text{ of }\cdot625}{48}$.

21. $\frac{2\cdot\dot{6}\text{ of }2\cdot8\dot{3}}{6\cdot\dot{2}\text{ of }\cdot\dot{8}5714\dot{2}}+\frac{4\frac{2}{3}\text{ of }4\cdot0\dot{3}\dot{6}}{3\cdot75\text{ of }1\cdot\dot{7}}$.

22. $\frac{2\cdot375}{3\cdot1\dot{6}}$ of $\frac{4\cdot\dot{4}}{\cdot0625}\div\frac{8\cdot\dot{8}}{7}$ of $\frac{16}{5\cdot625}$.

23. $\frac{4\cdot\dot{2}-3\cdot1\dot{4}}{1\cdot\dot{3}+2\cdot\dot{1}0\dot{2}}$ of $\frac{1\cdot3\text{ of }4}{\cdot37\text{ of }8\cdot\dot{8}\dot{1}}$.

24. $\frac{\cdot044\times2\cdot1}{\cdot000035}\div\frac{3\cdot\dot{0}7692\dot{3}}{2\cdot\dot{3}\times5\cdot\dot{6}}$.

25. $\frac{3\cdot30208\dot{3}}{16\cdot51041\dot{6}}+\frac{6\cdot\dot{6}\times\cdot375}{1\frac{6}{7}\text{ of }\cdot\dot{5}3846\dot{1}\text{ of }\frac{3}{4}}+\frac{2\cdot\dot{7}\dot{2}}{11\cdot\dot{0}\dot{9}}$.

26. $\frac{\cdot1\times\cdot1\times\cdot1+\cdot01\times\cdot01\times\cdot01}{\cdot2\times\cdot2\times\cdot2+\cdot02\times\cdot02\times\cdot02}$.

27. $\frac{\cdot375\times\cdot375-\cdot025\times\cdot025}{\cdot375-\cdot025}$.

28. $\frac{\cdot02\times\cdot9\times\cdot15-\cdot14\times\cdot06\times\cdot03+\cdot13\times\cdot01\times\cdot04}{\cdot05\times\cdot04\times\cdot03}$.

29. $\cdot\dot{6}$ of $3\cdot3$ of $\frac{1\cdot75}{2\cdot625}$ of $17+\cdot4$ of $5\cdot75-\frac{1\cdot\dot{7}1428\dot{5}}{2\cdot\dot{0}9523\dot{8}}$.

30. $\frac{2\cdot375}{3\cdot1\dot{6}}$ of $\frac{4\frac{4}{5}}{\cdot0625}\div\frac{8\cdot\dot{8}}{5\frac{5}{8}}$ of $\cdot\dot{5}7142\dot{8}-\left\{\frac{2\cdot8\text{ of }2\frac{3}{11}}{1\cdot1\dot{3}\dot{6}}-4\frac{8}{15}\text{ of }\frac{4\cdot\dot{4}-2\cdot8\dot{3}}{1\frac{2}{3}+2\cdot\dot{6}2\dot{9}}\right\}$.

XV. REDUCTION OF DECIMALS.

375. A general view having now been taken of decimals, we proceed to show how they may be made to change their denominations when they are considered as belonging to a particular unit; and in what ways they may be adapted to the particular computations in which they are most frequently employed.

376. Reduction of Decimals can conveniently be classed under the two following heads :—

(1) To reduce a **decimal** of one denomination to a lower denomination : and conversely.

(2) To reduce a quantity of one denomination to a **decimal** of a higher denomination.

377. Case I. To reduce a decimal of one denomination to a lower denomination. (**Descending Reduction**).

RULE. Multiply the decimal of the given denomination by the number which connects the lower denomination with one (or unit) of the given denomination.

Ex. Reduce *Rs.*7·15 to *pies*, and ·045 of £7 to *farthings*.

(1) *Rs.*7·15
16
*a.*114·40
12
*p.*1372·8
∴ the reqd. result = 1372·8*p.*

(2) £7
·045
£·315
20
*s.*6·300
12
*d.*75·6
4
*q.*302·4
∴ the reqd. result = 302·4*q.*

378. Case II. To reduce a quantity of one denomination to a decimal of a higher denomination. (**Ascending Reduction**).

RULE. Divide the number of the given denomination by the number which connects that denomination with one (or unit) of the higher denomination.

Ex. Reduce 3333 pies to the decimal of a *rupee*, and 21¾ grs. to the decimal of an *oz. Troy*.

(1) 12)3333*p.*
16) 277·75*a.*
Rs. 17·359375
∴ the reqd. decimal = *Rs.*17·359375.

(2) 24 { 8)21·75 grs.
3) 2·71875
20) ·90625 dwt.
·0453125 oz.
∴ the reqd. decimal = ·0453125 oz.

379. Sometimes we employ both the *descending* and the *ascending* process in reducing a decimal of one denomination to a decimal of another denomination.

Ex. Reduce ·78936 of a guinea to the decimal of £1.

·78936 gui.
21
20) 16·57656*s.*
£·828828 ∴ the reqd. decimal = £·828828.

Examples CXIV.

Reduce :—

1. £·02375 ; £·00375 ; £3·5478 ; £·00625 ; £·28125 to *pence*.

2. ·03125*s.* ; £·8947916̇ ; ·001 guinea ; £·47083̇ ; £·383̇ to *farthings*.

3. *Rs.*5·00625 ; *Rs.*2·76543 ; ·775625 of *Rs.*5 ; *Rs.*3·049 to *pies*.

4. *Re.*·972916 *Re.*·40972̇ ; *Re.*·68125 ; *Re.*·634375 ; *Re.*·3405̇ to *pice*

5. ·7859 cwt. to *ounces* ; 4·34954̇ miles to *yards* ; ·549675 days. to *seconds* ; 2·5384375 of a day to *seconds*.

6. ·6197916̇lb. Troy to *grains* ; ·678571428̇ week to *minutes*.

7. 3·6874 acres to *sq. yds.* ; ·0475 gallon to *pints* ; 2·274025 mds. to *chataks* . ·825 of a lea. to *yards*.

8. 8 4*d.* ; ·335*s.* ; 6·375*d.* ; ·4068*q.* to the decimal of £1.

9. 37·9872 sec. to the dec. of a *day* ; 420·8138 sq. yds. to the decimal of an *acre* ; 2·25 of 3·5 ac. to *poles*.

10. 47·733 lbs. to the dec. of a *ton* ; 1 oz. Avoir. to the dec. of 1 *oz. Troy* ; 3·6̇ cwt. to the dec. of a *ton*.

11. £·625 to the decimal of a guinea, and of half-a-guinea.

12. 527·3994 yds. to the dec. of a *mile* ; ·54375 lbs. Troy to *ounces Avoir.* ; 1 oz. to the dec. of a *cwt.*

---

380. The preceding two cases of Art. 376 enable us

(i) To reduce a decimal of one denomination to a **compound quantity** of lower denominations ; and

(ii) To reduce a **compound quantity** to a decimal of a higher denomination.

381. Case I. *To reduce a decimal of one denomination to a compound quantity of lower denominations.*

RULE. Multiply the decimal by the numbers which connect the successive denominations in order ; and the integral parts of the products *taken out*, as they occur, will be the value required.

Ex. 1. Find the values of *Rs.*3·46875 and £5·6125.

(1) *Rs.*3·46875	(2) £5·6125
16	20
*a.*7·50000	*s.* 12·2500
12	12
*p.*6·0	*d.*3·00
The reqd. value = *Rs.*3. 7*a.* 6*p.*	The reqd. value = £5. 12*s.* 3*d.*

Ex. 2. Find the values of 4·215 of *Rs.*7 and 31·258$\dot{3}$ of £2.

(1)
4·215
7
*Rs.*29·505
16
*a.*8·080
12
*p.*0·96
The reqd. value = *Rs.*29. 8*a.* 0·96*p.*

(2)
31·258$\dot{3}$
2
£62·5166...
20
*s.*10·3333...
12
*d.*3·9999...
The reqd. value = £62. 10*s.* 4*d.*

382. Case II. To reduce a compound quantity to a decimal of a higher denomination.

RULE. Divide the lowest denomination by the number which connects it with the next, and to the left of the quotient affix the number of this denomination; and continue the process till the required denomination is obtained.

Ex. 1. Express *Rs.*5. 1*a.* 6*p.* as the decimal of *Re.* 1, and £3. 18*s.* 11¼*d.* as the decimal of £1.

(1)
12)6*p.*
16)1·5*a.*
*Rs.*5·09375
The reqd. decimal = *Rs.*5·09375.

(2)
4) 1*q.*
12)11·25*d.*
20)18·9375*s.*
£3·946875
The reqd. decimal = £3·946875.

Ex. 2. Reduce 7 fur. 25 po. to the decimal of a *mile*, and 14⅖oz Avoir. to the decimal of 1 *oz. Troy.*

(1)
40)25 po.
8) 7·625 fur.
·953125 mi.
The reqd. decimal = ·953125 mi.

(2)
5) 2·
16)14·4 oz.
·9 lb.
7000
6300 grs.

24 {8)6300 grs.
  {3) 787·5
20) 262·5 dwts.
13·125 oz. Troy.
The reqd. decimal = 13·125 oz. Troy.

Examples CXV.

1. Find the values of :—

(1) *Rs.*5·07125 ; *Rs.*80·075 ; ·01$\dot{6}$ of a rupee ; 30·3$\dot{6}$ of ·75 of *Rs.*10.
(2) ·45 of £1 ; ·16875 of £3 ; 2·36875 of £6 ; £·5675 ; £·0484 ; £·$\dot{7}$.
(3) ·340625 of £1 ; ·615 of 1*s.* ; ·48$\dot{3}\dot{5}$ of £1 ; £5·6125 ; ·4375 of £1.
(4) ·375 of a guinea ; 1·025 of a guinea ; ·76354$\dot{1}\dot{6}$ of £1 ; ·458$\dot{3}$ of 1*s.*
(5) ·375 of a cwt. ; ·6875 of a yard ; 13·3375 acres ; ·655 of a day.
(6) *Rs.*5·798958$\dot{3}$; ·871$\dot{6}$ of a ton ; 2·5384375 days ; 22·25 of 17 half-crs.
(7) ·0000355113$\dot{6}\dot{3}$ mile ; ·10$\dot{7}$1428$\dot{5}$ of a cwt. ; ·09375 of an acre.
(8) ·00625 of 1 md. ; ·013$\dot{8}$ of 3·5 moidores ; 3·2$\dot{3}$ of 1½ acres.

2. Reduce :—

(1) $5\frac{1}{4}d.$; $\frac{3}{4}d.$; 8*s.* $11\frac{1}{4}d.$; 1*s.* $3\frac{1}{4}d.$; £1. 14*s.* $10\frac{1}{2}d.$ to the decimal of 1*s.*

(2) 12*s.* $6\frac{3}{4}d.$; 15*s.* $9\frac{3}{4}d.$; 17*s.* $0\frac{3}{4}d$; $1\frac{3}{4}d.$; £2. 15*s.* $9\frac{3}{4}d.$ to the dec. of £1.

(3) 7*a.* 6*p.* ; 8*a.* 3*p.* ; 13*a.* $6\frac{3}{4}p.$; *Rs.*53. 13*a.* 8*p.* to the decimal of 1 *Re.*

(4) 18*s.* $11\frac{1}{2}d.$ to the dec. of a guinea ; $4\frac{3}{4}$ guineas to the dec. of £50.

(5) *Rs.*2. 13*a.* 10*p.* to the dec. of *Rs.*5 ; *Rs.*35. 14*a.* 6*p.* to the dec. of *Rs.*25 ; *Rs.*6 6*a.* 8*p.* to the dec. of *Rs.*10. 8*a.*

(6) 12*s.* $6\frac{3}{4}d.$ to the decimal of £1, of £100 and of £·001.

(7) 10 oz. 11 dwts. $21\frac{1}{2}$grs. to the dec. of 1 lb. *Troy* ; and of 1 lb. *Avoir.*

(8) 9 cwt. 13 lbs. 4 oz. 3·84drs. to the dec. of a *ton* ; 4 cwt. 1qr. $10\frac{1}{4}$ lbs. to the dec. of 1cwt.; 17cwt. 3qrs. 17 lbs. 8·7 oz. to the dec. of a *ton.*

(9) 12hrs. 55min. $23\frac{1}{11}$sec. to the dec. of a *day* ; 5 days 12hrs. 25min. 37·92 sec. to the dec. of a *week* ; 1 cwt. 3 qrs. 4 lbs. to the dec. of a *ton.*

(10) 11yds. ; 3 fur. 66yds. and 6yds. 2 ft. $7\frac{1}{2}$in. each to the dec. of a *mile.*

(11) ·002 of 2·75 pag. to the dec. of *Rs.*3·46 ; 4 mds. 8sr. $1\frac{1}{2}$ch. to the dec. of 14mds. ; 3 sr. 4 ch. 2 to. 11 m. to the dec. of 1 *md.*

(12) 6 fur. 100 yds. 2 ft. 3 in. to the dec. of a *mile* ; 3 ro. 31 po. $16\frac{1}{2}$yds. to the dec. of an *acre* ; 13cub. ft. 1323 cub. in. to the dec. of a *cub. yard.*

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383. *To multiply or divide a quantity by a decimal, or to find the value of a decimal of a quantity.*

RULE. (1) Express the given quantity, when necessary, as a simple quantity, and perform the required operation ; or (2) reduce the decimal to a fraction in its lowest terms, and proceed as in fractions. (Arts. 302 and 303.)

Note. When the decimal is *recurring* and the value is required to be *exact*, the second method is advantageous.

Ex 1. Find the value of ·432 of *Rs.*6. 10*a.* 8*p.*

(1) *Rs.*6. 10*a.* 8*p.* × ·432 = 1280*p.* × ·432 = 552·96*p.* = *Rs.*2. 14*a.* 0·96*p.*

(2) ·432 of *Rs.*6. 10*a.* 8*p.* = $\frac{432}{1000}$ of *Rs.*6. 10*a.* 8*p.* = $\frac{54}{125}$ of *Rs.*6. 10*a.* 8*p.*
= $\frac{1}{125}$ of *Rs.*360 = *Rs.*$2\frac{22}{25}$ = *Rs.*2. 14*a.* 0·96*p.*

Ex. 2. Find the value of $4·23\dot{4}\dot{5}$ of £2. 15*s.*

$4·23\dot{4}\dot{5}$ of £2. 15*s.* = $4\frac{2322}{9900}$ of £2. 15*s.* = $4\frac{129}{550}$ of £2. 15*s.*
= £2. 15*s.* × 4 + £2. 15*s.* × $\frac{129}{550}$ = £11 + 55*s.* × $\frac{129}{550}$
= £11 + 12·9*s.* = £11. 12*s.* 10·8*d.*

Ex 3. Find the value of $3\cdot\dot{3}$ of $\dfrac{4\cdot\dot{4}}{\cdot 7\dot{3}\dot{5}}$ of 1 sq. ft. 3 sq. in.

Value required $= 3\frac{3}{9}$ of $\dfrac{4\frac{4}{9}}{\frac{735}{999}}$ of $1\frac{3}{144}$ sq. ft. $= 3\frac{1}{3} \times 4\frac{4}{9} \times \frac{999}{735} \times \frac{147}{144}$ sq. ft.

$= \frac{10}{3} \times \frac{40}{9} \times \frac{999}{735} \times \frac{49}{48}$ sq. ft. $= \frac{370}{18}$ sq. ft.

$= 20\frac{5}{9}$ sq. ft. $=$ <u>20 sq. ft. 80 sq. in.</u>

Ex. 4. Find the value of $2\cdot 8680\dot{5}$ of *Re.*1. 8*a.* + $\cdot 8\dot{3}$ of *Rs.*2 − 1·8. of *Rs.*2. 8*a.*

$2\cdot 8680\dot{5}$ of *Re.* 1. 8*a.* $= 2\frac{78125}{90000}$ of *Re.*1. 8*a.* $= 2\frac{125}{144}$ of *Re.*1. 8*a.*

$=$ *Re.*1. 8*a.* × 2 + *Re.*$\frac{3}{2} \times \frac{125}{144}$ = *Rs.*3 + *Re.*1. 4*a.* 10*p.*

$=$ *Rs.*4. 4*a.* 10*p.*

$\cdot 8\dot{3}$ of *Rs.*2 $= \frac{75}{90}$ of *Rs.*2 $= \frac{5}{6}$ of *Rs.*2 = *Re.*$\frac{5}{3}$ = *Re.*1. 10*a.* 8*p.*

1·8 of *Rs.*2. 8*a.* $= \frac{18}{10}$ of *Rs.*$2\frac{1}{2} = \frac{9}{5} \times$ *Rs.*$\frac{5}{2}$ = *Rs.*$\frac{9}{2}$ = *Rs.*4. 8*a.*

∴ value required = *Rs.*4. 4*a.* 10*p.* + *Re.*1. 10*a.* 8*p.* − *Rs.*4. 8*a.*

$=$ <u>*Re.*1. 7*a.* 6*p.*</u>

Examples CXVI.

1. Find the values of :—

(1) 1·85 of *Re.*1. 10*a.* 8*p.* ; 2·375 of *Rs.*6. 10*a.* 8*p.* ; ·775625 of *Rs.*50.
(2) ·925 of 6*s.* 8*d.* ; ·7365 of 6*s.* 8*d.* ; ·59375 of 19*s.* 4*d.* ; ·78125 of £6.
(3) ·00390625 of £1. 12*s.* ; ·0474609375 of £10. 13*s.* 4*d.* ; ·07 of £2. 10*s.*
(4) 6·156510416 of *Rs.*40 ; ·001953125 of *Rs.*400 ; 1·46875 of 3 bighas.
(5) ·046875 of 1 md. 8 sr. ; 4·106 of 4 mds. 32 sr. 8 ch. ; ·045 of 4 miles.
(6) ·7385 of 13*s.* 4*d.* ; 1·625 of 2 tons 4 cwt. ; 27·138 of 2 mi. 450 yds.
(7) ·3792 of £3. 18*s.* $1\frac{1}{2}$*d.* ; ·0013 of £3. 17*s.* $10\frac{1}{2}$*a.* ; ·365 of £1. 0*s.* 10*d.*
(8) £3. 14*s.* $6\frac{3}{4}$*d.* × 2·46875 ; £874. 13*s.* 4*d.* × 1·875.
(9) £1205. 6*s.* 8*d.* ÷ 51·2 ; £503. 12*s.* $6\frac{3}{4}$*d.* ÷ 26·312.
(10) *Rs.*47. 13*a.* × 24·5775 ; *Rs.*149. 5*a.* × 345·67 ; *Rs.*239. 9*a.* 6*p.* ÷ 13·53.
(11) ·2775 of 1 sq.yd. 3 ft. 72 in. ; ·9765625 of 2 tons 18 cwt. 3 qrs. 14lbs.
(12) 225 days 14 hrs. 36 min. ÷ 8·71846 ; 27lbs. 13oz. 15 drs. × ·4352.

2. Find the values of :—

(1) $\cdot\dot{5}$ of *Rs.*2. 6*a.* 4·8*p.* ; $3\cdot\dot{0}\dot{6}$ of *Rs.*2. 1*a.* ; $\cdot\dot{3}$ of *Rs.*3. 8*a.* 4*p.*
(2) $\cdot\dot{7}1428\dot{5}$ of 10*s.* 6*d.* ; $\cdot\dot{4}2\dot{8}$ of £3. 8*s.* ; $3\cdot 958\dot{3}$ of *Rs.*8.
(3) $\cdot 3\dot{4}8\dot{1}$ of £4. 18*s.* 8*d.* ; $4\cdot\dot{0}09\dot{9}$ of *Rs.*16. 13*a.* 4*p.* ; $\cdot 00015\dot{7}4\dot{0}$ of *Rs.*81.

(4) $\cdot5\dot{3}57142\dot{8}$ of 2 cwt. 3qrs. $17\frac{1}{2}$lbs.; $13\cdot26\dot{3}79\dot{8}$ of 3 mi. 7 fur. $22\frac{1}{2}$po.

(5) $\cdot208\dot{3}$ of $\cdot3\dot{4}2857\dot{1}$ of $2\frac{1}{2}$ cwt.; $1\cdot91\dot{6}$ of 8*s.*; $3\cdot0\dot{7}$ of 11*s.* 3*d.*; $3\cdot24\dot{2}$ of $7\frac{1}{2}$ bighas; $3\cdot\dot{6}$ of 4 qrs. 4 bus.

(6) $\cdot\dot{8}4615\dot{3}$ of $\cdot0\dot{8}\dot{1}$ of *Rs.*6. 8*a.*; $\cdot\dot{0}\dot{1}\times\cdot\dot{1}0\dot{1}$ of *Rs.*749. 4*a.*; $\cdot\dot{1}\times\cdot\dot{4}\dot{7}$ of *Rs.*3601. 2*a.*; $\cdot4\dot{6}9\dot{4}$ of *Rs.*5. 3*a.* 2*p.*

3. What is the value of $\cdot\dot{2}3\dot{4}$, when the unit is worth £20, and the worth of $\cdot\dot{3}$ of $\cdot\dot{3}$, when the unit is valued at *Rs.*108?

4. What is the value of $\cdot58\dot{3}$, when the unit is 3 oz. 5 dwts.?

5. Find the respective values of :—

(1) ·45 of *Rs.*35 + ·75 of *Rs.*2. 5*a.* 4*p.* + 3·245 of *Re.*1. 10*a.* 8*p.*

(2) 8·71875 of 5*a.* 4*p.* + 1·146875 of *Rs.*3. 5*a.* 4*p.* − ·0625 of *Rs.*10. 8*a.*

(3) ·375 of a guinea + ·1875 of a crown + ·3 of 7*s.* 6*d.* − ·875 of 2*d.*

(4) ·5*s.* + ·7 of a crown + £·125; £·6 + ·3125*s.* + $\cdot\dot{2}$ of a guinea.

(5) 1·125 of *Rs.*13. 8*a.* + 44·045 of 7*a.* 6*p.* − ·0625 of *Rs.*3. 12*a.* + 1·025 of 2*a.* 6*p.* − 2·56 of *Rs.*5. 7*a.* 6*p.*

(6) ·175 of 28 mds. + ·195 of 1 md. 16 sr. + ·145 of 14 sr. + ·15 of 8 ch.

(7) ·625 of £1. 1*s.* + $\cdot\dot{5}\dot{4}$ of 8*s.* 3*d.* + $\cdot0\dot{2}\dot{7}$ of £2. 15*s.*

(8) $\cdot\dot{7}$ of 7*s.* 6*d.* − $\cdot\dot{8}\dot{4}$ of 16*s.* 6*d.* + $\cdot9\dot{2}\dot{7}$ of £2. 10*s.* 5*d.*

(9) $\cdot\dot{2}8571\dot{4}$ of £30 + £$6\cdot\dot{8}5714\dot{2}$ + $\cdot\dot{6}$ of $\cdot\dot{7}1428\dot{5}$ of £·6 + $1\cdot\dot{3}$ of $\cdot\dot{4}2857\dot{1}$*s.*

(10) $\cdot\dot{8}5714\dot{2}$ of 2·0625 tons + $\cdot\dot{5}7142\dot{8}$ of 3·375 cwt. + $\cdot\dot{7}1428\dot{5}$ of 1·25 qrs. + $\cdot\dot{2}8571\dot{4}$ of 10·5 lbs.

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384. *To find what decimal one compound concrete quantity is of any other of the same kind.*

RULE. Express the first quantity as the fraction of the second, as in Art. 306, and then reduce this fraction to a decimal.

Ex. 1. Reduce 3*s.* $11\frac{1}{4}$*d.* to the decimal of £1. 19*s.* $4\frac{1}{2}$*d.*

3*s.* $11\frac{1}{4}$*d.* = $47\frac{1}{4}$*d.*, and £1. 19*s.* $4\frac{1}{2}$*d.* = $472\frac{1}{2}$*d.*

$47\frac{1}{4}d. \div 472\frac{1}{2}d. = \frac{189}{4} \times \frac{2}{945} = \frac{1}{10}$; ∴ the reqd. decimal = ·1. *Ans.*

Ex. 2. Express $\frac{3}{8}$ of *Rs.*3. 12*a.* + ·625 of *Rs.*5 − $\cdot5\dot{4}\dot{5}$ of *Rs.*4. 9*a.* 4*p.* as the decimal of *Rs.*100.

$\frac{3}{8}$ of *Rs.*3. 12*a.* = $\frac{3}{8} \times 60a. = 22\frac{1}{2}a.$ = *Re.*1. 6*a.* 6*p.*

·625 of *Rs.*5 = *Rs.*3·125 = *Rs.*3. 2*a.*

$\cdot5\dot{4}\dot{5}$ of *Rs.*4. 9*a.* 4*p.* = $\frac{540}{990}$ of $73\frac{1}{3}a. = \frac{6}{11} \times \frac{220}{3}a.$ = 40*a.* = *Rs.*2. 8*a.*

$\therefore$ the first quantity $= Re.1.\ 6a.\ 6p. + Rs.3.\ 2a - Rs.2.\ 8a.$

$= Rs.2.\ 0a.\ 6p. = Rs.2\frac{1}{32}.$

$\therefore$ the reqd. decimal $= 2\frac{1}{32} \div 100 = \cdot0203125.$ *Ans.*

Examples CXVII.

1. In the following Examples, reduce the first of the two given quantities to the decimal of the second :—

(1) *Rs.*11. 2*a.* 2*p.* ; *Rs.*178. 2*a.* 8*p.*

(2) *Rs.*12. 0*a.* 6*p.* ; *Rs.*25. 7*a*

(3) *Re.*1. 11*a.* ; *Rs.*2. 8*a.*

(4) *Rs.*2. 13*a.* 10*p.* ; *Rs.*50.

(5) 5*s.* ; 13*s.* 4*d.*

(6) 13*s.* $6\frac{3}{4}$*d.* ; 15*s.* 6*d.*

(7) £3. 6*s.* $8\frac{1}{2}$*d.* ; £7. 10*s.*

(8) $3\frac{1}{2}$ guineas ; £2. 15*s.* $5\frac{1}{2}$*d.*

(9) $1\frac{1}{2}$*d.* ; 7*s.* $10\frac{1}{2}$*d.*

(10) 7*s.* 8·1942*d.* ; 15*s.* 9*d.*

(11) $\frac{7}{8}$ of 10*s.* ; 13*s.* 4*d.*

(12) $\frac{3}{4}$ of 2*s.* 6*d.* ; $\frac{5}{8}$ of $1\frac{1}{2}$ guineas.

(13) 3·45 of 10*s.* 6*d.* ; half-a-crown.

(14) $\cdot052\dot{7}$ of £1. 7*s.* 6*d.* ; 13*s.* 4*d.*

(15) 3 hrs. 26 min. 37 sec. ; 13 days 20 hrs. 23 min.

(16) 1 cwt. 2 qrs. $3\frac{1}{4}$ lbs. ; 1 ton 4 cwt. 1 qr. 24 lbs.

(17) 10 lbs. 11 oz. 12 dwts. 7 grs. ; 9 lbs. 8 oz. Avoir.

(18) 5 ac. 3 ro. 15 po. ; 1 ac. 2 ro. 32 po.

(19) $3\frac{1}{3}$ of £4. 15*s.* 4*d.* ; $\cdot\dot{2}\dot{7}$ of 16*s.* 3*d.* (20) 2 sr. 4 ch. ; 1 md. 8 sr.

(21) $2\frac{23}{33}$ of £2. 6*s.* $5\frac{3}{4}$*d.* ; £18. 17*s.* $10\frac{3}{4}$*d.*

(22) ·101 of 1 lb. 5 oz. ; $\frac{7}{8}$ of 1 qr. 22 lbs. 8 oz.

(23) 1 bi. 11 k. 8 ch. ; 16 k. 14 ch.

(24) 1 md. 3 sr. $8\frac{8}{9}$ ch. ; 1 md. 16 sr.

2. Express 3*s.* $5\frac{11}{200}$*d.* as the decimal of a dollar of 4*s.* $1\frac{1}{2}$*d.*

3. Express £5·456 as the decimal of a rupee of 1*s.* 10*d.*

4. Express ·375 of a guinea + $\frac{3}{16}$ of a crown + ·3 of 7*s.* 6*d.* − $\frac{3}{8}$ of 2*d.* as the decimal of 16*s.*

5. *Find the value of* £·0375 + ·625*s.* + ·75*d.* + 2*s.* 3·5*d. and reduce* the result to the decimal of 7*s.* 6*d.*

6. Find the value of $\cdot\dot{2}4\dot{6}$ of *Rs.*4. 10*a* + $\cdot25\dot{9}$ of *Rs.*12. 8*a.* + $\cdot0\dot{2}$ of *Rs.*33. 12*a.* and reduce the result to the decimal of *Rs.*30.

7. Find the value of $\frac{3}{4}$ of $\dfrac{1}{9\frac{1}{2}}$ of £1. 18*s.* + $\frac{2}{3}$ of ·375 of 15*s.* + $\frac{2}{3}$ of $\cdot4\dot{2}\dot{9}$ of 8*s.* 3*d.* and express the result as the decimal of £5.

8. Express $\frac{3}{8}$ of 12*s.* 6*d.* + ·625 of 7*s.* 6*d.* − $\cdot5\dot{0}\dot{5}$ of 16*s.* 6*d.* as the decimal of £1.

9. Express £874. 13*s.* 4*d.* × 3·75 as the decimal of £1000.

10. What decimal of a crown is the difference between $6\frac{1}{2}$ half-guineas and £3·525 ?

11. Express the difference between $\cdot\dot{3}7\dot{8}$ of 13*s.* $10\frac{1}{2}d.$ and $\cdot 3\dot{7}\dot{8}$ of 16*s.* 6*d.* as the decimal of $\cdot 42\dot{6}$ of £1. 17*s.* 6*d.*

12. Express £·9 + 2·7*s.* + 3·6*d.* as the decimal of £(2 − ·2) + (6 − ·6)*s.* + (8 − ·8)*d.*

XVI. APPROXIMATION.

385. It has already been shewn in Art. 351, that in converting a vulgar fraction to a decimal, where the division does not terminate (which is often denoted by dots (...) placed at the end of the quotient, an **approximation** to its true value can always be found to any degree of accuracy). Thus $\frac{5}{17}$ = ·29411764...If we wish to *approximate* to the result by terminating the operation at the 5th place, we write $\frac{5}{17}$ = ·29412, but if at the fourth place, we write $\frac{5}{17}$ = ·2941, and so on. From this it is evident that we increase the last figure retained by 1, if the succeeding figure be 5, or greater than 5.

386. The reason for the above is obvious from the following considerations. If we take ·29412 to represent ·29411764..., instead of ·29411, it is clear that ·29412 is greater, and ·29411 less than the true value of the decimal; but ·29412 is greater than the true value by ·00000236..., and ·29411 is less than the true value by ·00000764...

Now ·00000236...is less than ·00000764...

Therefore ·29412 is nearer the true value than ·29411.

387. Contracted Addition and Subtraction. These methods have already been explained in Art. 369.

388. Contracted Multiplication. In multiplying one long decimal by another, it is generally required to get the product *approximately* correct, *i. e.* as far as a certain decimal place. The following Rule enables us to shorten the work.

Rule. Mark off in the decimal part of the multiplicand as many figures as is one more than the number of decimal places we are required to retain in the product: under the last of these marked figures place the *units' figure of the multiplier, writing the figures in* a reverse order. Omit decimal points of both the multiplicand and the multiplier and add 0's (if necessary) in the multiplicand, so that every figure of the multiplier shall have a figure above it. Begin the multiplication with the right-hand figure of the multiplier and multiply in succession by each of the others, in each case beginning the multiplication from the figure above the one we are multiplying by, but carrying to it the *nearest ten* from its product with the next figure on the right. Place the units' figure of all these partial products in the same vertical line; add as usual, and mark off the required number of decimal places in the result, striking out the last figure.

Note. In carrying the *nearest ten*, if the product is a number from 5 to 14 carry **1**; from 15 to 24 carry **2**; from 25 to 34 carry **3**; from 35 to 44 carry **4**; and so on. If the product is 4 or less than 4, reject it. (Art. 385.)

Ex. 1. Multiply 459·63524 by 25·4637, retaining 3 places; ·00040635 by 241·6358, retaining 6 places; and ·453 by ·01694, retaining 4 places of decimals.

(1)	(2)	(3)
4596352,4	4063,50	4530,0
736452	8536142	496100
91927048	812700	453
22981762	162540	272
1838541	4064	41
275781	2438	2
13789	122	·0076$\cancel{8}$
3217	20	
11704·013$\cancel{8}$	3	
	·098188$\cancel{7}$	

for ·01694 may be written as 0·01694.

Ex. 2. Multiply 3·2567834 by 4·2089542, retaining 7 places, and $1\cdot8\dot{2}35\dot{7}$ by $\cdot0\dot{7}8\dot{5}$, retaining 6 places of decimals.

(1)	(2)
325678340	18235723,57
24598024	758758700
1302713360	1276500
65135668	145886
2605426	9118
293109	1276
16284	146
1303	9
65	1
13·7076521$\cancel{5}$	·143293$\cancel{6}$

389. Contracted Division. In dividing one decimal by another where the quotient is required to be approximately correct only to a certain number of decimal places, we use the following Rule :—

RULE. Make the divisor a whole number; and determine first of all—by inspection or by taking one step in the ordinary way—the highest number of *integral* figures in the quotient, and then the whole number of figures in the quotient; from the left of the divisor cut off this number of figures, and one more for *approximation*; and strike out the rest. Proceed one step with this new divisor, but in multiplying its first figure by the quotient figure, carry the *nearest ten* from its product with the next figure on the right. Instead of bringing down a figure to the remainder, strike off another figure from the divisor, and proceed as before, until no figure is left in the divisor.

If the number of figures in the divisor, be less than the number of figures to be cut off, proceed in the ordinary way until the number

of figures still to be found in the quotient is one less than the number of figures in the divisor, and then apply the Rule.

Ex. 1. Divide 2508·928065051 by 92·410357 approximately correct to 4 places of decimals.

9,2,4,1,0,3,~~5~~)2508928065·051(27·1498
1848207
660721
646872
13849
9241
4608
3696
912
832
80
74

Making the divisor a whole number, we find by inspection that there will be **2** figures in the *integral* part of the quotient; and **4** places of decimals are to be retained. Hence, **6** figures are retained in the divisor and **1** more for *approximation*, so that the divisor is 924103,5. In the next stage the divisor is 92410,3; 3 being retained for *approximation*, and so on.

Ex. 2. Divide ·257917 by 2·03458 approximately correct to 7 places of decimals.

2,0,3,4,5,8)257917·70(·1267667
203458
544590
406916
137674
122075
15599
14242
1357
1220
137
122
15
14

Here, by inspection, we find that the quotient will contain no integral part; and as **7** places of decimals are to be retained, *the divisor must consist of* **8** figures, with **1** for approximation. But as there are only **6** figures in the divisor, proceed in the usual way of division for **2** figures in the quotient, when the number of figures still to be obtained will be one less than the number of figures in the divisor. Then apply the Rule.

Ex. 3. Divide ·549532676 by 931·2167, retaining 7 places of decimals.

9,3,1,2,1)5495·32676(·0005901
46561
8392
8381
11
9

By inspection, we determine that there will be **3** ciphers after the decimal point in the quotient; hence only $(7-3)$ or **4** figures are required in the quotient. Therefore we retain **5** figures in the divisor, one for approximation.

390. Series. The value of a *Series* is frequently required to be obtained correct to a certain number of decimal places. In such cases proceed as in the following Examples.

Ex. 1. Find the value, correct to 7 places of decimals, of

$$1+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\&c.$$

1	$=1$	$=1\cdot$	
$\frac{1}{1.2}$	$=\frac{1}{2}$	$=\cdot 5$	
$\frac{1}{1.2.3}=\frac{1}{3}\times\frac{1}{2}$	$=\frac{1}{3}\times\cdot 5$	$=\cdot 1666666$	67
$\frac{1}{1.2.3.4}=\frac{1}{4}\times\frac{1}{1.2.3}$	$=\frac{1}{4}\times\cdot 166666667$	$=\cdot 0416666$	67
$\frac{1}{1.2.3.4.5}=\frac{1}{5}\times\frac{1}{1.2.3.4}$	$=\frac{1}{5}\times\cdot 041666667$	$=\cdot 0083333$	33
$\frac{1}{1.2.3.4.5.6}=\frac{1}{6}\times\frac{1}{1.2.3.4.5}$	$=\frac{1}{6}\times\cdot 008333333$	$=\cdot 0013888$	89
$\frac{1}{1.2.3.4.5.6.7}$	$=\frac{1}{7}\times\cdot 001388889$	$=\cdot 0001984$	12
$\frac{1}{1.2.3.4.5.6.7.8}$	$=\frac{1}{8}\times\cdot 000198412$	$=\cdot 0000248$	01
$\frac{1}{1.2.3.4.5.6.7.8.9}$	$=\frac{1}{9}\times\cdot 000024801$	$=\cdot 0000027$	56
$\frac{1}{1.2.3.4.5.6.7.8.9.10}$	$=\frac{1}{10}\times\cdot 000002756$	$=\cdot 0000002$	76
$\frac{1}{1.2.3.4.5.6.7.8.9.10.11}$	$=\frac{1}{11}\times\cdot 000000276$	$=\cdot 0000000$	25
		$1\cdot 7182818$	26

The next and the following terms need not be considered, as they will all give 0's only up to the 7th decimal place.

Ex. 2. Find the value, correct to 5 places of decimals, of

$$\tfrac{3}{13}+(\tfrac{3}{13})^2+(\tfrac{3}{13})^3+(\tfrac{3}{13})^4+\ldots\ldots\text{to infinity.}$$

Let s denote the sum of the given series.

$$\text{Then } s=\tfrac{3}{13}+(\tfrac{3}{13})^2+(\tfrac{3}{13})^3+(\tfrac{3}{13})^4+\ldots\ldots\ldots$$

$$\therefore\ \tfrac{13}{3}s=1+\tfrac{3}{13}+(\tfrac{3}{13})^2+(\tfrac{3}{13})^3+(\tfrac{3}{13})^4+\ldots\ldots\ldots$$

Hence by subtraction, we get

$$(\tfrac{13}{3}-1)s=1\text{ ; or }\tfrac{10}{3}s=1\text{ ;}\qquad\therefore\ s=\tfrac{3}{13}=\underline{\cdot 23076.}$$

Ex. 3. Find the value, correct to 7 decimal places, of

$$\frac{1}{3.5}+\frac{2}{3^2.5^3}+\frac{3}{3^3.5^5}+\frac{4}{3^4.5^7}+\&c.$$

Let s denote the sum of the series,

$$\text{then } s = \frac{1}{3\cdot 5} + \frac{2}{3^2\cdot 5^3} + \frac{3}{3^3\cdot 5^5} + \frac{4}{3^4\cdot 5^7} + \&c.$$

$$\therefore \frac{1}{3\cdot 5^2}s = \qquad \frac{1}{3^2\cdot 5^3} + \frac{2}{3^3\cdot 5^5} + \frac{3}{3^4\cdot 5^7} + \&c.$$

By subtraction, we have

$$\left(1 - \frac{1}{3\cdot 5^2}\right)s \text{ or } \frac{74}{75}s = \frac{1}{3\cdot 5} + \frac{1}{3^2\cdot 5^3} + \frac{1}{3^3\cdot 5^5} + \frac{1}{3^4\cdot 5^7} + \&c.$$

$$\therefore \frac{74}{75}s \times \frac{1}{3\cdot 5^2} = \qquad \frac{1}{3^2\cdot 5^3} + \frac{1}{3^3\cdot 5^5} + \frac{1}{3^4\cdot 5^7} + \&c.$$

Again, by subtraction, we get

$$\frac{74}{75}s \times \left(1 - \frac{1}{3\cdot 5^2}\right) = \frac{1}{3\cdot 5}, \text{ or } \frac{74}{75}s \times \frac{74}{75} = \frac{1}{15}.$$

$$\therefore s = \frac{75 \times 75}{74 \times 74 \times 15} = \frac{375}{5476} = \underline{\cdot 0684806\ldots}$$

391. Abbreviated method of dividing a number by 9, 99, 999, &c.

RULE. Point off in the dividend as many decimal places (counting from the right) as there are *nines* in the divisor; then again twice as many decimal places, next three times as many, and so on. Then add these several numbers as in Addition of Decimals. The integral part will give the quotient and the recurring part the remainder.

Ex. Divide 578921 by 99 by the abbreviated method.

5789·21	$578921 \div 99 = 578921 \times \frac{1}{99} = 578921 \times \cdot\dot{0}\dot{1}$
57·8921	$= 578921 \times \left\{\frac{1}{100} + \frac{1}{(100)^2} + \frac{1}{(100)^3} + \&c.\right\}$
·578921	$= \frac{578921}{100} + \frac{578921}{10000} + \frac{578921}{1000000} + \&c.$
·00578921	$= 5789{\cdot}21 + 57{\cdot}8921 + {\cdot}578921 + {\cdot}00578921$
·0000578921, &c.	$= 5847{\cdot}6868\ldots \qquad + \&c.$
<u>5847·6868681021</u>	

Hence the quotient is <u>5847</u> and remainder <u>68</u>.

Examples CXVIII.

1. Multiply (by the *contracted method*) :—

(1) ·43429448 by ·6931472 retaining 7 places of decimals.
(2) 459·63524 by 25·4637..........................6.......... ....
(3) 583·26784 by ·00985..........................2.............................
(4) ·0008127 by 483·2716.......... ....6.............................
(5) 3670·257 by 12·61158.......... ....3..............................
(6) ·86858896 by 1·0986123. .... .5..................................

(7) $52{\cdot}687640812$ by $18{\cdot}703216231$ retaining 6 places of decimals.
(8) $1{\cdot}050625$ by itself ....4.................
(9) $27{\cdot}54\dot{3}\dot{6}$ by $8{\cdot}34\dot{7}$ ....5.................
(10) ${\cdot}012\dot{3}4\dot{5}$ by $49{\cdot}\dot{3}\dot{6}$ ....5.................

2. Divide (by the *contracted method*) :—

(1) $3789{\cdot}436$ by $265{\cdot}5984$ retaining 2 places of decimals.
(2) $742{\cdot}876315$ by $4967{\cdot}358$ ....4.................
(3) $185{\cdot}37612$ by ${\cdot}08764032$ ....4.................
(4) $154{\cdot}362904$ by ${\cdot}000541398$ ....7.................
(5) $10{\cdot}926954$ by ${\cdot}3547808034$ ....3.................
(6) 2 by $15{\cdot}314865$ ....5.................
(7) 1 by $3{\cdot}1415926535$ ....6.................
(8) $2{\cdot}347\dot{2}\dot{1}$ by $3{\cdot}279\dot{2}\dot{4}$ ....7.................
(9) $176{\cdot}804\dot{3}\dot{2}$ by $25{\cdot}123456$ ....3.................
(10) $66{\cdot}02\dot{0}3\dot{7}$ by $248{\cdot}7\dot{2}\dot{2}$ ....5.................

3. Find the respective values of :—

(1) $\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+\&c.$ .... to infinity.

(2) $\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+\&c.$ ....to...... ..

(3) $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\&c.$ ....to........

(4) $1+\frac{1}{1.3}+\frac{1}{1.3.5}+\frac{1}{1.3.5.7}+\&c.$...to infinity to 7 places of decimals.

(5) $\frac{1}{5}+\frac{1}{3}\times\frac{1}{5^3}+\frac{1}{5}\times\frac{1}{5^5}+\frac{1}{7}\times\frac{1}{5^7}+\&c.$......to 6.....................

(6) $\frac{1}{10^2}\times\left\{1-\frac{3}{10^2}+\frac{3\cdot4}{1.2}\times\frac{1}{10^4}-\frac{3\cdot4\cdot5}{1.2.3}\times\frac{1}{10^6}+\&c.\right\}$ to 6..............

(7) $16\times\left\{\frac{1}{5}-\frac{1}{3}\times\frac{1}{5^3}+\frac{1}{5}\times\frac{1}{5^5}-\frac{1}{7}\times\frac{1}{5^7}+\&c.\right\}-\frac{4}{239}$ to 6..............

(8) $\frac{1}{4}+\frac{3}{4^2}+\frac{1}{4^3}+\frac{3}{4^4}+\frac{1}{4^5}+\frac{3}{4^6}+\&c.$ ....to infinity.

(9) $5\times\left\{1-\frac{1}{50}-\frac{1}{2\times(50)^2}-\frac{1\times3}{6\times(50)^3}-\frac{1\times3\times5}{24\times(50)^4}\right.$
$\left.-\frac{1\times3\times5\times7}{120\times(50)^5}-\&c. \text{ to inf.}\right\}$ to 5 places of decimals.

(10) $\frac{2}{5}+\frac{4}{5^2}+\frac{2}{5^3}+\frac{4}{5^4}+\frac{2}{5^5}+\frac{4}{5^6}+\&c.$ ....to infinity.

4. Divide (by the *abbreviated* method) :—

(1) 2916438 and 75061382 separately by 9.

(2) 51647901 and 7204561 separately by 99.

(3) 7204561 and 580844 separately by 999.

(4) 591608 and 7391684 separately by 9999.

(5) 236916 by 9999 and 720532876 by 99999.

Examples worked out.

Ex. 1. A man owns $\frac{3}{16}$ of a house, and sells $\cdot 1\dot{3}5\dot{1}$ of his share; what fraction of the house does he still own?

He sells $\cdot 1\dot{3}5\dot{1}$ of $\frac{3}{16} = \frac{1350}{9990}$ of $\frac{3}{16} = \frac{15}{111}$ of $\frac{3}{16}$.

$\therefore$ he has left $(1 - \frac{15}{111})$ of $\frac{3}{16} = \frac{96}{111}$ of $\frac{3}{16} = \frac{6}{37}$. *Ans.*

Ex. 2. A vessel's cargo, $\frac{2}{3}$ of which is worth £$6666\cdot\dot{6}$, gets damaged, and the owner in consequence sells $\dfrac{\cdot 8\dot{3} + \cdot 041\dot{6}}{1\cdot 05}$ of it for half the original value of the whole cargo. What is the value of the remainder at the same rate and what the loss on the whole cargo?

The whole cargo is worth $\frac{3}{2}$ of £$6666\cdot\dot{6}$ = £$9999\cdot\dot{9}$ = £10000.

$$\text{He sells } \frac{\cdot 83333\ldots + \cdot 04166\ldots}{1\cdot 05} = \frac{\cdot 874\dot{9}}{1\cdot 05} = \frac{\cdot 875}{1\cdot 05} = \frac{875}{1050} = \frac{5}{6}.$$

$\therefore$ he has remaining $(1 - \frac{5}{6})$ or $\frac{1}{6}$.

Now since $\frac{5}{6}$ of the cargo sells for $\frac{1}{2}$ of £10000 = £5000;

$\therefore$ $\frac{1}{6}$ of the cargo must sell for $\frac{1}{5}$ of £5000 = £1000.

Hence loss = £(10000 − 5000 − 1000) = £4000.

Ex. 3. A woman had a certain number of eggs; she sold ·25 of the number and 3 more to one person, ·375 of the remainder to a second, and $\cdot\dot{6}$ of what still remained to a third, when she had only 15 left. How many had she at first.

After selling $\cdot\dot{6}$ or $\frac{2}{3}$ of the second remainder, she had $(1 - \frac{2}{3})$ or $\frac{1}{3}$ of the eggs left. Therefore $\frac{1}{3}$ of the second remainder = 15; $\therefore$ the second remainder = $15 \times 3 = 45$.

Again, ·375 or $\frac{3}{8}$ of the first remainder being sold, $\frac{5}{8}$ remained;

$\therefore$ $\frac{5}{8}$ of the first remainder = 45; $\therefore$ the first remainder = $45 \times \frac{8}{5} = 72$.

Next, after selling ·25 or $\frac{1}{4}$ of what she now had and 3 more, she had 72 left; $\therefore$ $\frac{3}{4}$ of the number = $72 + 3 = 75$.

$\therefore$ the whole number of eggs = $75 \times \frac{4}{3} = 100$.

Ex. 4. *A* owns $\cdot 58\dot{3}$ of an estate and *B* the rest. If $\frac{3}{5}$ of *B*s share is *Rs.* 5000 less than *A*'s, what is the worth of the whole estate?

Since A's share $=\frac{525}{900}$ or $\frac{7}{12}$ of the estate ;
$\therefore$ B's share $=(1-\frac{7}{12})$ or $\frac{5}{12}$ of the estate.

$\therefore$ $\frac{3}{5}$ of B's share $=(\frac{3}{5}\times\frac{5}{12})$ or $\frac{1}{4}$ of the estate ; and the difference of their shares $=(\frac{7}{12}-\frac{1}{4})$ or $\frac{1}{3}$ of the estate.

Therefore $\frac{1}{3}$ of the estate $=$ *Rs.*5000 ;
$\therefore$ the whole estate $=$ *Rs.*$5000\times 3=$ *Rs.*15000.

Miscellaneous Examples IV.

1. Find the sum, difference, product and two quotients of 30·33 and ·0337 ; and find the sum of all the results.

2. Reduce ($\frac{3}{25}$ of 2·45 $-\frac{1}{100}$ of ·02) $\div$ 1000 to a decimal.

3. Find the sum of 3·102 + ·00071 + 5·876 + 1·2 + ·31907 + ·027 + 310·68 + ·0000743 + 38·691 + ·1041457.

4. Which is the greater, ·39 of a guinea, or $\cdot 40\dot{9}\dot{9}$ of £1 ?

5. Divide the sum of 8·25 and 4·125 by their difference.

6. Divide the product of 1·075 and ·0101 by ·43.

7. Divide the difference between 3·1047 and ·0731 by the sum of 1·27 and 11·384.

8. If ·3 of an estate is sold for *Rs.* 4504, find the value of ·48 of it at the same rate.

9. A man, who possesses $\cdot\dot{2}\dot{7}$ of a ship, sells $\cdot 41\dot{6}$ of his share for *Rs.* 32400 ; what is the ship worth ?

10. In a school of 200 children there are 4 classes, of which the first contains ·24, the second ·36, and the third ·18 of the whole ; of how many does the fourth class consist ?

11. If $\cdot\dot{6}$ of the number of apples in a basket exceeds ·6 of the number by 57·4 ; find the number of apples.

12. Divide 8·064 by $\cdot 84\dot{6}+\frac{20}{7}$ of $\cdot 291\dot{6}$.

13. Divide $\frac{\cdot 052}{1\cdot 3}$ of 1·56 by $\frac{\cdot 0624}{14\cdot 4}$ of 25·92.

14. A butcher bought an equal number of calves and sheep for £265 ; for the calves he gave £3·75 a head, and for the sheep £2·875 a head ; how many did he buy of each kind ?

15. A gentleman having given ·5 of the money in his purse for a horse, and ·375 of the remainder for a sheep, had £1·6875 still left ; what sum had he at first ?

16. Divide *Rs.*870 between A, B and C, so that ·75 of C's share shall $=$ ·5 of A's $=$ ·6 of B's.

17. A coal-dealer bought 198 mds. of coal for *Rs.*32·5875, of which he sold 100 mds. for *Re.* ·2375 a maund. At what price per seer must he sell the remainder so as to gain *Rs.*2·1875 by his bargain ?

18. *A* had *Rs.*2568. 11*a.* 4*p.*, which was *Rs.*$431.88541\dot{6}$ less than $.6$ of $.\dot{7}$ of 2.5 times *B*'s money. How much money had *B*?

19. How many oranges at £$.084375$ a dozen ought to be given or 378 eggs at $.0625$*s.* each?

20. What number must be subtracted from the product of 9.27 and 8.0003 to give the sum of 19, 27.9652, $.003$, 5.0267 and 17.09?

21. *A* has shares in an estate to the amount of $.25$ of it and of $.\dot{3}\dot{6}$ of it. *B* has shares in the same estate to the amount of $.25\dot{7}\dot{2}$ of it; find the difference in value between the properties of *A* and *B*, when $.\dot{3}\dot{6}$ of the estate is worth *Rs.*50000.

22. Divide 9.614 by $.0000019$ and $\dfrac{2\frac{1}{5}}{5\frac{1}{2}}$ by $.000\dot{3}$ and multiply the sum of the quotients by $.0005$.

23. Express the value of $\dfrac{133\frac{1}{7}}{83\frac{8}{9}} \div \left(1+\dfrac{2}{3+\dfrac{4}{5+\frac{6}{7}}}\right)$ of a rupee in decimals of £1, when the value of the rupee is 1*s.* $5\frac{1}{2}$*d.*

24. Simplify $.0576 \times 1.97 + .142857 \div 2\frac{1}{7} + .0454864$.

25. Divide 1001 by 390625; $.1001$ by $.000390625$ and 10.01 by 390.625. Multiply $1.1\dot{8}$ by $.\dot{5}3846\dot{1}$.

26. Find the value (to three places of decimals) of

$$1+\tfrac{1}{4}+\frac{1\times 3}{1\times 2}(\tfrac{1}{4})^2+\frac{1\times 3\times 5}{1\times 2\times 3}(\tfrac{1}{4})^3+\frac{1\times 3\times 5\times 7}{1\times 2\times 3\times 4}(\tfrac{1}{4})^4+\text{\&c. to infinity}$$

27. Simplify :—

$$\frac{3.\dot{3}}{6.0625} \text{ of } \frac{9.7}{2.4\dot{2}} \div \frac{2.5}{1.09}(7.25+2.75)\times\frac{\text{£}3.\ 6s.\ 8d.}{\text{£}10.\ 13s.\ 4d.}$$

28. Subtract $.03$ from $.\dot{0}\dot{3}$ and divide the result by $.1\dot{0}\dot{2}$.

29. Find the value of $.01\dot{6}$ of *Rs.*260. 2*a.* 6*p.* + $.\dot{3}5\dot{1}$ of *Rs.*13. 14*a.* + $1.0\dot{0}03\dot{3}$ of *Rs.*7. 14*a.* 3*p.*

30. Find how much more than $\dfrac{.0338184}{.03416}$ of $1.1\dot{6}$ of $.6$ of $.\dot{5}8\dot{7}$ of *Rs.*52. 1*a.* 4*p.* I need to pay a bill of *Rs.*21. 4*a.*

31. A person owns $\frac{7}{25}$ of an estate, and sells $.\dot{3}57142\dot{8}$ of his share; what part of the whole estate has he still left?

32. *A* and *B* can do a piece of work in 15.75 days, *B* and *C* can do it in $18.\dot{6}$ days, and *A* and *C* in $16.\dot{3}$ days. In what time would *A*, *B* and *C* singly perform the whole work?

33. There is a number which, when multiplied by 4.255 and divided by $.0016$, gives 851; find the number.

34. Shew that, whether the value of $3\frac{1}{4}+4\frac{2}{5}-5\frac{1}{2}+16\frac{5}{8}-\frac{11}{24}+10-14\frac{5}{6}$ be found by vulgar fractions or by decimals, the results coincide.

35. The owner of $\cdot 375$ of a mine sold $\cdot\dot{6}$ of his share for *Rs.*25200; find the value of $\cdot 875$ of the mine.

36. A cistern of water lost $\cdot 12$ of its contents by leakage, then 26 gals. were drawn off, and it was then $\cdot 75$ full; how many gals. did it contain at first?

37. In a cricket match, one side of 11 men made a certain number of runs, one player obtained $\cdot 25$ of the number, each of three others $\cdot 1$, each of two others $\cdot 0625$, and the rest 39 amongst them; find the whole number of runs.

38. Reduce to their simplest forms :—

(1) $\dfrac{\cdot 005}{\frac{1}{15}\text{ of }11\frac{1}{7}}$ of $\dfrac{49\frac{1}{21}}{\frac{1}{4}\text{ of }2\cdot 25}\div\left(\dfrac{1}{21}+\dfrac{1}{27}\right)$. (2) $\cdot 70\frac{4}{5}$. (3) $\dfrac{4}{\frac{9}{10}}-\dfrac{\frac{4}{9}}{10}$.

39. Five bells which toll at intervals of 1·2, 1·5, 1·75, 1·8, 2·1 seconds respectively, begin tolling simultaneously; how long after will they all toll simultaneously again?

40. Reduce £24. 16*s.* $4\frac{1}{2}$*d.* and £167. 10*s.* $6\frac{1}{4}$*d.* $\frac{1}{2}$*q.* to decimals of the same denomination, so as to find how often the former is contained in the latter.

41. Find the value of $\dfrac{\cdot 0931\dot{8}}{\cdot 56\dot{8}\dot{1}}$ of $2\frac{1}{12}$ of 2·5 days.

42. A woman has a certain number of eggs; she sells $\cdot\dot{3}$ of the number and one more to one person, $\cdot\dot{3}$ of the remainder to a second person, and $\cdot 5$ of the remainder to a third person; after these sales she has 15 eggs left. How many had she at first?

43. A clerk copied $\cdot 55$ of *Rs.*50 instead of 5·5 of *Rs.*50; what was the amount of the error?

44. From a rod 2·078 miles long, portions are cut off each equal to $\cdot 0037$ of an inch, how many such portions can be cut off and what will be the remainder?

45. Express the sum of $\cdot\dot{5}7142\dot{8}$ of a vis, $\frac{3}{8}$ of $\dfrac{1}{3\frac{3}{8}}$ of $\frac{317}{384}$ of a maund and $\frac{3801}{10136}$ of a cwt. as the decimal of 1 ton. (a vis = 3lbs. 2 oz.; one maund = $82\frac{2}{7}$lbs. Avoir.)

46. The difference in the values of the two shares into which a certain property is divided is *Rs.*48·575, and one share is $\cdot 51$ of the whole. Find the value of the property and of each share.

47. *A* has an income = ($\cdot 6$ of $\cdot 8\dot{3}\div 3\cdot 5$) of *B*'s income. If *A* after spending *Rs.*645 per annum, find that he has exceeded his income by $\cdot 075$ of it, find *B*'s income.

48. *A* can reap $\cdot\dot{4}$ of a field in $2\cdot\dot{6}$ days and *B* can reap $\cdot 6$ of it in 4·5 days; *A* and *B* work together till they have reaped $\cdot 75$ of

the field. A then leaves, and B completes the work. If A earn $Rs.2.\ 8a.$ a day, what ought the reaping of the field to cost?

49. Out of a bag of silver, I take $Rs.25$ more than $\cdot 5$ of the whole sum which it contained; then $Rs.15$ more than $\cdot 2$ of what then remained; and then $Rs.10$ more than $\cdot 25$ of what then remained; after this $Rs.5$ remained. What did the bag contain at first?

50. A has shares in an estate to the amount of $\cdot 15 \div \cdot 3\dot{6}$ of it, B has shares in the same estate to the amount of $\cdot 4\dot{7}\dot{2}$ of it; find the difference in value between the properties of A and B, when $\cdot 0\dot{5}\dot{6}$ of the estate is worth £373·3.

CHAPTER VII.

Rules of Practice and Invoices.

392. We shall here shew how the primitive fractions, as defined in Art. 228, may be applied to the *practical* calculation of prices, when the price of a unit of any denomination is supposed to be given; and the tediousness of the *enunciations* of the rules at length, will be a sufficient excuse for the mere *indications* of the processes to be employed, by means of examples.

393. An **aliquot part** of a number is such that we may make up the number by taking the part a certain *integral* number of times. Its relation with the whole can therefore be expressed by a fraction which has unity for its numerator and an integer for its denominator.

Thus, $5a.\ 4p.$, being $\frac{1}{3}$ of $Re.1$, is an *aliquot* part of a rupee; $10s.$, being $\frac{1}{2}$ of £1, is an *aliquot* part of a pound.

Table of Aliquot Parts.

Of a **Rupee.**		Of a **£.**		Of a **Maund.**	
$8a.$	$= \frac{1}{2} Re.$	$10s.$	$= \frac{1}{2}$ £.	20 sr.	$= \frac{1}{2}$ md.
$5a.\ 4p.$	$= \frac{1}{3} Re.$	$6s.\ 8d.$	$= \frac{1}{3}$ £.	10 sr.	$= \frac{1}{4}$ md.
$4a.$	$= \frac{1}{4} Re.$	$5s.$	$= \frac{1}{4}$ £.	8 sr.	$= \frac{1}{5}$ md.
$2a.\ 8p.$	$= \frac{1}{6} Re.$	$4s.$	$= \frac{1}{5}$ £.	5 sr.	$= \frac{1}{8}$ md.
$2a.$	$= \frac{1}{8} Re.$	$3s.\ 4d.$	$= \frac{1}{6}$ £.	4 sr.	$= \frac{1}{10}$ md.
$1a.\ 4p.$	$= \frac{1}{12} Re.$	$2s.\ 6d.$	$= \frac{1}{8}$ £.	2 sr. 8 ch.	$= \frac{1}{16}$ md.
$1a.$	$= \frac{1}{16} Re.$	$2s.$	$= \frac{1}{10}$ £.	2 sr.	$= \frac{1}{20}$ md.
		$1s.\ 8d.$	$= \frac{1}{12}$ £.	1 sr. 4 ch.	$= \frac{1}{32}$ md.
		$1s.\ 4d.$	$= \frac{1}{15}$ £.	1 sr.	$= \frac{1}{40}$ md.
		$1s.\ 3d.$	$= \frac{1}{16}$ £.		
		$1s.$	$= \frac{1}{20}$ £.		

Of an **Anna**.

$6p.$	$= \frac{1}{2}a.$
$4p.$	$= \frac{1}{3}a.$
$3p.$	$= \frac{1}{4}a.$
$2p.$	$= \frac{1}{6}a.$
$1p.$	$= \frac{1}{12}a.$
$2ps.$	$= \frac{1}{2}a.$
$1ps.$	$= \frac{1}{4}a.$

Of a **Ton**.

10 cwt.	$= \frac{1}{2}$ ton.
5 cwt.	$= \frac{1}{4}$ ton.
4 cwt.	$= \frac{1}{5}$ ton.
2 cwt. 2qr.	$= \frac{1}{8}$ ton.
2 cwt.	$= \frac{1}{10}$ ton.
1 cwt. 1qr.	$= \frac{1}{16}$ ton.
1 cwt.	$= \frac{1}{20}$ ton.

Of a **Bigha**.

10 kathas	$= \frac{1}{2}$ big.
5 k.	$= \frac{1}{4}$ big.
4 k.	$= \frac{1}{5}$ big.
2 k. 8ch.	$= \frac{1}{8}$ big.
2 k.	$= \frac{1}{10}$ big.
1 k. 4 ch.	$= \frac{1}{16}$ big.
1 k.	$= \frac{1}{20}$ big.

Of an **Acre**.

2 ro.	$= \frac{1}{2}$ ac.
1 ro.	$= \frac{1}{4}$ ac.
20 po.	$= \frac{1}{8}$ ac.
16 po.	$= \frac{1}{10}$ ac.

Of a **Week**.

$3\frac{1}{2}$ da.	$= \frac{1}{2}$ wk.
$1\frac{3}{4}$ da.	$= \frac{1}{4}$ wk.

Of a **Shilling**.

$6d.$	$= \frac{1}{2}s.$
$4d.$	$= \frac{1}{3}s.$
$3d.$	$= \frac{1}{4}s.$
$2d.$	$= \frac{1}{6}s.$
$1\frac{1}{2}d.$	$= \frac{1}{8}s.$
$1\frac{1}{3}d.$	$= \frac{1}{9}s.$
$1d.$	$= \frac{1}{12}s.$

Of a **Cwt**.

2 qrs.	$= \frac{1}{2}$ cwt.
1 qr.	$= \frac{1}{4}$ cwt.
16 lbs.	$= \frac{1}{7}$ cwt.
14 lbs.	$= \frac{1}{8}$ cwt.

Of a **Katha**.

8 ch.	$= \frac{1}{2}$ k.
4 ch.	$= \frac{1}{4}$ k.
2 ch.	$= \frac{1}{8}$ k.
1 ch.	$= \frac{1}{16}$ k.

Of a **Rood**.

20 po.	$= \frac{1}{2}$ ro.
10 po.	$= \frac{1}{4}$ ro.
8 po.	$= \frac{1}{5}$ ro.
5 po.	$= \frac{1}{8}$ ro.
4 po.	$= \frac{1}{10}$ ro.
2 po.	$= \frac{1}{20}$ ro.
1 po.	$= \frac{1}{40}$ ro.

Of a **Month**.

1 wk.	$= \frac{1}{4}$ mo.
2 wk.	$= \frac{1}{2}$ mo.
15 da.	$= \frac{1}{2}$ mo.
10 da.	$= \frac{1}{3}$ mo.

Of a **Seer**.

8 ch.	$= \frac{1}{2}$sr.
4 ch.	$= \frac{1}{4}$sr.
2 ch.	$= \frac{1}{8}$sr.
1 ch.	$= \frac{1}{16}$sr.

Of a **Quarter**.

14 lb.	$= \frac{1}{2}$ qr.
7 lb.	$= \frac{1}{4}$ qr.
4 lb.	$= \frac{1}{7}$ qr.
3 lb. 8 oz.	$= \frac{1}{8}$ qr.
2 lb.	$= \frac{1}{14}$ qr.
1 lb. 12 oz.	$= \frac{1}{16}$ qr.
1 lb.	$= \frac{1}{28}$ qr.

Of a **lb.** Avoir.

8 oz.	$= \frac{1}{2}$ lb.
4 oz.	$= \frac{1}{4}$ lb.
2 oz.	$= \frac{1}{8}$ lb.
1 oz.	$= \frac{1}{16}$ lb.

Of an **Oz.** Avoir.

8 dr.	$= \frac{1}{2}$ oz.
4 dr.	$= \frac{1}{4}$ oz.
2 dr.	$= \frac{1}{8}$ oz.
1 dr.	$= \frac{1}{16}$ oz.

Of a **Mile**.

4 fur.	$= \frac{1}{2}$ mi.
2 fur.	$= \frac{1}{4}$ mi.
1 fur.	$= \frac{1}{8}$ mi.

Of a **Furlong**.

110 yd.	$= \frac{1}{2}$ fur.
55 yd.	$= \frac{1}{4}$ fur.

394. Practice is a short method of finding the value of any quantity by means of *aliquot parts*, when the value of a unit of any denomination is given. It is therefore another method of solving questions in *Compound Multiplication*.

395. Practice may be either **Simple** or **Compound**.

It is *Simple Practice*, when the value of one unit of a certain denomination is given, and the value of a number of these units is

required ; but in *Compound Practice*, the given quantity is not wholly expressed in the same denomination as the unit whose value is given.

Thus, to find the value of 350 articles at 15*a.* 8*p.* each is *Simple Practice* ; and to find the value of 14 mds. 15 sr. 7 ch. at *Rs.*2. 5*a.* 8*p.* per maund is *Compound Practice*.

I. SIMPLE PRACTICE.

396. The RULE for Simple Practice will be best understood by the following Examples.

Ex. 1. Find the value of 1298 things at *Rs.*8. 14*a.* 6*p.* each.

If the cost of a thing be *Re.* 1 ; then the total cost is *Rs.*1298.

	Rs.	*a.*	*p.*	
8*a.* = $\frac{1}{2}$ of *Re.*1.	1298	0	0	= price @ *Re.*1 each.
			8	
	10384	0	0	= price @ *Rs* 8 ......
4*a.* = $\frac{1}{2}$ of 8*a.*	649	0	0	= price @ 8*a.* ...
2*a.* = $\frac{1}{2}$ of 4*a.*	324	8	0	= price @ 4*a.* ......
6*p.* = $\frac{1}{4}$ of 2*a.*	162	4	0	= price @ 2*a.* ......
	40	9	0	= price @ 6*p.* ......
	*Rs.*11560	5	0	= price @ *Rs* 8. 14*a.* 6*p.* each.

Note 1. It is generally most convenient, when possible, to use the *aliquot part* of the denomination next superior to the highest denomination of the price proposed.

Here, *Rs.*8. 14*a.* 6*p.* is less than *Rs.*9 by 1*a.* 6*p.* Hence the calculation may be shortened thus :—

1*a.* = $\frac{1}{16}$ of *Re.*1	*Rs.*1298. 0*a.* 0*p.*		= price at *Re.*1 each.
	9		
	*Rs.*11682 0 0		= price at *Rs.*9 each.
6*p.* = $\frac{1}{2}$ of 1*a.*		*Rs.*81. 2*a.*	= price at 1*a.* ...
		*Rs.*40. 9*a.*	= price at 6*p.* ......
	121 11 0		= price at 1*a.* 6*p.* ....
	*Rs.*11560 5 0		= price at *Rs.*8. 14*a.* 6*p.*

Ex. 2. Find the cost of 345 things at £3. 17*s.* 10$\frac{1}{2}$*d.* each.

	£.	*s.*	*d.*	
10*s.* = $\frac{1}{2}$ of £1.	345	0	0	= cost @ £1 each.
			3	
	1035	0	0	= cost @ £3 each.
5*s.* = $\frac{1}{2}$ of 10*s.*	172	10	0	= cost @ 10*s.* ......
2*s.* 6*d.* = $\frac{1}{2}$ of 5*s.*	86	5	0	= cost @ 5*s.* ......
3*d.* = $\frac{1}{10}$ of 2*s.* 6*d.*	43	2	6	= cost @ 2*s.* 6*d.* ... ..
1$\frac{1}{2}$*d.* = $\frac{1}{2}$ of 3*d.*	4	6	3	= cost @ 3*d.* ......
	2	3	1$\frac{1}{2}$	= cost @ 1$\frac{1}{2}$*d* .....
	£1343	6	10$\frac{1}{2}$	= cost @ £3. 17*s.* 10$\frac{1}{2}$*d.* each.

Otherwise thus :—As £3. 17*s*. $10\frac{1}{2}$*d*. is the difference between £4. and 2*s*. $1\frac{1}{2}$*d*., we can simplify the process thus :—

	£.	s.	d.	£.	s.	d.	
2*s*. = $\frac{1}{10}$ of £1.	345	0	0	345	0	0	= cost @ £1 each.
$1\frac{1}{2}$*d*. = $\frac{1}{16}$ of 2*s*.	34	10	0			4	
	2	3	$1\frac{1}{2}$	1380	0	0	= cost @ £4 each.
	£36	13	$1\frac{1}{2}$	36	13	$1\frac{1}{2}$	= cost @ 2*s*. $1\frac{1}{2}$*d*......
				£1343	6	$10\frac{1}{2}$	= cost @ £3.17*s*.$10\frac{1}{2}$*d*......

Note 2. Sometimes by introducing a *subsidiary* aliquot part, we can easily find the required aliquot part ; thus, taking the preceding example, we have

	£.	s	d.	
2*s*. = $\frac{1}{10}$ of £1.	345	0	0	= cost at £1 each.
$\cancel{6}$*d*. = $\frac{1}{4}$ of $\cancel{2}$*s*.	34	10	0	= cost at 2*s*. ......
$1\frac{1}{2}$*d*. = $\frac{1}{4}$ of 6*d*.	$\cancel{8}$	$\cancel{12}$	$\cancel{6}$	
	2	3	$1\frac{1}{2}$	= cost at $1\frac{1}{2}$*d*. ......
	36	13	$1\frac{1}{2}$	= cost at 2*s*. $1\frac{1}{2}$*d*. ...

Ex. 3. Find the value of $456\frac{5}{8}$ mds. at *Rs*.8. 5*a*. 10*p*. per maund.

Since *Re*.$\frac{5}{8}$ = 10*a*., the cost of $456\frac{5}{8}$ mds. at *Re*.1 is *Rs*.456. 10*a*. ; we therefore proceed as before, thus :—

	Rs.	a.	p.	
4*a*. = $\frac{1}{4}$ of *Re*.1.	456	10	0	= value @ *Re*.1 each.
			8	
	3653	0	0	= value @ *Rs*.8......
1*a*. = $\frac{1}{4}$ of 4*a*.	114	2	6	= value @ 4*a*. .....
6*p*. = $\frac{1}{2}$ of 1*a*.	28	8	$7\frac{1}{2}$	= value @ 1*a*. ......
3*p*. = $\frac{1}{2}$ of 6*p*.	14	4	$3\frac{3}{4}$	= value @ 6*p*. ......
1*p*. = $\frac{1}{3}$ of 3*p*.	7	2	$1\frac{7}{8}$	= value @ 3*p*. ......
	2	6	$0\frac{5}{8}$	= value @ 1*p*. ......
	Rs.3819	7	$7\frac{3}{4}$	= value @ *Rs*.8. 5*a*. 10*p*. each.

Ex. 4. Find the cost of $2864\frac{3}{7}$ cwt. at 9*s*. $10\frac{3}{4}$*d*. per cwt.

Since £$\frac{3}{7}$ would introduce a fraction of a farthing, it will be better to find separately the cost of 2864 cwt. and of $\frac{3}{7}$ of a cwt. and then add.

	£.	s.	d.	
5*s*. = $\frac{1}{4}$ of £1.	2864	0	0	= cost @ £1 each.
4*s*. = $\frac{1}{5}$ of £1.	716	0	0	= cost @ 5*s*. ......
10*d*. = $\frac{1}{6}$ of 5*s*.	572	16	0	= cost @ 4*s*. ......
$\cancel{6}$*d*. = $\frac{1}{10}$ of $\cancel{5}$*s*.	119	6	8	= cost @ 10*d*. ......
	$\cancel{71}$	$\cancel{12}$	$\cancel{0}$	
$\frac{3}{4}$*d*. = $\frac{1}{8}$ of 6*d*.	8	19	0	= cost @ $\frac{3}{4}$*d*. ......
	£1417	1	8	= cost @ 9*s*. $10\frac{3}{4}$*d*. each.
		4	$2\frac{25}{28}$	= cost of $\frac{3}{7}$ of a cwt.
	£1417	5	$10\frac{25}{28}$	= cost @ 9*s*. $10\frac{3}{4}$*d*. per cwt.

	s.	d.
	9	$10\frac{3}{4}$
		3
7)	29	$8\frac{1}{4}$
	4	$2\frac{25}{28}$

Ex. 5. Find the price of 2108 cwt. of sugar at £1. 6*s*. $2\frac{3}{8}$*d*. each.

	£.	*s*.	*d*.	
4*s*. = $\frac{1}{5}$ of £1.	2108	0	0	= price at £1 each.
2*s*. = $\frac{1}{2}$ of 4*s*.	421	12	0	= price at 4*s*. ....
2*d*. = $\frac{1}{12}$ of 2*s*.	210	16	0	= price at 2*s*. ....
$\frac{1}{4}$*d*. = $\frac{1}{8}$ of 2*d*.	17	11	4	= price at 2d. ....
$\frac{1}{8}$*d*. = $\frac{1}{2}$ of $\frac{1}{4}$*d*.	2	3	11	= price at $\frac{1}{4}$*d*. ....
	1	1	$11\frac{1}{2}$	= price at $\frac{1}{8}$*d*. ....
	£2761	5	$2\frac{1}{2}$	= price at £1. 6*s*. $2\frac{3}{8}$*d*. each.

Examples CXIX.

Find by Practice the values of the following articles :—

1.	3467 at 2*a*. 6*p*.	**2.**	659 at 13*a*. 2*p*.
3.	1448 at 10*a*. 8*p*.	**4.**	1281 at 5*a*. 4*p*.
5.	2370 at 13*a*. 4*p*.	**6.**	659 at 1*s*. $7\frac{3}{4}$*d*.
7.	1250 at 2*s*. $3\frac{3}{4}$*d*.	**8.**	328 at 8*s*. $5\frac{1}{2}$*d*.
9.	7351 at 14*s*. $9\frac{1}{4}$*d*.	**10.**	2345 at *Rs*.2. 14*a*. 8*p*.
11.	1600 at *Rs*.2. 5*a*. 6*p*.	**12.**	140321 at 13*a*. $11\frac{1}{2}$*p*.
13.	632 at *Rs*.14. 5*a*. 6*p*.	**14.**	7777 at 17*s*. $8\frac{3}{4}$*d*.
15.	1298 at 17*s*. $9\frac{3}{4}$*d*.	**16.**	537 at £1. 7*s*. $2\frac{1}{2}$*d*.
17.	2937 at £2. 11*s*. $10\frac{3}{4}$*d*.	**18.**	1684 at £8. 5*s*. $1\frac{1}{2}$*d*.
19.	412 at £5. 14*s*. $5\frac{1}{2}$*d*.	**20.**	6439 at *Rs*.16. 15*a*. $7\frac{3}{4}$*p*.
21.	295 at *Rs*.5. 11*a*. $7\frac{3}{7}$*p*.	**22.**	3655 at £9. 16*s*. $10\frac{1}{2}$*d*.
23.	3546 at £5. 15*s*. $7\frac{3}{4}$*d*.	**24.**	65437 at *Rs*.4. 13*a*. 2*p*.
25.	$1449\frac{3}{7}$ at *Rs*.11. 6*a*. 6*p*.	**26.**	$237\frac{1}{2}$ at 13*a*. 8*p*.
27.	$1128\frac{2}{5}$ at *Rs*.2. 15*a*. 11*p*.	**28.**	$7432\frac{1}{2}$ at *Rs*.6. 12*a*. 4*p*.
29.	$6147\frac{3}{4}$ at 17*s*. $6\frac{1}{2}$*d*.	**30.**	$2763\frac{3}{8}$ at 13*s*. $6\frac{3}{4}$*d*.
31.	$217\frac{1}{4}$ at £2. 17*s*. $7\frac{1}{2}$*d*.	**32.**	$769\frac{3}{5}$ at *Rs*.16. 4*a*.
33.	$674\frac{5}{8}$ at £3. 19*s*. $6\frac{1}{2}$*d*.	**34.**	22·6 at 5*a*. 1*p*.
35.	169·875 at £2. 17*s*. $10\frac{1}{4}$*d*.	**36.**	359·3125 at £1. 6*s*. 2*d*.
37.	3764·6 at *Rs*.27. 4*a*. 10*p*.	**38.**	$178\cdot\dot{6}$ at *Rs*.3. 5*a*. 2*p*.
39.	$821\cdot\dot{5}$ at *Rs*.6. 15*a*. 2*p*.	**40.**	861 at *Rs*.5. 7*a*. $5\frac{4}{7}$*p*.
41.	45656 at 6*a*. $2\frac{3}{5}$*p*.	**42.**	2841 at 5*s*. $10\frac{8}{11}$*d*.
43.	2731 at £4. 8*s*. $9\frac{3}{8}$*d*.	**44.**	567384 at 5*a*. $10\frac{1}{2}$*p*.
45.	30000 at *Rs*.4. 2*a*. $4\frac{2}{3}$*p*.	**46.**	$5109\frac{9}{13}$ at £4. 16*s*. $4\frac{1}{4}$*d*.

II. COMPOUND PRACTICE.

397. The Rule for Compound Practice will be easily shewn by the following Examples.

Ex. 1. Find the value of 8mds. 6sr. 12ch. at *Rs.*5. 6*a*. 8*p*. per md.

	Rs.	*a.*	*p.*	
5 sr. = $\frac{1}{8}$ of 1 md.	5	6	8	= value of 1 md.
			8	
	43	5	4	= value of 8 mds.
1 sr. = $\frac{1}{5}$ of 5 sr.		10	10	= value of 5 sr.
8 ch. = $\frac{1}{2}$ of 1 sr.		2	2	= value of 1 sr.
4 ch. = $\frac{1}{2}$ of 8 ch.		1	1	= value of 8 ch.
			$6\frac{1}{2}$	= value of 4 ch.
	*Rs.*44	3	$11\frac{1}{2}$	= value of 8 mds. 6 sr. 12 ch.

Ex. 2. What is the price of 3 cwt. 2 qrs. 16 lbs. at £3. 7*s*. 8*d*. per cwt.?

	£.	*s.*	*d.*	
2 qrs. = $\frac{1}{2}$ of 1 cwt.	3	7	8	= price of 1 cwt.
			3	
	10	3	0	= price of 3 cwt.
14 lbs. = $\frac{1}{4}$ of 2 qrs.	1	13	10	= price of 2 qrs.
2 lbs. = $\frac{1}{7}$ of 14 lbs.		8	$5\frac{1}{2}$	= price of 14 lbs.
		1	$2\frac{1}{2}$	= price of 2 lbs.
	£12	6	6	= price of 3 cwt. 2 qrs. 16 lbs.

Ex. 3. Find the value of 11 mds. 4 sr. 8 ch. at *Re.*1. 14*a*. 4*p*. per seer.

	Rs.	*a.*	*p.*	
8 ch. = $\frac{1}{2}$ of 1 sr.	1	14	4	= value of 1 seer.
4 sr. = 1 sr. × 4.			4	
1 md. = 4 sr. × 10.	7	9	4	= value of 4 sr.
			10	
	75	13	4	= value of 1 md.
			11	
	834	2	8	= value of 11 mds.
	7	9	4	= value of 4 sr.
		15	2	= value of 8 ch.
	*Rs.*842	11	2	= value of 11 mds. 4sr. 8ch.

Ex. 4. Find the value of 365 mds. 37 sr. 8ch. at *Rs.*126. 6*a*. 8*p*. per maund.

	Rs.	*a.*	*p.*	
20 sr. = $\frac{1}{2}$ of 1 md.	126	6	8	= value of 1 md.
			10	
	1264	2	8	= value of 10 mds.
			10	
	12641	10	8	= value of 100 mds.
			3	
	37925	0	0	= value of 300 mds.
	7585	0	0	= value of 60 mds.
	632	1	4	= value of 5 mds.
10 sr. = $\frac{1}{3}$ of 20 sr.	63	3	4	= value of 20 sr.
5 sr. = $\frac{1}{2}$ of 10 sr.	31	9	8	= value of 10 sr.
2 sr. 8 ch. = $\frac{1}{2}$ of 5 sr.	15	12	10	= value of 5 sr.
	7	14	5	= value of 2 sr. 8 ch.
	*Rs.*46260	9	7	= value of 365mds. 37sr. 8ch.

Ex. 5. Find the rent of 71 bighas 6 kat. 14 ch. at *Rs.*8. 12*a.* per bigha.

	Rs.	*a.*	*p.*	
4 kat. = $\frac{1}{5}$ of 1 big.	8	12	0	= rent of 1 bigha.
			10	
	87	8	0	= rent of 10 bighas.
			7	
	612	8	0	= rent of 70 bighas.
	8	12	0	= rent of 1 bigha.
2 kat. = $\frac{1}{2}$ of 4 kat.	1	12	0	= rent of 4 kat.
8 ch. = $\frac{1}{4}$ of 2 kat.		14	0	= rent of 2 kat.
4 ch. = $\frac{1}{2}$ of 8 ch.		3	6	= rent of 8 ch.
2 ch. = $\frac{1}{2}$ of 4 ch.		1	9	= rent of 4 ch.
			$10\frac{1}{2}$	= rent of 2 ch.
	*Rs.*624	4	$1\frac{1}{2}$	= rent of 71 big. 6 kat. 14 ch.

Examples CXX.

Find by Practice the value, rent, &c. (as the case may be) of :—

1. 15 mds. 25 sr. 11 ch. at *Rs.*12. 10*a.* 8*p.* per maund.
2. 8 mds. 11sr. 7 ch. at *Rs.*6. 10*a.* 8*p.* per maund.
3. 18 mds. 5 sr. 6 ch. at *Rs.*27. 14*a.* 8*p.* per maund.
4. 777 mds. 20 sr. 12 ch. at *Rs.*40. 10*a.* 8*p.* per maund.
5. 373 mds. 39 sr. 7 ch. at *Rs.*25. 2*a.* 4*p.* per maund.
6. 3 cwt. 2 qrs. 17 lbs. at £1. 5*s.* 8*d.* per quarter.
7. 57 cwt. 3 qrs. 14 lbs. at £5. 9*s.* 6*d.* per cwt.
8. 45 oz. 6 dwts. 7 grs. at 5*s.* 10*d.* per oz.
9. 37 cwt. 3 qrs. 2 lbs. at £3. 14*s.* $7\frac{1}{2}$*d.* per cwt.

10. 72 cwt. 3 qrs. 17 lbs. at 6*s.* $1\frac{1}{2}$*d.* per quarter.
11. 15 tons 11 cwt. 3 qrs. 18 lbs. at £3. 7*s.* 6*d.* per cwt.
12. 6 tons 12 cwt. 3 qrs. $10\frac{1}{2}$ lbs. at £3. 14*s.* $8\frac{1}{4}$*d.* per cwt.
13. 5 ac. 2 ro. 4 po. $4\frac{1}{2}$ yds. at *Rs.*10 per rood.
14. 16 yds. 2 ft. 10 in. at 2*s.* $6\frac{1}{2}$*d.* per yard.
15. 196 miles 3 fur. $137\frac{1}{2}$ yds. at *Rs.*363. 4*a.* 8*p.* per mile.
16. 7 mds. 2 sr. 14 ch. at 3*a.* 6*p.* per seer.
17. 38 mds. 25 sr. 10 ch. at 10*a.* 6*p.* per seer.
18. 53 big. 12 kat. 2 ch. at *Rs* 19. 12*a.* per bigha.
19. 155 big. 1 kat. 4 ch. at *Rs.*89. 8*a.* 4*p.* per bigha.
20. 44 ac. 2 ro. 25 po. at £55. 16*s.* $7\frac{1}{2}$*d.* per acre.
21. 35 qrs. 7 bus. $3\frac{1}{4}$ pks. at 58*s.* 6*d.* per quarter.
22. 9 cub. yds. 21 ft. 432 in. at £4. 14*s.* 6*d.* per cub. yard.
23. 5 lbs. 10 oz. 12 dwts. $6\frac{3}{4}$ grs. at £3. 17*s.* 11*d.* per oz.
24. 17 tons 12 cwt. 3 qrs. 18lbs. at £6. 15*s.* 9*d.* per cwt.
25. 6231 cwt. 2 qrs. 11 lbs. 15 oz. at £3. 14*s.* 8*d.* per cwt.
26. 191 ac. 3 ro. 37 po. at £42. 3*s.* 4*d.* per acre.
27. 18 gals. 3 qts. $1\frac{1}{2}$ pts. at 17*s.* $10\frac{1}{2}$*d.* per gallon.
28. 8 kan. 4 mds. 32 palm. at *Rs.*3. 7*a.* 5*p.* per md.
29. 45 kan. 14 mds. 28 sr. at *Rs.*33. 13*a.* 7*p.* per kandi.
30. 5 ac. 2 ro. 7 po. 88 sq. yds. at £161. 6*s.* 8*d.* per acre.
31. 7 mo. 2 wks. 5 days at *Rs.*24. 2*a.* 8*p.* per month.
32. 9 mo. 1 wk. 6 days at *Rs.*11. 6*a.* per week.
33. 48 sq. yds. 8 ft. 114 in. at 13*s.* $7\frac{3}{4}$*d.* per sq. yd.
34. 28 yds. 2 qrs. $1\frac{3}{4}$ nl. at £1. 11*s.* $1\frac{1}{2}$*d.* per yard.
35. 7 mds. 7 vis. 39 palm. at *Rs.*2. 15*a.* 6*p.* per md.

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398. The method of Practice may conveniently be applied to such examples as the following :—

Ex. 1. Find the dividend on *Rs.*57201. 12*a.* at 5*a.* $4\frac{1}{2}$*p.* in the Rupee.

	Rs.	*a.*	*p.*	
4*a.* = $\frac{1}{4}$ of *Re.*1.	57201	12	0	= amount of debts in full.
1*a.* = $\frac{1}{4}$ of 4*a.*	14300	7	0	= amt. at 4*a.* in the *Re.*
3*p.* = $\frac{1}{4}$ of 1*a.*	3575	1	9	= amt. at 1*a.* ....
$1\frac{1}{2}$*p.* = $\frac{1}{2}$ of 3*p.*	893	12	$5\frac{1}{4}$	= amt. at 3*p.* ....
	446	14	$2\frac{5}{8}$	= amt. at $1\frac{1}{2}$*p.* ....
	*Rs.*19216	3	$4\frac{7}{8}$	= amt. at 5*a.* $4\frac{1}{2}$*p.* in the Rupee.

Ex. 2. Find the rent for 3 mo. 3 wks. 4 days from January 1, at *Rs.*106. 12*a.* per month.

The month of April for which rent is due for 3 wks. 4 days or 25 days, contains 30 days.

	Rs.	*a.*	*p.*	
15 days $=\frac{1}{2}$ of 30 days.	106	12	0	= rent of 1 month.
			3	
10 days $=\frac{1}{3}$ of 30 days.	320	4	0	= rent of 3 months.
	53	6	0	= rent of 15 days.
	35	9	4	= rent of 10 days.
	*Rs.*409	3	4	= rent of 3 mo. 25 days, or 3 mo. 3 wks. 4 days.

Ex. 3. Find the value of 35 chests of tea, each containing 1 md. 17 sr. 9 ch. at *Rs.*80. 12*a.* per maund.

	Rs.	*a.*	*p.*	
10 sr. $=\frac{1}{4}$ of 1 md.	80	12	0	= value of 1 md.
5 sr. $=\frac{1}{2}$ of 10 sr.	20	3	0	= value of 10 sr.
2 sr. 8 ch. $=\frac{1}{2}$ of 5 sr.	10	1	6	= value of 5 sr.
1 ch. $=\frac{1}{40}$ of 2 sr. 8 ch.	5	0	9	= value of 2 sr. 8 ch.
		2	$0\frac{9}{40}$	= value of 1 ch.
Rs.	116	3	$3\frac{9}{40}$	= value of 1 md. 17 sr. 9 ch. or of 1 chest.
$35=5\times7$			35	
	*Rs.*4067	2	$4\frac{7}{8}$	= value of 35 chests.

Ex. 4. Find to the nearest pie the rent of 275·365 bighas at *Rs.*3. 7*a.* 9*p.* per bigha.

	Rs.		
4*a.* $=\frac{1}{4}$ of 1 *Re.*	275·365		= rent at *Re.*1 per bigha.
	3		
	826·095		= rent at *Rs.*3............
2*a.* $=\frac{1}{2}$ of 4*a.*	68·841	25	= rent at 4*a*............
1*a.* $=\frac{1}{2}$ of 2*a.*	34·420	625	= rent at 2*a*...... ...
6*p.* $=\frac{1}{2}$ of 1*a.*	17·210	3125	= rent at 1*a*... ....
3*p.* $=\frac{1}{2}$ of 6*p.*	8·605	15625	= rent at 6*p*............
	4·302	578125	= rent at 3*p*............
	*Rs.*959·474	921875	= rent at *Rs.*3. 7*a.* 9*p.* per bigha,

and *Rs.*959·475 = *Rs.*959. 7*a.* 7*p.* the required rent.

Examples CXXI.

1. A bankrupt pays 10*a.* 6*p.* in the rupee ; find the dividend on a debt of *Rs.*3471.

2. Find the price of 5222 yds. at *Rs.*29. 13*a.* for a dozen yards.

3. How much income tax must be paid on an income of £756. 18*s.* 6*d.* at 1*s.* 2*d.* in the pound ?

4. Find the price of 256479 articles at £4. 12*s.* $6\frac{3}{4}$*d.* per 100.

5. Find the price of 265 sheep at £63. 3*s.* $1\frac{1}{2}$*d.* per score.

6. How much will the carriage of 5 packages, each containing 4 cwt. 3 qrs. 21 lbs., come to, at *Rs.*6. 4*a.* per ton?

7. What is the dividend on *Rs.*57348. 5*a.* 4*p.* at 7*a.* 6*p.* in the rupee?

8. What is the dividend on £1710. 14*s.* 6*d.* at 13*s.* $4\frac{1}{2}$*d.* per £?

9. Find the price of 111 things at £11. 11*s.* 11*d.* per every 11.

10. Find the weight of 2697 packages, each weighing 19 lbs. 10 oz. 18 dwts. 22 grs.

11 What distance will a train travel in 3 hours 39 min. 22 sec. at a speed of 49 miles 7 fur. 52 yds. per hour?

12. Find the rent for 11 mo. 2 wks. 6 days from March 1, 1889 at *Rs.*38. 4*a.* 6*p.* per month.

13. Find the produce of 14 bighas 18 k. 2 ch. at 12 mds. 8 sr. per bigha.

14. Find the rent of 375·3675 bighas at *Rs.*29. 15*a.* per bigha.

15. Find the value of 143·7526 gallons of spirit at *Rs.*11. 14*a.* per gallon.

16. A bankrupt owes *Rs.*7953·75 and pays 12*a.* 3*p.* in the rupee; what is the value of his assets?

17. A bankrupt's debts amount to *Rs.*35483. 5*a.* 4*p.*; find what his creditors will lose, if he pay 10*a.* $3\frac{1}{2}$*p.* in the rupee.

18. When exchange is at 2*s.* $1\frac{1}{2}$*d.* per rupee, what is the value of *Rs.* 4032. 8*a.* 8*p.* in English money?

19. Find the rent for 7 mo. 3 wks. 4 days from Feb. 1, at *Rs.*60 per month.

20. If 1 lb. Avoir. is 1 lb. 2 oz. 11 dwts. 16 grs. Troy, what is the weight (Troy) of 1 cwt. 2 qrs. 25 lbs. 10 oz. 6 drs.?

III. INVOICES.

399. Every tradesman sells his goods at two prices, **cash** and **credit**. When payment is made at the time of purchase, it is called *cash*, but otherwise *credit*. Both these sales are **entered** in a book called the **Day-Book**, in the order in which they occur in the course of the day.

The *entries* in the *Day-Book* are **posted** at short intervals in the **Ledger**, the index of which contains a list of customers' names in alphabetical order. For facility of reference, opposite each name is the page of the *Ledger* in which is collected all the dealings which have taken place with that particular customer.

400. When a buyer has completed his purchases he is presented with a **Bill** containing in detail a written list of the goods bought with a statement of the cost of them attached. An **Invoice** is a copy of the *Bill* which is sent home with the goods or forwarded to a customer living at a distance. Each separate entry in an *Invoice* or a *Bill* is called an **Item**.

401. An **Account** is a statement sent by the seller to the buyer at the end of the term of a credit shewing the totals and dates of each *Invoice* and the sum total of the whole. In such a case the *account* is said to be **rendered** (*i. e.*) sent to the buyer. If the details of the goods are also given, it is called a **Detailed Account** or **Bill of Parcels**.

(i) SPECIMEN OF AN INVOICE.

INVOICE, Calcutta, *4th April*, 1897.
From S. C. AUDDY, ESQ.,
58, Wellington Street.

		Rs.	*a.*	*p.*
16 copies of Hall and Steven's Euclid ...	at *Rs.*3. 1*a.* 6*p.*	49	8	0
14 copies of Todhunter's Euclid.......	at *Rs.*2. 6*a.* 6*p.*	33	11	0
25 copies of Lock's Arithmetic..........	at *Rs.*3. 1*a.* 6*p.*	77	5	6
10 copies of Dicken's Novels	at *Re.*1. 4*a.*	12	8	0
		173	0	6

(ii) SPECIMEN OF AN ACCOUNT.

K. C. SETT & CO. Calcutta, *May 4th*, 1897.
Bought of KHETTER MOHUN DEY & CO.,
45, Radha Bazar Street, Calcutta.

1897			*Rs.*	*a.*	*p.*
January	5	To goods as per invoice................	48	10	6
February	12	To goods as per invoice................	59	7	3
March	18	To.............ditto....................	85	12	0
April	4	To.............ditto......	72	6	6
			266	4	3

(iii) SPECIMEN OF A DETAILED ACCOUNT.

H. BALFOUR, ESQ. Calcutta, *July 24th*, 1897.
Bought of MOORE & CO.,
Dhurmtollah Street.

1897					*Rs.*	*a*	*p*	*Rs.*	*a.*	*p.*
April 21	40yds.	Irish linen........	at *Re.*1. 4*a.* 8*p.*		51	10	8			
	1doz.	Dusters... ... ... „	4*a.* 8*p.*		3	8	0			
	25yds.	Towelling........ „	7*a.* 4*p.*		11	7	4	66	10	0
May 4	23yds.	Flannel............ „	*Re.*1. 3*a.* 4*p.*		27	12	8			
	15yds.	Brown Holland...., „	7*a.* 8*p.*		7	3	0	34	15	8
June 20	14yds.	Calico .... „	3*a.* 8*p.*		3	3	4			
	22yds.	Brussels Carpet .... „	*Rs.*2. 4*a.* 8*p.*		50	6	8			
	2 Rugs,	*Rs.*10. 8*a.*, *Rs.*18. 8*a.*			29	0	0	82	10	0
								184	3	8

Examples CXXII.

Make out invoices for the following :—

1. 10 sr. of sugar at $3a.$ $9p.$ per sr. ; 6 sr. of tea at $15a.$ $3p.$ per sr.; 8 sr. of coffee at $14a.$ $3p.$ per sr. ; 12 sr. of wheat at $1a.$ $2p.$ per sr. ; 10 sr. of rice at $1a.$ $3p.$ per sr. ; and 9 sr. of cream at $11a.$ per sr.

2. $4\frac{1}{2}$ yds. of long cloth at $2a.$ $9p.$ per yd. : $7\frac{1}{4}$ yds. of cambric at $4a.$ $6p.$ per yd. ; 6 pairs of socks at $1a.$ $3p$ per pair ; 3 pairs of hose at $4a.$ $6p.$ per pair ; 1 doz. pairs of socks at $2a.$ $9\frac{1}{2}p.$ per pair ; and $5\frac{1}{2}$ yds. of flannel at $8a.$ $11p.$ per yd.

3. 5 lbs. of black tea at *Re.*1. $5a.$ $4p.$ per lb. ; $2\frac{1}{4}$ lbs. green tea at *Rs.*2. $4a.$ per lb. ; $15\frac{1}{2}$ lbs. of lump sugar at $3a.$ $8p.$ per lb.; 17 lbs. of moist sugar at $2a.$ $8p.$ per lb. ; $7\frac{1}{2}$ lbs. of raisins at $7a.$ $4p.$ per lb. ; and 4 lbs. of currants at $4a.$ $4p.$ per lb.

4. $15\frac{1}{2}$ yds. of flannel at $2s.$ $3d.$ per yd. ; 29 yds. of calico at $8\frac{1}{2}d.$ per yd. ; 25 yds of Irish linen at $2s.$ $4d.$ per yd. ; 17 yds. of towelling at $1s.$ $2d.$ per yd. ; $12\frac{1}{2}$ yds of brown holland at $11\frac{1}{2}d.$ per yd. ; and $3\frac{1}{2}$ doz. handkerchiefs at $9s.$ $10d.$ a doz.

5. $39\frac{1}{2}$ yds. of Brussels carpet at *Rs.*2. $10a.$ $8p.$ per yd. ; $62\frac{3}{4}$ yds. of Kidderminster carpet at *Re.*1. $12a.$ per yd. ; 27 yds. of cocoa-nut matting at $9a.$ $4p.$ per yd. ; $34\frac{1}{2}$ yds. of drugget at *Re.*1. $2a.$ per yd. ; and $43\frac{1}{2}$ yds. of India matting at $8a.$ $8p.$ per yd.

6. $17\frac{5}{8}$ yds. of calico at $6a.$ $6p.$ per yd. ; $35\frac{9}{16}$ yds. of flannel at $14a.$ $2p.$ per yd. ; $96\frac{8}{11}$ yds. of sheeting at *Re.*1. $0a.$ $4p.$ per yd. ; $104\frac{5}{8}$ yds. of holland at $8a.$ $6p.$ per yd. ; and $12\frac{3}{8}$ yds. of ribbon at $5a.$ $7p.$ per yd.

7. $17\frac{1}{2}$ mds. of coal at *Rs.*8. $14a.$ per md. ; carriage of ditto at *Re.*1. $2a.$ per md. ; 2 mds. of coke at *Rs.*14. $9a.$ $4p.$ per md. ; 62 mds. of gram at *Rs.*2. $2a.$ per md. ; 23 sr. of seed at $9a.$ per seer ; and 136 mds. of grain at *Rs.*3. $11a.$ per md.

8. 24·25 yds. of cloth at *Rs.*5. $4a.$ per yd. ; 13 yds. of flannel at $15a.$ $4p.$ per yd. ; 43·75 yds. of calico at $6a.$ per yd. ; 12·75 yds. of drugget at *Re.*1. $6a.$ per yd. ; 37 yds. of Brussels carpet at *Re.*1. $13a.$ $4p.$ per yd. ; and 25·5 yds. of Kidderminster do. at *Re.*1. $4a.$ $8p.$ per yd.

9. $3\frac{1}{2}$ pharas of lime at *Rs.*2. $3a.$ $4p.$ per phara ; 15 sr. of ghee at *Rs.*20. $8a.$ per md. ; $2\frac{3}{4}$ sr. of tea at *Re.*1. $0a.$ $8p.$ per seer ; 20 sr. of flour at *Rs.*2. $3a.$ per md. ; $3\frac{1}{2}$ yds. of flannel at *Re.*1. $2a.$ per yd. ; and 29 yds. of calico at $9a.$ $7p.$ per yd.

10. Calcutta, June 16th, 1885.—W. Godfrey, Esq. bought of Ghose and Co., 500 envelopes at $14a.$ $8p.$ per 100 ; 3 boxes of elastic bands at $11a.$ per box ; $\frac{1}{2}$ a gross of penholders at $6a.$ $4p.$ per doz. ; $2\frac{1}{4}$ reams of Foolscap at $7a.$ per quire ; 4 dozen quill pens at $3a.$ per doz. ; 13 note-books at $9a.$ each ; and 250 official envelopes at *Re.*1 per 100. Make out a copy of the bill and find its amount.

CHAPTER VIII.

Involution and Evolution.

402 **A power** of a number is the number which arises from successive multiplications by itself; the operation by which it is obtained is termed **involution**; and the **degree** or **order** of the power is denoted by the *number* of factors employed.

Thus, taking the number 2, we shall have the *powers* of it as follows :—

$2=2$, the first power of 2 ; $2\times2=4$, the second power of 2 ;

$2\times2\times2=8$, the third power of 2 ;

$2\times2\times2\times2=16$, the fourth power of 2 ;

$2\times2\times2\times2\times2=32$, the fifth power of 2 ;

$2\times2\times2\times2\times2\times2=64$, the sixth power of 2 ;

and so on, as far as we please ;
but instead of expressing these multiplications at *length*, which would soon become inconvenient, we denote the same operations by means of **indices** or small figures placed a little above the line to the right of the quantities whose powers are intended to be exhibited ; thus, what is put down above may be denoted by

$2^1=2$; $2^2=4$; $2^3=8$; $2^4=16$; $2^5=32$; $2^6=64$; &c. ;
where the **index** *sometimes called the* **exponent** *is equal to the* number of *factors* and is greater by *one* than the number of *operations.*

403. The *second* powers of the *nine* digits are expressed thus :—

$1^2=1$; $2^2=4$; $3^2=9$; $4^2=16$; $5^2=25$; $6^2=36$;

$7^2=49$; $8^2=64$; $9^2=81$;

and their *third* powers as follows :—

$1^3=1$; $2^3=8$; $3^3=27$; $4^3=64$; $5^3=125$; $6^3=216$;

$7^3=343$; $8^3=512$; $9^3=729$.

The *second* and *third* powers of numbers are styled their **squares** and **cubes** ; and the operations by which *all* powers are obtained are merely those of multiplication.

404. A power of a fraction is equal to the fraction formed by **raising** both its numerator and denominator to the power, and the power of a quantity formed by factors is found by raising each factor to the power.

Thus, $(\frac{2}{3})^2=\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$; $(\frac{2}{3})^3=\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=\frac{8}{27}$; and so on.

Also $(2\times7)^2=2^2\times7^2=4\times49=196$; $(2{\cdot}5)^3=2{\cdot}5\times2{\cdot}5\times2{\cdot}5=15\ 625$.

Note. A mixed quantity must be represented as a simple fraction or as a decimal, before the process can be applied.

405. A **root** of a number is such a number as being multiplied into itself one or more times produces it; and the operation by which this root is obtained is called **evolution.**

Thus, the second or **square root** of 16 is 4, because the *square* of 4 is 16, or $4^2=4\times4=16$. The third or **cube root** of 512 is 8, since the *cube* of 8 is 512, or $8^3=8\times8\times8=512$; and similarly of vulgar fractions and decimals.

406. This operation is expressed by the sign $\sqrt{\ }$ which is called the **radical sign,** with a small figure placed on its left to *particularize* the root intended: thus,

$$\sqrt[2]{16}=4;\quad \sqrt[3]{512}=8;\text{ and } \sqrt[5]{32}=2;$$

but the *square root* is denoted by the sign $\sqrt{\ }$ *only*, without the small figure, as being of most frequent occurrence.

These operations are also indicated by means of the primitive fractions $\frac{1}{2}$, $\frac{1}{3}$, &c., used as **indices** so that the *indices* $\frac{1}{2}$, $\frac{1}{3}$, &c., denote operations exactly the reverse of those expressed by the *indices* 2, 3, &c., respectively: thus,

$$4^2=16;\ 16^{\frac{1}{2}}=4 \text{ and } 8^3=512;\ 512^{\frac{1}{3}}=8.$$

I. EXTRACTION OF THE SQUARE ROOT.

407. A **perfect square** is a number whose square root can be expressed exactly either by an integer or by a fraction.

Thus, 16 is a *perfect square*, for its square root is 4.

408. In squaring a number we see that its square has the same units' figure as the square of its units' figure, and if a number ends with 0, its square also ends with 0. Now as the squares of the simple numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and also 10 are respectively 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100, it follows that the square of every number (integral or decimal) must end with either 1, 4, 5, 6, 9 or an even number of 0's. Hence *no number ending with* (i) *the digits 2, 3, 7, 8,* (ii) *an odd number of 0's, can be a perfect square.*

409. By the help of the Multiplication Tables we can immediately obtain the square root of a number not exceeding 400.

Thus, $9\times9=81$, and $15\times15=225$; $\therefore\ \sqrt{(81)}=9$, and $\sqrt{(225)}=15$.

410. When a number can be easily resolved into its prime factors, its square root can be determined by inspection. In a perfect square, every prime factor that occurs must occur an *even* number of times. Thus,

Ex. 1. Find the square root of 1764.

$$1764=2\times2\times3\times3\times7\times7=2^2\times3^2\times7^2=(2\times3\times7)^2.$$

$$\therefore\ \sqrt{1764}=2\times3\times7=42.\ \ \textit{Ans.}$$

Ex. 2. Obtain the square root of 705600.

$$705600 = 10 \times 10 \times 2 \times 2 \times 6 \times 6 \times 7 \times 7 = (10^2 \times 2^2 \times 6^2 \times 7^2)$$
$$= (10 \times 2 \times 6 \times 7)^2.$$

$$\therefore \sqrt{705600} = 10 \times 2 \times 6 \times 7 = \underline{840}. \quad \textit{Ans.}$$

Ex. 3. What is the *least number* which, when multiplied into 51425, will make the product a perfect square?

$$51425 = 11 \times 11 \times 425 = 11 \times 11 \times 25 \times 17 = 11^2 \times 5^2 \times 17.$$
$$\therefore \text{ the required number} = \underline{17}. \quad \textit{Ans.}$$

Examples CXXIII

1. Find the respective values of :—

(1) 31^3. (2) $39^2 \times 48^3$. (3) 925^2. (4) $(31{\cdot}5)^3$.
(5) $(806^2+31^2) \times 59$. (6) $(2\frac{1}{4})^4$. (7) $506^2+506^3-307^3$.
(8) $(15^2-1{\cdot}31^2) \div 15$. (9) $502^3+18^3-1376^2$.
(10) $(7{\cdot}03^2 \times {\cdot}19)^2 \div (3{\cdot}14 \times {\cdot}02)^3$. (11) $1{\cdot}03(4{\cdot}07+3{\cdot}16)^2$.

2. Find (*by inspection*) which of the following are square numbers ;—

(1) 27 ; 96 ; 524 ; 9450 ; 7805 ; 9604 ; 12321 ; 494208.
(2) 4000 ; 75720 ; 388129 ; 582168 ; 12343225 ; 38812900.

3. Find the square roots of (*using factors*) :—

(1) 49 ; 196 ; 289 ; 361 ; 324 ; 256 ; 121 ; 400 ; 144.
(2) 625 ; 529 ; 900 ; 1296 ; 17424 ; 63504 ; 99225.
(3) 680625 ; 48024900 ; 12446784 ; 2480625 ; 57153600.

4. Find the *least numbers* which, when multiplied into the following numbers, will make the products perfect squares :—

175 ; 693 ; 1456 ; 3465 ; 3456 ; 4536 ; 28413 ; 750750.

5. What must be the least number of soldiers in a regiment, to admit of its being drawn up 2, 3, 4, 5, or 6 deep, and also of its being formed into a solid square?

---

411. *From the number of figures in any proposed quantity, to find the number of figures in its square root.*

Since, the square root of 1 is 1 ;
the square root of 100 is 10 ;
the square root of 10000 is 100 ;
the square root of 1000000 is 1000 ; &c. ;
we see immediately that the square root of a number of fewer than 3 figures must consist of only 1 figure ; that of a number of more than 2 figures and fewer than 5, of 2 figures ; that of a number of

more than 4 figures and fewer than 7, of 3 figures, and so on; whence it follows, that if a dot or full point be placed over every alternate figure, beginning at the *units'* place, the number of such points will be the same as the number of figures in the square root. This is called the **Rule of pointing.**

Thus, the square root of $\dot{1}9\dot{8}$ consists of 2 figures in its integral part; the square root of $3\dot{1}4\dot{2}5\dot{6}$ consists of 3 figures in its integral part; and so on.

412. The extraction of the square root of a number depends on the principle illustrated by the following examples :—

$$\text{Since } 28^2=(20+8)^2=20^2+2\times 20\times 8+8^2 ;$$
$$\therefore\ 28^2-20^2=2\times 20\times 8+8^2 ;$$
$$\therefore\ 28^2-20^2\div(2\times 20)=8+\text{a proper fraction.}$$

413. *To extract the square root of a whole number.*

RULE. Point the alternate figures of the number proposed, beginning at the place of units, so as to form as many periods of two figures each as possible, and remember that each period consists of the figure over which the dot is placed and the figure to its left. (The first period may consist of one figure only).

Find the greatest square number contained in the first period on the left hand, put down its root on the right as in division, and subtract it from that period. To the remainder bring down the next period for a dividend, double the root just found for a divisor (called the *trial divisor*), and find how often it is contained in this dividend exclusive of the figure on its right hand; annex this quotient to the figures in both the quotient and divisor. Multiply the divisor thus *completed* by the last figure of the quotient, and if the product be not greater than the dividend, subtract it from the dividend, but if the product be greater, use a *lower* number for the root figure until it becomes less; subtract the product as before. To this remainder bring down the period which comes next in order; take twice the number in the root, and see how often it is contained in this dividend with its last figure omitted; and proceed precisely as before. Repeat the process till every period in succession is disposed of, and the root will thus be obtained.

Note. If at any step the quotient figure is o, set down o in the root, annex it to the trial divisor, bring down the next period and proceed as before.

Ex. 1. Find the square root of 8649.

```
     8649( 93.
     81
183| 549
    | 549
```

Place dots over 9 and 6, so that the number is divided into two periods, 86 and 49.

The number whose square is immediately below 86 is 9 (for $9^2=81$ which is next below 86). Hence **9** is put in the root and 81 subtracted from 86.

To the remainder 5 is brought down the next period 49; thus the new dividend is 549. Now $2 \times 9 = 18$, is the *trial divisor*, which goes into 54 (549 with 9 omitted) 3 times. Hence **3** is put after 9 in the root and also annexed to 18. Multiply 183 by 3 and the product is 549, which subtracted from the dividend leaves nothing. Therefore 93 is the root obtained.

Ex. 2. Extract the square roots of 804609; 12809241; and 21224449.

```
(1)   804609 ( 897.     (2)   12809241 ( 3579.     (3)   21224449 ( 4607.
      64                      9                          16
  169|1646                 65|380                     86|522
     |1521                   |325                       |516
 1787|12509               707|5592                  9207|64449
     |12509                  |4949                      |64449
                         7149|64341
                             |64341
```

414. When an integer (which is a perfect square) ends with an even number of ciphers, it would be sufficient to extract the square root of the significant figures and then to annex to the root *one* cipher for every *two* ciphers in the proposed number.

Ex. Extract the square root of 841000000.

```
841000000 ( 29000.
4
49|441
  |441
```

Here are 6 ciphers in the given number: therefore we add 3 ciphers to 29, the square root of 841.

415. Again, since the square root of ·01 is ·1;
the square root of ·0001 is ·01;
the square root of ·000001 is ·001; &c.;

we infer that the quantity proposed must first be made to have an *even number of decimal places, and then the pointing must* proceed from the place of *units* towards the right hand over every alternate figure as before; and the number of such points will be the same as the number of decimal places in the square root.

416. If there be no whole number or integral part in the proposed number, we must, in pointing, begin with the *second* figure from that which would be the *units*' place, if there were a whole number, and place dots successively over every alternate figure to the right. If there be a whole number as well as a decimal fraction, it would be the safest method to begin at the *units*' place and point over every alternate figure to the *right and left* of it. The number of dots over the whole numbers and decimals will shew the number of figures in the integral and decimal parts of the root respectively.

Ex. Extract the square roots of 93·7024, ·02819041 and ·00822649.

(1)	(2)	(3)
93·7024(9·68.	·02819041(·1679.	·00822649(·0907.
81	1	81
186) 1270	26) 181	1807) 12649
1116	156	12649
1928) 15424	327) 2590	
15424	2289	
	3349) 30141	
	30141	

Examples CXXIV.

1. Find the square roots of :—

(1) 676 ; 1444 ; 16129 ; 21025 ; 288369 ; 998001 ; 71289.
(2) 2025 ; 692224 ; 54756 ; 822649 ; 97574884 ; 10004569.
(3) 33016516 ; 45859984 ; 5774409 ; 62805625 ; 4020025.
(4) 6512490000 ; 5777216064 ; 95481000000 ; 3915380329.
(5) 8260628544 ; 93870306991561 ; 78702684186368o889.

2. Extract the square roots of :—

(1) 22·09 ; 33·64 ; 1082·41 ; 22·8484 ; 187·4161 ; ·128881.
(2) ·0064 ; ·005329 ; ·00053361 ; ·00038025 ; 3659·0401.
(3) 1164·1744 ; 136966·6081 ; 240168·6049 ; 236·144689.
(4) 41605·800625 ; ·00501361708761 ; ·0000004911 2064.

3. A certain number of boys spent *Rs.*90. 4*a.*, each spending as many four-anna pieces as there were boys ; what was the number of boys ?

4. A square pavement contains 20736 square stones, all of the same size ; what number composes one of its sides ?

5. A society collected among themselves for certain purposes a fund of *Rs.*459. 6*a.* ; each person paid twice as many pies as there were members in the whole society. Find the number of members.

6. A general, trying to mass his army of 15410 men into a square found he had 34 men over ; required the number of men in the front.

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417. If the number is not a perfect square, we can find an approximation to its square root to any required number of decimal places by affixing ciphers to the right hand of the proposed number and bringing down periods of 2 ciphers each.

Ex. Find the square roots of 11 and ·4, each to 4 places of decimals.

(1)

```
        11·00000000 ( 3·3166...
         9
  63|200
    |189
  661|1100
     | 661
  6626|43900
      |39756
  66326|414400
       |397956
         16444
```

(2)

```
     ·40000000 ( ·6324...
      36
  123 400
      369
  1262|3100
      |2524
  12644|57600
       |50576
         7024
```

418. When the proposed number is a recurring decimal, extend the recurring part by a repetition of its period and then proceed as in decimals.

Thus, to extract the square root of $4\cdot3\dot{1}5\dot{7}$ to four places of decimals, first extend the recurring part 157 and put 4·3157157157... for $4\cdot3\dot{1}5\dot{7}$, and then proceed as usual.

419. When the number of figures to be found in the decimal part of the root is *large*, we may obtain in the usual way one more than half the required number of figures in the root, and then the remaining figures by dividing the last remainder by the last divisor, as in Art. 389.

Ex. Extract the square root of 10 to 8 places of decimals.

```
        10· (3·16227766...
         9
  61|100                      6,3,2,4,4)49116(7766
    | 61                                44271
  626|3900                               4845
     |3756                               4427
  6322|14400                              418
      |12644                              379
  63242|175600                             39
       |126484                             38
         49116
```

The first 5 figures are obtained in the usual way, and the last 4 by Contracted Division.

Examples CXXV.

1. Find the square roots of (each to 4 places of decimals) :—

(1) 2 ; 3 ; 5 ; 6 ; 7 ; 8 ; 12 ; 13 ; 18 ; 20 ; 32 ; 38.

(2) 44 ; 51 ; 72 ; 80 ; 95 ; 638 ; 796 ; 801 ; 1000.

(3) 5713; 363 ; 35120 ; 8837 ; 822646 ; 72471438 ; 7432.

2. Extract the square roots of (each to 4 places of decimals) :—

(1) $\cdot1$; $\cdot2$; $\cdot3$; $\cdot4$; $\cdot5$; $\cdot6$; $\cdot7$; $\cdot8$; $\cdot9$; $1\cdot2$; $1\cdot6$.

(2) $\cdot05$; $5\cdot1$; $4\cdot9$; $\cdot16$; $\cdot016$; $\cdot01$; $\cdot51$; $\cdot051$; $4\cdot03$.

(3) $\cdot002$; $\cdot225$; $\cdot021$; $\cdot\dot{3}$; $4\cdot\dot{5}$; $34\cdot85$; $321\cdot73025$; $18\cdot\dot{7}$.

(4) $3\cdot14159$; $175\cdot250564$; $12\cdot56636$; $29\cdot41275$; $7894\cdot61\dot{9}\dot{3}$.

(5) $2\cdot3\dot{6}\dot{1}$; $5\cdot0\dot{1}3\dot{2}$; $4\cdot02\dot{9}8\dot{1}$; $49\cdot005\dot{2}\dot{1}$; $\cdot\dot{0}\dot{7}$; $57\cdot\dot{5}\dot{7}$; $\cdot019\dot{8}$.

3. Find to 10 decimal places the square roots of :—

(1) $\cdot001728$; $9\cdot79$; $\cdot0683$; $\cdot3467$; $44\cdot284$; $1\cdot57$; $75\cdot347$.

(2) $\cdot85$; $\cdot07$; 3 ; $97\cdot9$; $\cdot0003532$; $27\cdot773$; $\cdot0365$.

420. The square root of a fraction may be obtained by finding the square roots of its numerator and denominator separately.

(1) If the denominator of the given fraction, or of the fractional part of the mixed number, be *a perfect square*, we apply the Rule directly, whether the numerator be a perfect square or not.

$$\text{Thus, } \sqrt{\frac{144}{169}}=\frac{\sqrt{144}}{\sqrt{169}}=\frac{12}{13}\,;\quad \sqrt{8\tfrac{17}{64}}=\sqrt{\frac{529}{64}}=\frac{\sqrt{529}}{\sqrt{64}}=\frac{23}{8}=2\tfrac{7}{8}.$$

$$\sqrt{\frac{29}{64}}=\frac{\sqrt{29}}{\sqrt{64}}=\frac{5\cdot385164\ldots}{8}=\cdot673145\ldots$$

$$\sqrt{1278\tfrac{7}{25}}=\sqrt{\frac{31957}{25}}=\frac{\sqrt{31957}}{\sqrt{25}}=\frac{178\cdot7652\ldots}{5}=35\cdot7530\ldots$$

(2) But if the denominator of the given fraction or of the fractional part of the mixed number be *not a perfect square*, we reduce *the fraction or the mixed number either* (i) *to an equivalent* fraction whose denominator is a perfect square and extract the square root of both numerator and denominator, or (ii) to a decimal, and proceed in the usual way.

$$\text{Thus, } \sqrt{\tfrac{5}{7}}=\sqrt{\frac{5\times7}{7\times7}}=\sqrt{\frac{35}{49}}=\frac{\sqrt{35}}{\sqrt{49}}=\frac{5\cdot91607\ldots}{7}=\cdot84515\ldots$$

$$\text{or}=\sqrt{(\cdot\dot{7}1428\dot{5})}=\cdot84515\ldots$$

$$\sqrt{25\tfrac{8}{11}}=\sqrt{\frac{283}{11}}=\sqrt{\frac{283\times11}{11\times11}}=\sqrt{\frac{3113}{121}}=\frac{\sqrt{3113}}{11}=\frac{55\cdot794265\ldots}{11}$$

$$\text{or}=\sqrt{(25\cdot\dot{7}\dot{2})}=5\cdot072205\ldots \qquad =5\cdot072205\ldots$$

421. If a recurring decimal is a *perfect square*, it would be convenient to reduce it to a vulgar fraction and proceed as in Art. 420 (1), above.

$$\text{Thus, } \sqrt{(1\cdot\dot{7})}=\sqrt{\frac{16}{9}}=\frac{\sqrt{16}}{\sqrt{9}}=\tfrac{4}{3}=1\cdot\dot{3}.$$

Examples CXXVI.

1. Find the square roots of :—

(1) $\frac{4}{25}$; $\frac{169}{256}$; $\frac{361}{1369}$; $\frac{841}{1024}$; $\frac{2304}{3481}$; $\frac{7056}{9216}$; $\frac{38642}{31752}$.

(2) $4\frac{25}{36}$; $10\frac{39}{49}$; $345\frac{24}{25}$; $32\frac{32}{49}$; $32\frac{1}{9}$; $41\frac{1288}{1575}$; $564\frac{220}{289}$.

(3) $3083\frac{214}{225}$; $72002\frac{7}{9}$; $30789\frac{268}{289}$; $15061\frac{119}{121}$; $154751\frac{77}{169}$.

2. Find the square roots of (each to 4 places of decimals where the root does not come out exactly) :—

(1) $\frac{5}{8}$; $9\frac{1}{2}$; $6\frac{2}{5}$; $\frac{1 \cdot 28}{12 \cdot 5}$; $\frac{4 \cdot 41}{\cdot 64}$; $76\frac{14}{17}$; $\frac{5 \cdot 04}{\cdot 012}$; $\frac{\cdot 00841}{1000}$.

(2) $\frac{3}{5}$; $21\frac{8}{11}$; $4\frac{5}{21}$; $\frac{12}{13}$; $\frac{16}{17}$; $21\frac{35}{117}$; $\frac{17}{123}$; $27\frac{27}{128}$.

(3) $\frac{18}{49}$; $\frac{129}{400}$; $\frac{7}{72}$; $\frac{14}{243}$; $287\frac{5}{8}$; $\frac{1}{303}$; $367\frac{2}{7}$; $756 \cdot 28\frac{18923}{20736}$.

3. Find the square roots of :—

(1) $\cdot\dot{1}$; $\cdot 02\dot{7}$; $18\cdot\dot{7}$; $3\cdot 36\dot{1}$; $28\cdot\dot{4}$; $\cdot 0002\dot{7}$; $4738\cdot 02\dot{7}$.

(2) $\cdot 0013\dot{4}$; $\cdot 071\dot{1}$; $53\cdot\dot{7}$; $\cdot 00\dot{4}$; $5\cdot\dot{4}$; $\cdot 01\dot{7}$; $\cdot\dot{0}4938271\dot{6}$.

II. EXTRACTION OF THE CUBE ROOT.

422. A **perfect cube** is a number whose cube root can be expressed exactly either by an integer or by a fraction.

The cubes of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9,
are respectively 1, 8, 27, 64, 125, 216, 343, 512, 729,
and it is important that these last numbers and the corresponding roots should be committed to memory.

423. *Given the number of figures in a number, to find the number of figures in its cube root.*

Since, the cube root of 1 is 1 ;
the cube root of 1000 is 10 ;
the cube root of 1000000 is 100 ; &c. ;

it follows that the cube root of any number between 1 and 1000 must lie between 1 and 10, *i.e.* will have 1 figure in the integral part ; of any number between 1000 and 1000000 must lie between 10 and 100, *i.e.* will have 2 figures in the integral part ; of any number between 1000000 and 1000000000 must lie between 100 and 1000 *i. e.* will have 3 figures in the integral part, and so on. Hence, if a point be placed over the *units'* figure of the number, and thence over every third figure to the left of that place, the number of points will shew the number of figures in the integral part of the root. This rule may manifestly be extended to Decimals.

424. *To extract the cube root of a given number.*

RULE. Place a point over the *units'* figure of the given number and thence over every *third* figure to its left, and also to its right if the number be a decimal, adding ciphers, if necessary, to get periods of three ; and remember that each period consists of the figure over

which the dot is placed and the two figures to its left, if there are so many (for the first period may contain 1, 2, 3 figures).

Find the number whose cube is either equal to, or next less than the *first* period on the left hand and place it as the *first* figure of the root. Subtract its cube from the first period, and to the remainder bring down the *next* period.

Multiply the square of the root already obtained by 300 for a *trial* or *partial* divisor, and then find how often this divisor is contained in the dividend; this quotient gives the *next* figure of the root. Then, multiply this quotient figure by the product of the previous figure of the root by 30, and place the result below the partial divisor. Below these place the square of this last quotient figure and add the three together for a *Complete* divisor. Multiply this complete divisor by the last figure of the root and subtract. To the remainder bring down the next period to form the next dividend.

Multiply the square of the root already obtained by 300, and find how often this trial divisor is contained in the dividend. Put this quotient as the third figure of the root. Then, multiply the figures of the root already obtained by 30 and the product by the last quotient figure, and place the product below the partial divisor. Then place the square of the last quotient figure, and add the three together, for a complete divisor. Multiply this divisor by the last figure and subtract, and bring down the next period to form the next dividend. Proceed in this way till all the periods have been brought down.

Note. If at any step, the dividend is less than the divisor, put a cipher to the root, two ciphers to the trial divisor, and bring down the next period.

Ex. 1. Find the cube root of 21952.

	21952(28
	8
$2^2 \times 300 = 1200$	13952
$2 \times 30 \times 8 = 480$	
$8^2 = 64$	
1744	13952

Here, first divide into periods beginning with the first period on the left contains only 2 figures. The trial divisor 1200 goes into the dividend 13952, 8 times.

Ex. 2. Extract the cube root of 12812·904.

	12812·904(23·4
	8
$2^2 \times 300 = 1200$	4812
$2 \times 30 \times 3 = 180$	
$3^2 = 9$	
1389	4167
$23^2 \times 300 = 158700$	645904
$23 \times 30 \times 4 = 2760$	
$4^2 = 16$	
161476	645904

First divide into periods of three beginning with 2, both left and right. The first period is 12 and the greatest cube root in 12 is 2.

The trial divisor 1200 goes into the dividend 4812, 3 times.

The trial divisor 158700 goes into 645904, 4 times.

We may shorten the process a little as below :—

		$\dot{1}2\dot{8}1\dot{2}\cdot90\dot{4}(23\cdot4$
63	12	8
6	189	4812
694	1389	
	9	4167
	1587	645904
	2776	
	161476	645904

In column II. instead of writing ciphers, put 9 the units' figure of 189 two places further to the right; in the same manner, write 2776 in the second step.

To find the trial divisor in the second step, take the sum of $189+1389+3^2$, which is equal to the product of 69×23.

425. If the number is not a perfect cube, we may obtain its cube root to any required number of decimal places by annexing ciphers and bringing down periods of three ciphers each.

Ex. Extract the cube root of ·3 to 3 decimal places.

		$\cdot30\dot{0}00\dot{0}00\dot{0}(\cdot669\ldots$
186	108	216
12	1116	84000
1989	11916	
	36	71496
	13068	12504000
	17901	
	1324701	11922309
		581691

Since the root is to be extracted to 3 places of decimals, there must be 3 periods of 3 figures in the decimal part; therefore we must affix 8 ciphers to 3.

426. When one more than a half of the figures required in the root have been obtained by the ordinary method, the rest can be found by Contracted Division, as in Art. 389.

427. In extracting the cube roots of vulgar fractions, if the denominator of the fraction be *a perfect cube*, find the cube roots of both the numerator and the denominator separately; but if the denominator of the fraction be *not a perfect cube*, either reduce the fraction to an equivalent fraction whose denominator is a perfect cube and then extract the cube root of numerator and denominator, or reduce the fraction to a decimal and proceed in the ordinary way.

$$\text{Thus, } \sqrt[3]{\frac{27}{64}}=\frac{\sqrt[3]{27}}{\sqrt[3]{64}}=\frac{3}{4}\,;\ \sqrt[3]{\frac{29}{64}}=\frac{\sqrt[3]{29}}{\sqrt[3]{64}}=\frac{3\cdot072317\ldots}{4}=\cdot768079\ldots$$

$$\sqrt[3]{8\frac{5}{7}}=\sqrt[3]{\frac{61}{7}}=\sqrt[3]{\frac{2989}{343}}=\frac{\sqrt[3]{2989}}{\sqrt[3]{343}}=\frac{14\cdot4048\ldots}{7}=2\cdot0578\ldots$$

$$\text{or}=\sqrt[3]{(8\cdot\dot{7}1428\dot{5})}=2\cdot0578\ldots$$

Examples CXXVII.

1. Find the cube roots of :—

(1) 1331; 15625; 46656; 2197; 185193; 117649.

(2) 704969 ; 912673 ; 33076161 ; 15069223 ; 105823817.

(3) 873722816 ; 197767717056 ; 702121283072.

(4) 17·576 ; 132·651 ; 493·039 ; 64481·201 ; 18·609625.

(5) ·007645373 ; ·876467493 ; ·001030301 ; ·000026730899.

(6) $\frac{64}{343}$; $\frac{729}{140608}$; $49\frac{8}{27}$; $7558\frac{197}{512}$; $465\frac{31}{64}$; $57\frac{132}{343}$.

(7) 1034 ; 5·912 ; 5 ; ·078759 ; 3·467 (each to 4 decimal places).

(8) $\frac{3}{7}$; $\frac{8}{13}$; $\frac{2}{7}$; $7\frac{7}{8}$. $18\frac{7}{12}$; $18\frac{499}{729}$ (each to 4 decimal places).

(9) 002 ; ·003 ; ·013 ; ·024 ; 2·187 (each to 8 places of decimals).

2. Find the cube roots of :—

(1) $\frac{5·12}{33·75}$; $\frac{5030·912}{65536}$; $\frac{5·12}{·03375}$; $\frac{1257·728}{16384}$.

(2) 3845·296 ; ·037 ; 1587·962 ; ·8 ; ·27 ; ·325142 ; 81·812703 (the last four to 4 decimal places).

3. A cubical block of stone contains 50653 solid feet ; find the length of its side.

4. Extract the cube root of 233·744896, and derive the cube root of this number multiplied by ·008.

III. EXTRACTION OF SOME OTHER ROOTS.

428. The directions already employed may by a little management be rendered available for the discovery of some other roots, as will be evinced in the following notes.

(1) The **Fourth** root of a number is found by extracting its square root, and then the square root of its square root.

(2) The **Sixth** root of a number is found by extracting its cube root, and then the square root of its cube root ; or by extracting its square root, and then the cube root of its square root.

(3) The **Eighth** root of a number is found by extracting its square root, then the square root of its square root, and lastly the square root of that square root.

(4) The **Ninth** root of a number is found by extracting its cube root, and then the cube root of its cube root.

Ex. Find the *fourth* root of 1679616 and the *sixth* root of 308·915776.

(1) Here the *square* root is found to be 1296 ; and the *square* root of 1296 is 36. Therefore the *fourth* root of 1679616 is 36.

(2) Here the *square* root is found to be 17·576 ; and the *cube* root of 17·576 is 2·6. Therefore the *sixth* root required is 2·6.

Examples CXXVIII.

1. Find the fourth roots of 104976 ; 1500625 ; 4323738·0096.

2. Find the sixth roots of 2985984 ; 24·137569 ; 17596·287801.

3. Find the eighth roots of 214358881 ; 21035·8 ; ·003532 ; $57\frac{57}{64}$ last three to 5 decimal places).

4. Find the ninth roots of 262144 ; ·134217728 ; 5159780352.

5. Find the fourth root of $21\frac{204}{2401}$; and the sixth roots of 66121 and 260184053769595201.

Miscellaneous Examples V.

1. What are the prime factors in 45090045, and what is the allest whole number by which it must be multiplied in order to ıke it a perfect square ?

2. What is the difference between the values of

$$\frac{2\frac{4}{9} \text{ of } 5\frac{1}{8}}{27\frac{1}{2}} \text{ of } Rs.11.\ 4a. \text{ and } \frac{5\frac{1}{12}}{2\frac{1}{2}+3\frac{3}{4}} \text{ of } Rs.6.\ 4a. ?$$

3. A chain, 11 yds. long, is divided into 50 equal parts, called nks ; find how many square links there are in an acre.

4. Bring $\left\{\left(\frac{5\frac{1}{4}-\frac{1}{3} \text{ of } 2\frac{5}{8}}{\frac{3}{8}\times 4\frac{1}{6}+\frac{1}{12}}+\frac{2\frac{3}{7}}{4\frac{3}{7}}\right)\div 21\frac{28}{29}\times 3\frac{19}{208}\right\}$ cwt. to the fraction of $4\frac{1}{7}$ ton.

5. A merchant bought 264 gallons of spirit at *Rs.*12. 8*a.* $4\frac{1}{2}$*p.* per gal. ; 378 gallons at *Rs.*9. 10*a.* 7*p.* per gal. ; and 420 gallons at *Rs.*12. 15*a.* $6\frac{1}{4}$*p.* per gal. If he sell the whole quantity at *Rs.*12. 4*a.* per gal., what profit will he make by the transaction ?

6. Two numbers have for their G. C. M. 179 and for their L. C. M. 56385. What must the greater number be, if the less

$$=105 \text{ times } \frac{2\frac{2}{7}}{4\frac{3}{7}} \text{ of } \frac{363{\cdot}37}{8{\cdot}4} ?$$

7. Which is the greater

$$\frac{3}{5}\text{ of }\frac{13}{16}-\frac{1\frac{4}{5}}{6\frac{2}{3}}\text{ of }\frac{19}{20}+\frac{3}{7}\text{ of }\frac{6\frac{5}{12}}{3\frac{3}{7}}\text{ or }\frac{5}{3}\text{ of }\frac{13}{16}+\frac{6\frac{2}{3}}{1\frac{2}{3}}\text{ of }\frac{19}{20}-\frac{7}{3}\text{ of }\frac{6\frac{5}{12}}{3\frac{2}{3}} ?$$

and express the difference as a decimal.

8. Express *Rs.*6. 5*a.* $10\frac{2}{3}$*p.* as the fraction of *Rs.*9. 8*a.* 10*p.*

9. Find by Practice the value of 29764 articles at *Re.*1. 11*a* $9\frac{3}{4}$*p.* each.

10. Simplify $\frac{2}{5}(6\frac{2}{3}+2\frac{1}{2})£+\frac{2\frac{1}{4}-\frac{2}{3}\text{ of }1\frac{5}{8}}{\frac{1}{5}\text{ of }3\frac{1}{3}+\frac{13}{36}}\times{\cdot}95\text{ of }5s.+\frac{16{\cdot}8}{{\cdot}024}d.$

11. Simplify $(25{\cdot}4)^2+(24{\cdot}6)^2-12{\cdot}7\times 98{\cdot}4+({\cdot}6)^2$.